Researching the Socio-Political Dimensions of Mathematics Education

Issues of Power in Theory and Methodology

Edited by
Paola Valero
Robyn Zevenbergen

Mathematics Education Library

Kluwer Academic Publishers

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VOLUME 35

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Paola Valero
Aalborg University, Denmark

and

Robyn Zevenbergen
Griffith University, Australia

KLUWER ACADEMIC PUBLISHERS
NEW YORK, BOSTON, DORDRECHT, LONDON, MOSCOW

eBook ISBN: 1-4020-7914-1
Print ISBN: 1-4020-7906-0

Created in the United States of America

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CONTRIBUTORS

Bill Atweh is a Senior Lecturer in mathematics education at the Queensland University of Technology, Australia. His main research interests are in the areas of social context of mathematics education, including gender and class; the use of action research; globalisation in mathematics education; and, more generally, social justice issues in education. He has served six years as Vice President (Publications) for the Mathematics Education Research Group of Australasia. He is the co-editor of two volumes of research reviews in Australasia as well as the *Action research in practice: Partnerships for social justice in education* (1998, Routledge), *Research and supervison in mathematics and science education* (1998, Lawrence Erlbaum), and *Sociocultural research in mathematics education: An international perspective* (2001, Lawrence Erlbaum). Bill's address is: Education —QUT, Locked Bag, Red Hill, QLD, Australia. E-mail: b.atweh@qut.edu.au

Hannah Bartholomew is a Research Associate at King's College London. Her research interests are in mathematics education, particularly issues relating to equity. Recent projects with which she has been involved include a study into the effects of ability grouping on students' experiences and attainment in secondary mathematics, and a study into the experiences of undergraduate mathematics students. Hanna's address: Department of Education and Professional Studies; King's College London, Franklin-Wilkins Building (Waterloo Bridge Wing), Waterloo Road, London SE1 9NN, UK. E-mail: Hannah.bartholomew@kcl.ac.uk

Alan Bishop has been Professor of Education at Monash University since 1992 after spending most of his life in the UK. In 1969, after completing his PhD, he was appointed Lecturer in the Department of Education at Cambridge University, UK where he worked for 23 years. He edited (1978-90) the international research journal, *Educational Studies in Mathematics,* published by Kluwer, and has been an Advisory Editor since 1990. He is Managing Editor of the research book series *Mathematics Education Library,* also published by Kluwer (1980-present). He is also the Chief Editor of the *International handbook of mathematics education* (1996) and of the *Second handbook on mathematics education* (forthcoming) published by Kluwer. He has been involved with research in mathematics education all his working life, and his Cambridge experience led him to focus attention during the last 20 years on the cultural, social and political issues involved in mathematics education. He is the author or editor of several influential books, reports, articles and chapters on

mathematics education, and he has frequently assisted with research projects in other countries concerning at-risk mathematics learners. Alan's address is: Faculty of Education, Monash University, P.O. Box 6 Melbourne VIC 3800, Australia. E-mail: Alan.Bishop@education.monash.edu.au

Marcelo C. Borba is the Chair of the Graduate Program of Mathematics Education at the State University of São Paulo, Rio Claro, SP, Brazil. He was awarded his Ph.D. at Cornell University, USA in 1993. He has published in different languages —English, Spanish and Portuguese— papers, chapters and books about ethnomathematics, computer technology and mathematics education and research methodology. He is a member of different boards of journals and research centres worldwide. Marcelo's address is: Graduate Program in Mathematics Education, State University of Sao Paulo at Rio Claro, Av. 24A, 1515; 13506-700 Rio Claro-SP, Brazil. E-mail: mborba@rc.unesp.br

Anna Chronaki is Assistant Professor at the University of Thessaly, in Volos, Greece, where she teaches didactics of mathematics, research methods and new technologies. Having earned grants for postgraduate studies, she completed her M.Ed. and Ph.D. at the University of Bath in the field of mathematics education. She worked for a number of years at the School of Education of the Open University and University of Bath in the UK as a research fellow and part-time lecturer. She has also worked as a researcher at the Computer Technology Institute and as a lecturer at the School of Primary Education at the University of Athens, Greece. Her current research theorises the role of resources —including technology and texts— and social interactions in mathematics classrooms as well as the broader social issues influencing mathematics teaching and learning. She has published several papers in refereed journals, congress proceedings and in edited books. Anna's address is: Faculty of Human Sciences, Department of Early Childhood Education, University of Thessaly, Argonauton & Fillelinon, Volos 38221, Greece. E-mail: chronaki@ece.uth.gr

Tony Cotton is a Principal Lecturer in primary mathematics education at Nottingham Trent University in England. His research interests are around issues of social justice within and through mathematics education. He is also interested in developing research methodology that takes note of the voices and stories of those we research with. He is the programme leader for undergraduate primary teacher education programmes at Nottingham Trent University and writes pupil and teachers materials as well as within the research literature. He has also led several European education projects developing citizenship and multicultural curriculum materials. Tony's address is: Faculty of Education, Nottingham Trent University, Ada Byron King Building, Clifton lane, Nottingham NG11 8NS, UK. E-mail: tony.cotton@ntu.ac.uk

Paul Ernest studied mathematics, logic and philosophy at Sussex and London universities and became a mathematics teacher in the 1970s. He is currently Professor of Philosophy of Mathematics Education at the University of Exeter,

where he runs the masters and doctoral degree programmes in mathematics education He is the editor of the *Philosophy of Mathematics Education Journal* (web location: http://www.ex.ac.uk/~PErnest/ contributions always welcome!). His main research interests concern fundamental questions about the nature of mathematics and how it relates to teaching, learning and society. His most recent academic book is *Social constructivism as a philosophy of mathematics* (SUNY Press, 1998). Paul's address is: University of Exeter, School of Education, Heavitree Road, Exeter EX1 2LU, UK. E-mail: p.ernest@ex.ac.uk

Núria Gorgorió is a Senior Lecturer in the Faculty of Education at the Universitat Autònoma de Barcelona, Spain, which she joined after working as a mathematics teacher in a secondary school in a deprived area of Barcelona for five years. Her areas of teaching include mathematics and mathematics education to pre-service and in-service teachers, and research methods and other courses to masters and doctoral students. At present, her research work concentrates on mathematics education in multicultural classrooms, being her interests those related to linguistic, cultural, social and political dimensions as intertwined. Núria's address is: Facultat de Ciències de l'Educació G-5 142, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain. E-mail: nuria.gorgorio@uab.es

Tansy Hardy is a Senior Lecturer at the Maths Education Centre, Sheffield Hallam University, UK. She has taught for the last 10 years in Higher Education on teacher training courses for both teachers of primary school and secondary school children. She is also responsible for professional development work with teachers. Prior to this, she was involved in inservice training and advisory work, and as head of a mathematics department in a secondary school. Throughout her teaching career she has been actively involved in research. One of her current interests is in the process of teacher development —exploring how teachers translate guidance on curriculum and pedagogy into patterns of practice in their own professional work, and how this relates to the strategies we use in initial and inservice teacher training. Tansy's address is: Department of Education, Sheffield Hallam University, 25 Broomgrove Road, Sheffield, UK. E-mail: T.Hardy@shu.ac.uk

Gelsa Knijnik is Professor of the Graduate Program on Education of the Universidade do Vale do Rio dos Sinos, Brazil. Her studies in the field of mathematics education are focused in its cultural, social and political dimensions and are connected to the ethnomathematics taught. Since 1991 she has been researching with the Brazilian Landless Movement. She is well known internationally for her work in this area and has published a number of important papers and books in relation to this work. Gelsa's address is: Rua Andre Puente 414/301, 90035-150 Porto Alegre, Brazil. E-mail: gelsak@portoweb.com.br

Stephen Lerman was a secondary school teacher of mathematics for many years. Whilst working on his Ph.D. he began lecturing in mathematics education at the Institute of Education. He is now Professor of Mathematics Education at South Bank University in London. He is a former President of the International Group for the

Psychology of Mathematics Education (PME). His research interests include classroom research, socio-cultural theories, equity issues, and teacher education. Stephen's address is: Division of Education South Bank University, 103 Borough Road, London SE1 0AA, UK. E-mail: lermans@sbu.ac.uk

Tamsin Meaney is a Lecturer in Linguistics at the University of Otago, which she joined after working for many years as a mathematics educator in diverse situations such as remote communities in the Northern Territory of Australia and as a secondary mathematics teacher in New Zealand. Her research interests include how language influences the learning of mathematics, especially when the language of instruction is not the students' first language. She has previously written under the name Tamsin Roberts. Tamsin's address is: Linguistics Programme, University of Otago, PO Box 56, Dunedin, New Zealand. E-mail: tamsin.meaney@stonebow.otago.ac.nz

Núria Planas is teaching and researching at the Universitat Autonoma de Barcelona, Spain, as a Research Associate. Her main areas of interest are social theories of learning and mathematical learning as a process of social participation. As a mathematics teacher for some years in a high schools with many immigrant students, she has been intimately involved with the cultural and social complexity of certain schools, and is fully conscious of the need to develop an equitable but mathematically rich curriculum and teaching program for all the students. Núria's address is: Facultat de Ciencies de l'Educacio, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain. E-mail: Nuria.Planas@uab.es

Thomas S. Popkewitz is a Professor in the Department of Curriculum & Instruction, The University of Wisconsin-Madison, USA. His studies explore the systems of reason that order teaching, teacher education and the educational sciences. Some of his most recent books are *Struggling for the soul; The politics of education and the construction of the teacher,* comparative studies of reform in *Educational knowledge; Changing relationships between the State, civil society, and the educational community* (edited), and *Cultural history and education, Critical studies on knowledge and schooling* (edited with B. Franklin and M. Pereyra). He is currently writing a book on the reason of cosmopolitanism, the construction of the modern citizen, and the governing of the child, as well as editing books on educational partnership, changes in the welfare state and child care, and historically examining John Dewey as the indigenous foreigner. Thomas' address is: Department of Curriculum & Instruction, University of Wisconsin-Madison, 225 N. Mills Street, Madison WI 53706, USA. E-mail: tspopkew@facstaff.wisc.edu

Diane Reay is a Senior Lecturer in Research Methods in the School of Social Science and Public Policy, King's College London, UK. She taught in inner London primary schools for twenty years before moving into higher education. Since then she has published widely in the areas of gender, social class and ethnicity. Her book *Class work: Mothers' involvement in their children's schooling* (University College Press, 1998) is an analysis of the inequalities arising out of parental involvement in

schooling. Diane's address is: Department of Education and Professional Studies, King's College London, Franklin-Wilkins Building (Waterloo Bridge Wing), Waterloo Road, London SE1 9NN, UK. E-mail: diane.reay@ kcl.ac.uk

Ole Skovsmose is a Professor at the Department of Education and Learning, Aalborg University, Denmark. His works concentrate on critical mathematics education. His publications include *Towards a philosophy of critical mathematics education* and *Educacão mathemática crítica.* He is member of the editorial board of *Nordic Studies in Mathematics Education, For the Learning of Mathematics, Mathematics Education Research Journal* and *Mathematics Education Library,* Kluwer. He has been co-ordinator of the Centre for Research of Learning Mathematics, a co-operative project among Roskilde University, The Danish University of Education and Aalborg University. Ole's address is: Aalborg University, Langagervej 8, 9220 Aalbog East, Denmark. E-mail: osk@learning.aau.dk

Paola Valero is Assistant Professor at the Department of Education and Learning, Aalborg University, Denmark. She worked in 'una empresa docente', the research centre in mathematics education at the University of Los Andes, in Bogotá, Colombia. There she participated in different research programs in mathematics education related to the teaching of pre-calculus with the use of graphic calculators, and mathematics teachers' professional development in secondary school. She was awarded her Ph.D. in mathematics education at the Danish University of Education. Her research is an interdisciplinary work exploring the political dimension of mathematics education in relation to secondary school mathematics and mathematics teacher education. She has been co-author in various books and has published several research papers. Paola's address is: Aalborg University, Langagervej 8,9220 Aalbog East, Denmark. E-mail: paola@learning.aau.dk

Renuka Vithal is Director of the School of Educational Studies at the University of Durban-Westville, South Africa. Her areas of teaching and research include mathematics education, teacher education and educational research with a strong orientation toward social, cultural and political perspectives and practices. She works with students at all levels from undergraduate programmes for students planning to become teachers of mathematics to practicing teachers involved in postgraduate studies. Renuka's address is: School of Educational Studies, University of Durban-Westville, Private Bag X54001, Durban 4000, South Africa. E-mail: rvithal@pixie.udw.ac.za

Dylan Wiliam is Professor of Educational Assessment and Assistant Principal of King's College London, UK, which he joined after working as a mathematics teacher in inner-city schools in London for eight years. He teaches research methods and educational assessment to masters and doctoral students, and divides his research time between mathematics education and research in educational assessment. Outside work, he spends as much time as he can watching baseball, scuba diving and reading. Dylan's address is: Department of Education and Professional Studies,

King's College London, Franklin-Wilkins Building (Waterloo Bridge Wing), Waterloo Road, London SE1 9NN, UK. E-mail: dylan.wiliam@kcl.ac.uk

Robyn Zevenbergen is an Associate Professor at Griffith University. She lectures in the areas of primary mathematics education and action research. Her research interests focus on issues of social justice with a particular focus on working-class and indigenous students. Most of her research is conducted in schools and/or workplaces, using qualitative methods. She is currently working on two large projects funded by the Australian Research Council, along with a middle years school-based reform project. Robyn's address is: Education, Griffith University, PMB 50, GCMC, Bundall, QLD 9726, Australia. E-mail: r.zevenbergen@mailbox.gu.edu.au

PETER MCLAREN

PREFACE

Mathematics education research as a discipline is situated at the confluence of an array of diffuse, seemingly incommensurable, and radically divergent discourses. Research claims that have grown out of mathematics education are wide-ranging and antagonistic rather than circumscribed by hidebound disciplinary frames. While there has never been a unified, totalising discipline of knowledge labelled 'mathematics education research', and while it has always been a contested terrain, it is fair to say that the master paradigm out of which this field has been generated has been that of cognitive psychology. Mainstream mathematics education knowledges refracting the master discourse of psychology —whereby cognition serves as the central privileged and defining concept— clearly delimits its possibilities for serving as a social tool of democratic transformation. The central point of departure of this new collection is that mathematics education research is insufficiently univocal to support the type of uncompromising interpretation that cognitive psychologists would bring to it.

The hallmark contribution of this pathbreaking volume edited by Paola Valero and Robyn Zevenbergen is the paradigmatic shift the authors have effected in the field of mathematics education research, taking up a position at the faultline of socio-cultural analysis and critical pedagogy. Approaching mathematics knowledges as various modes of understanding *in, between,* and *among* larger socio-political and cultural contexts —as well as modes of intervention *against,* and *alongside* them—, the contributors to this book have risen to the occasion by providing a serious reconsideration and reconceptualisation of mathematics and mathematics education and their potential to foster both understanding and transformation of contemporary educational problems. With the advent of this book, mathematics education stands ever more squarely in the camp of the critical tradition.

Clearly there are vested interests for certain kinds of mathematical knowledges that provide ballast for regnant regimes of capitalist exploitation that give rise to asymmetrical relations of power and privilege centred around race, class, and gender relations and affiliations, and that give license to pursue narrow neoliberal approaches to educational policy and pedagogy. Whereas much educational criticism has degenerated into sound-byte rebukes of the corporatisation of the educational lifeworld, and the technocractic and instrumental rationality of school reform mandates, the authors contributing to this volume display a precient and

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systematic awareness of how school mathematical knowledges generated across various institutional bases have become functionally advantageous for particular modes of governmentality and social control. Working within a zone of endeavour that marks a political shift in mathematics education in which hegemonic attitudes of objectivity and facticity are challenged for their insinuation into certain normative educational policies and practices, the authors of *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* undress conceptual distinctions among knowledge, power, economics, and values within current instantiations of mathematics research in an attempt to unseal the hatches of their seemingly watertight boundaries. Yolking together methodology, politics, knowledge and values in an attempt to reconceptualise mathematics education as much as a political project as a disciplinary practice, this volume never the less carefully avoids rhetorical excess as well as ideological and dogmatic imposition. More nuanced formulations of mathematical knowledges along the lines suggested by the authors of this volume can help researchers and teachers identify and transform those mathematical knowledges that integrate us into the larger social totality of contemporary capitalist society.

The concept of power is the load-bearing category of the book, demonstrating a debt to and an affinity with the critical education tradition, especially the socio-political strand that grew in tandem with the sociology of knowledge school in the early 1980s. The political leitmotif that connects all of these essays is the attempt to construct a more progressive mode of mathematics education through a hermeneutical, phenomenological and dialectical engagement with socio-political approaches to meaning-making and praxis. The book itself is a lesson in Freirean pedagogy as it attempts to set the context for a critical, self-reflexive understanding of mathematics education as a field of inquiry. That the editors are able impressively to achieve a clarity of exposition and a precision of conceptual delineation, even as they challenge the reader with a dialogical structure that positions both authors and readers in a network of plurivocal discourses, is a testament to Valero and Zevenbergen's ingenuity and creativity. The result is a set of filigreed discussions about mathematics education research and a systematic and coherent articulation of mathematical knowledges that have not been widely available for mathematics education postgraduate students and mathematics researchers. In redefining the debate over mathematical knowledges and social practices, this landmark new book joins the pioneer efforts of critical educators to speak knowledge to power.

Peter McLaren
University of California, Los Angeles
USA

INTRODUCTION

SETTING THE SCENE OF THIS BOOK

This book emerged from the desire for bringing together two main concerns, namely, the need for reflecting on the implications of adopting socio-political approaches on the understanding of mathematics education phenomena, and the need for evidencing the implications of those frames on the research process itself. In other words, the chapters in this collection reflect on and link mathematics education as a 'practice' and as a 'field of knowledge' —to use Ernest's (1998) terminology— from socio-political perspectives. It is precisely the combination of these two interests that makes this collection an original, needed contribution to international literature in the discipline, in general, and to the growing research on the social and political dimensions of mathematics education, in particular.

In the last decade, the international community of mathematics education researchers has increasingly recognised the importance of studies on the social dimensions of mathematics education. These studies not only represent a considerable shift in perspective from the dominant paradigm of psychology that has supported most of the conceptual development of mathematics education research. They also provide new insights that have allowed us to improve our understanding of mathematics education. Within the approaches that claim to focus on the 'social', there can be found many different interpretations of the term. For some researchers, following the ideas of social constructivism, the 'social' refers to the nature of mathematics as a knowledge that is constructed in an interpersonal interaction between the teacher and students and also among students themselves. The exchange among the participants in a learning situation is the core of the 'social' dimension. Some other researchers have gone further in considering how those interpersonal interactions form patterns of relationships in the classroom and how the whole classroom environment constitutes a micro-community in which learning takes place. The 'social' in this case is conceived in terms of a 'classroom culture' and the evolving learning practices of that closed classroom community. Cognitive processes are not independent of the meanings that emerge in those communities and of the ways of communicating meaning in those social and cultural situations. These points of view have clearly shifted our attention from conceptions of mathematical learning as an individual, mental, decontextualized construction.

These two types of understanding of the 'social' are at the core of what might be called the 'social turn' in mathematics education (Lerman, 2000). They have been at

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the centre of a prolific area in the last decade, illustrated by books such as Boaler (2000) and Burton (1999). We find that these definitions of the 'social' and the ways of researching the phenomena of mathematics education can be broadly categorised as socio-cultural-psychological understandings of the 'social' in mathematics education. Often, these perspectives resonate with the work of Vygotsky and are referred to as a 'socio-cultural' perspective where there has been a concerted effort to unite the social and psychological domains in order to understand their interaction and impact on learning mathematics.

However, it is possible to identify another trend strongly rooted in sociology, critical theory and the politics of education. This trend stands on the assumption that mathematics education is, in essence, a *social* and *political practice.* This practice is social because it is historically constituted in complex systems of action and meaning and in the intermesh of multiple contexts such as the classroom, the school, the community, the nation and even the globalised world. This practice is political because the exercise of power, both in it and through it, is one of its paramount features. In mathematics education it is always possible to ask whose knowledge is being represented in society, schools and classrooms, and with what effects for the different participants in it. The recognition of the different and multiple positions that social actors can adopt in relation to and with the use of (school) mathematical knowledge is at the core of discussions of equity, social justice and democracy in mathematics education. The politics of mathematics education is essential to an understanding of mathematics education practices in large social contexts and form this broader understanding of the 'social'.

Pioneer studies from this perspective are the work of Mellin-Olsen (1987) and Skovsmose (1994). More recent contributions in this trend are the book by Dowling (1998) and some of the chapters in Athew, Forgasz and Nebres (2001). The proceedings of the three meetings of the Mathematics Education and Society Conference (Gates & Cotton, 1998; Matos & Santos, 2000; Valero & Skovsmose, 2002) also evidence the expansion of this area of research. Even though publications within this trend have been appearing intermittently throughout journals and books, the production is not yet so extensive when compared with the volume of publications following dominant research approaches. This collection presents a new contribution to international research literature in the socio-political trend of mathematics education.

The book, though, does not gather papers reporting research carried out from socio-political perspectives. Rather, authors have been invited to present a reflection on the research process in mathematics education when adopting such a position. As a preliminary working definition, we conceive *research* as a social and political practice in which people engage in a process of systematic inquiry, which involves creating an interpretative bridge between situations being observed by a researcher and previous formulations about such situations. Research is done through a constant exchange between many intervening actors and factors, such as the *researchers* who build and experience the whole process; the *situations* which are approached; the *theoretical tools* adopted by the researcher to tackle the situations; and the *methodologies* or ways of connecting the situations with the theoretical tools in order to produce interpretations of former. A socio-political perspective in

mathematics education does not only offer possible theoretical tools and interpretations, but also emphasises the researcher's awareness of the research process and on how he/she privileges —and silences— diverse aspects of the research activity. In this sense, examining the process of research and its elements —and evidencing the power relationships involved in them— becomes one of the central features of socio-political approaches in mathematics education research. In other words, the adoption of socio-political approaches in mathematics education research also invites the researcher to be critical towards his/her own activity as a researcher. While this is a feature of a number of chapters, other authors have addressed it in a minor way in their submissions to this book —but have considered these issues in other papers related to their research.

In the international literature on mathematics education research, little attention has been paid to the examination of the research process. The few publications presenting theoretical and methodological discussions have been imbued with either psychological or socio-cultural psychological terms of reference. Examples of such publications are Sierpinska and Kilpatrick (1998), Kelly and Lesh (2000) and more recently English (2002). Understanding the implications of adopting a socio-political perspective to inquire the teaching and learning of mathematics demands, among others, examining issues such as the limitations of existing research results, the implications of adopting new theoretical tools, the characteristics of doing research in highly conflictive environments, the positioning of the researcher in the research process, and the implications of adopting socio-political methodological frameworks. The chapters in this book address one or more of these issues as part of their 'meta-reflection' on the research process in mathematics education.

Given that we acknowledge the dynamic nature of research, in which exchange and conversation are paramount, we have structured this book around the idea of *dialogue.* Through dialogue we engage in practice and, thus, in the transformation of our world. We create discourses that allow us to position ideas, activities and ourselves in larger arrangements of social action. In this book, this dialogic interaction is manifested concretely in the effort that the authors and editors have made to relate ideas in their chapters to those presented in other chapters, in search of a multi-vocal discourse to refer to the socio-political dimensions of mathematics education. First, this *introduction* brings forward a chapter by Paola Valero in which there is a discussion of the meaning of power in mathematics education. The chapter attempts to establish a dialogue between existing dominant research literature and the socio-political approach that this book adopts. This section also presents an overall dialogic relation among the different chapters and units in the book. Secondly, the subsequent chapters are organised in *five dialogic units.* A dialogic unit is a cluster of chapters addressing similar aspects of the research process. The first dialogic unit, including the chapters by Dylan Wiliam, Hannah Bartholomew and Diane Reay and by Stephen Lerman and Robyn Zevenbergen, adopts a critical position towards existing research results, based on the adoption of socio-political approaches in the examination of key research areas such as assessment and classroom communication. The second dialogic unit, comprising the chapters by Paul Ernest and by Tony Cotton and Tansy Hardy, addresses the implications of adopting alternative theoretical tools in researching mathematics education. The

third dialogic unit, covering the chapters by Núria Gorgorió, Núria Planas and Alan Bishop and by Gelsa Knijnik, discusses the tensions emerging while doing research in highly conflictive environments. The fourth dialogic unit, containing the chapters by Tamsin Meaney and by Anna Chronaki, discusses the issue of the researchers' positioning with respect to the research participants. The fifth dialogic unit, including the chapters by Bill Atweh, by Ole Skovsmose and Marcelo Borba, and by Renuka Vithal, deals with the adoption of socio-political methodological frameworks. Finally, the *epilogue,* with a chapter by Thomas Popkewitz, represents an exchange between mathematics education and general sociology and education, through his reading, critique and interpretation of all the chapters in the five dialogic units. With this structure and organisation of the chapters, we expect to capture the multiple connections that the authors and editors of this book have experienced in different spaces and during the years in which we worked together on this publication.

We expect that this collection of papers will be of interest not only to established researchers in mathematics education, but also to research students who are seeking alternative tools to engage in the task of understanding the enormous complexity of the acts of mathematical learning and teaching in diverse settings.

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PAOLA VALERO

SOCIO-POLITICAL PERSPECTIVES ON MATHEMATICS EDUCATION

In the short editorial note to the introductory section of this book the editors argued in favour of an emerging research trend in mathematics education called the *socio-political perspective.* They have briefly mentioned how this trend pays particular attention to the fields of practice and of academic research of mathematics education. My intention in this first chapter is to make a case for the existence and need to attend to such an approach. I explore possible reasons why the adoption of a socio-political perspective to research mathematics education practices, as unfolded in different educational environments, provides fruitful ways of conceiving the phenomena of the teaching and learning of mathematics. These conceptions open possibilities to understand other aspects of those practices and phenomena that other well-established research trends have not substantially considered. I also engage in a 'meta-reflection' about the practice of researching mathematics education. I contend that part of the development of a socio-political approach involves posing critical questions to the way in which we as researchers, in our activity, build theories, construct 'objects' of study and influence our world with the knowledge that we produce. In other words, I ask questions about the way in which we exercise power with our research in mathematics education.

REVISITING THE CORE OF MATHEMATICS EDUCATION RESEARCH

Mathematics education researchers, through their activity and in their discourse, have provided definitions of their field of study. These definitions start by characterising —and therefore constructing (see Popkewitz, this volume)— the 'objects' that form a part of its gaze. Even though it is possible to claim that mathematics education is a 'new' discipline or field of academic research, there have been multiple ways of defining its objects of study. There has also been a wide range of 'lenses' that researchers have chosen to give an account of those objects. Unity in approach has not characterised the evolution of the field, particularly in recent times. This diversity has raised a number of debates including whether or not mathematics education can in fact be considered a discipline (e.g., the discussion in Working Group 4 in Sierpinska & Kilpatrick, 1998, p. 25), whether there is a search

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of identity (e.g., Sierpinska & Kilpatrick, 1998) or, if in fact, the field has widespread, multiple identities (e.g., Vithal & Valero, 2003), and whether the various approaches —including the theories and methodologies— represent 'valuable' contributions to both the practical and theoretical concerns of the field (e.g., Lesh, 2002). In all these discussions, there has been a constant creation of discourses about what mathematics education research is, as well as what the practices of mathematics education in schools and other spaces are. This means that as researchers, we create the 'objects' of our study while we engage in the practice of researching those objects.

This remark is of crucial importance when attempting to define the elements of a research approach in mathematics education —in relation to other possible approaches— because it allows us to see that what we choose to research and the ways in which we carry out that research are constructions determined, among other factors, by who we are and how we choose to engage in academic inquiry. In other words, there are considerable 'subjective' and 'ideological' grounds —rather than 'objective' reasons— to engage in particular ways of conceiving and conducting research in mathematics education. Furthermore, there are many socially, economically and historically grounded reasons for understanding the development of this field of study and its constructions.

When tracing the trajectory of mathematics education research in English speaking countries, many authors have pointed to the origins of the discipline emerging mainly in the intersection of mathematics and psychology (e.g., Kilpatrick, 1992). Even though other fields of study have also played a role in contributing to the core of the field, these two have shown the most prolific developments. This means that the dominant definitions of mathematics education practices and mathematics education research have mainly arisen from the work of mathematicians whose focus has been on mathematics, or of psychologists who have a strong propensity towards learning and cognition. The result of this has been the emergence of definitions of mathematics education as the field of study which 'investigate[s] and [develops] the teaching of mathematics at all levels including its premises, goals and societal environment' (Wittmann, 1998, p. 87). Mathematics as 'an original and natural element of human cognition' (p. 90) is the starting point of this endeavour. The approaches adhering to this type of definition have spawned a considerable body of knowledge about the teaching and learning of school mathematics —each with its own nuances and assumptions. The focus has been predominantly on how individuals learn (school) mathematics and, in so doing, this research has developed an improved understanding of the ways in which practices in schools may enhance the learning and mathematical thinking capacity of individual students. Today, one only has to attend the many mathematics education conferences throughout the world —whether these are for teachers or researchers— to notice the dominance of this focus. Similarly, surveys of publications including books, journals and conference proceedings strongly support the claim that mathematics education is dominated by these two views of the field (Chassapis, 2002; Gómez, 2000; Skovsmose & Valero, 2002).

When looking at the evolution of the field in relation to the historical time in which it started flourishing —namely, around the middle of the 20th century—, it is

not surprising that the association between mathematics and psychology had supported mathematics education research. If we look at the history of mathematics education, as seen from the USA point of view (see Schoenfeld, 2002, pp. 437-443), we find a relationship between the expansion of different schools in psychology — such as the Gestalt psychology, behaviourism, and constructivism— and the different types of studies and directions that research has taken[1]. It is not implausible to hypothesize that the people who started doing research on different phenomena related to the teaching and learning of mathematics have found it fruitful to borrow theories and research tools from psychology. Such fruitfulness could be seen in the possibility to concentrate on understanding and improving individual learning and achievement, one of the main justifications and characteristics of general educational reforms in the USA since the 1960's (e.g., Lieberman, 1992); and to focus on mathematical thinking and cognition, one of the banners of mathematics education movements from the time of the New Maths. Some trends of psychological research could have been regarded as the most appropriate to reach educational as well as mathematics-related research aims. It is also worth reminding that in search for academic recognition, mathematics educators could have found in psychology and mathematics good allies to legitimate their work, as Lerman (2000, p. 22) argues: 'Both the disciplines of mathematics and psychology have high status in universities, and locating mathematics education within either group is seen as vital in some countries in terms of status and therefore funding and respectability'.

Besides the internal reasons for the fruitfulness of this alliance, it could be interesting to look at its possible social significance and 'functionality' in relation to broader frames of reasoning. Popkewitz (2002) has argued that the development of a particular kind of schooling and of the educational sciences —including curricular studies and subject-matter didactics such as mathematics education— has to be understood in relation to the consolidation of the modern state, one of whose main concerns is the administration of its citizens:

> The school subjects are not merely identifying and organizing academic disciplines of mathematics, science, history, geography, music or art into formats that children learn. If one historically considers the school subjects in the beginning of the 20th century, they installed standards that were to make the child's conduct legible, *easily administrable,* and equal. The logic underlying the teaching of mathematics and other curriculum, for example, was less related to the academic discipline than to a romantic, even spiritual hope of the future of a liberal democracy and a fear of deviance engendered in the hope [...] Pedagogy was to fabricate the new child who embodied the political principles of action and participation and, to paraphrase curriculum writers of the time, to prevent the barbarians from knocking at the American door. (pp. 37-38)

[1] An interesting historical exercise to research the history of the discipline, at least in the USA, could be to analyse the trends of the large number of volumes produced by the National Council of Teachers of Mathematics (NCTM) and track the emerging construction of the objects 'mathematics education practices' and 'mathematics education research'. This is in part what Schoenfeld (2002) does for the early developments at the beginning of the 20th century.

The role of psychology, more than any other social science, has been essential in the process of making the child 'administrable' through (mathematics) education because it has provided tools to name, describe and measure the way in which students are expected to think and behave within particularly socially valued systems of reason. Part of the administration of the child exercised through mathematics education is the reduction of the student —a fully socially-grounded child— to a *cognitive subject* whose dimensions of interest for the study of mathematics education are her or his mathematical thinking processes. This cognitive subject, which the discourse of mathematics education research construes as a 'schizomathematicslearner' (Valero, 2002b), embodies the main features that are of interest to mould: reasoning and thinking within the frames of reference of a discipline that has proven to be effective in the development of the type of society in which we live today. Mathematics and school mathematics are strongly linked with dominant systems of reason in the white, Western world. Lerman (2000, p. 21), adopting some of the main claims in Walkerdine (1988), states that mathematics is 'seen as a marker of general intellectual capacity', whose symbolic power allows to preserve 'its gendered and Eurocentric character, creating through its discursive practices the reasoning logical norm'. It is precisely this characteristic what connects mathematics —and mathematics education— to the systems of reason, which constitute founding pillars of the enterprise of modernity, such as the primacy of scientific rationality, objectivism, utilitarianism, progress and even democracy.

The recognition that mathematics and mathematics education have been central to the construction and consolidation of these systems of reason —which go together with systems of production— has helped in sustaining the advance of mathematics education, including the interest in developing effective ways of enhancing educational processes in this subject area. This key role is associated with the construction of 'myths' around mathematics education. Dowling (1998) has referred to the *myth of participation* as the conviction that people are handicapped to participate in society if they do not understand and are not able to use mathematics in a critical way. Just as an illustration of how this myth features in mathematics education research literature, let me draw on the justification that Malloy (2002) offers for the need of developing research and educational practices concerned with 'democratic access to powerful mathematical ideas' —one of the key notions in English (2002):

> An ideal education in which students have democratic access to powerful mathematical ideas can result in students having the mathematical skills, knowledge, and understanding to become educated citizens who use their political rights to shape their government and their personal futures. They see the power of mathematics and understand that they can use mathematical power to address ills in our society. Education of this sort addresses political aspects of democratic schooling, the social systems of nations, and often has as its focus the social betterment of nations and the world [...] The crux of democratic access to mathematics is *our* understanding and researching new ways to think about mathematics teaching and learning that has a moral commitment to the common good, as well as to individual needs. (Malloy, 2002, p. 17, emphasis in the original)

In this fragment mathematics education is clearly identified as a key element in educating citizens who are competent in dealing with the basic challenges of current societies. Of course, research and practices have to face the facts of underachievement and exclusion of some groups of students because it is desirable that *all* students —around the world, not only in the USA— acquire the intrinsically good qualities of mathematics education. It is not difficult to find in literature assertions, such as the one mentioned above, that clearly reveal the almost blind trust of researchers in the intrinsic goodness of mathematics education. To put it in other terms, the discourses of mathematics education have resonated with the discourses of mathematics and psychology, as well as with the discourse of modernity in the construction of a particular research discipline, with particular theories and methods, supporting the constitution of practices in the classroom that fulfil essential social functions, which help in sustaining a certain kind of social organisation.

My intention here is not to deny the contributions of the core of research in mathematics education. From this research, we know much more about how the acquisition of different mathematical ideas by different students in diverse formal and informal educational settings takes place. We also know more about the mathematics curriculum, its planning and implementation in the classroom. We have also gathered significant knowledge about the challenges of assessment, the introduction of information technologies and the creation of more attractive, friendly learning environments for students who otherwise would have 'suffered' in the mathematics classroom. We know more about mathematics teachers, their education and their working environments. Research has also shown that all this knowledge has helped in improving the practices of mathematics education in educational institutions. I would not suggest that this research has been misleading or that it has been deliberately used to manipulate and alienate students. Rather, I contend that the kind of discourse that mathematics education research and practices have constructed responds to the conditions of the social space in which it has originated and developed. What has been conceived as the progressive consolidation of a discipline, with a central 'research problématique' (see Adda, 1998, referring to Freudenthal's list of thirteen problems for mathematics education research —in Freudenthal, 1983), cannot be isolated from the social, political and economic frame in which that scientific endeavour has been carried out. Furthermore, I contend that the dominant discourse has been associated to a certain research focus and, therefore, has underplayed other elements that are as significant as individual mathematical cognition to reaching an understanding of mathematics education practices. In what follows I will examine how these alternative approaches have entered the field.

EXAMINING THE SOCIAL TURN IN MATHEMATICS EDUCATION RESEARCH

In more recent times there has been an increasing amount of research with a different emphasis compared to the core of mathematics education research as

defined above. In an overview of such research, Lerman (2000) referred to it as the 'social turn' in mathematics education and identified it as a trend that clearly appeared in published literature in English by the end of the 1980s. Lerman highlighted the fact that even within the mathematical-psychological mainstream of mathematics education research there was a degree of recognition of some 'social' factors such as interaction among cognitive subjects —named 'social' interaction in, for example, constructivist theories—, or humanistic and democratic concerns — named 'social' concerns— of researchers and teachers. However, the 'social turn' does not take on these meanings of the term 'social' but adheres to a more 'essential' meaning. Lerman defined this social turn as 'the emergence into the mathematics education research community of theories that see [mathematical] meaning, thinking, and reasoning as products of social activity' (p. 23). According to Lerman, the 'real' social turn occurred with the recontextualisation of socio-cultural psychology, of anthropological theories about cognition in practice, and of sociological theories of the construction of learning practices in the social spaces of schooling. This has led to the production of new knowledge within the field of mathematics education. Such knowledge differs fundamentally from the conceptual creations in mainstream mathematics education in the conception of the nature of what it means to learn, think and come to know (school) mathematics. In saying that 'meaning, thinking, and reasoning [are] products of social activity' we are moving away from the conception of these 'objects' as emerging from and within the mind of decontextualised cognitive subjects —with or without interaction with others. In fact the assertion implies that these 'objects' —or processes— are constituted in the encounter between contextualised, historically grounded human beings and their activity in particular settings and spaces that are socially structured. In this way (mathematical) meaning, thinking and reasoning emerge from and within this encounter and not from a mental process located somewhere in the 'head' of cognitive subjects.

Another element of central importance to the shift from mainstream mathematics education is the advance in the sociology of mathematics and in its formulations of mathematics as a social construction —as opposed to previous philosophies of mathematics, which did not significantly distance the ontology of mathematics from the basic 'Platonic illusion' (Rav, 1993, cited in Restivo, 1999). I think it is critical to point out that, even though the ideas of mathematics as a social, changing and fallible activity have been behind a great deal of the developments of mathematics education from, for example, constructivist perspectives, it is within the social turn that there really appears to be a theoretical consistency between a sociologically inspired philosophy of mathematics and mathematics education. Consider, for example, Restivo's work on the sociology of mathematics (e.g., 1992, 1998, 1999). In his many interactions with mathematics education researchers —such as in the Mathematics Education and Society Conference (see Gates & Cotton, 1998; Matos & Santos, 2000; and Valero & Skovsmose, 2002)— Restivo constantly insists that it is not easy for mathematics educators to leave behind the 'Platonic illusion', even though we profess some of the ideas of the social turn. For it seems, in the first place, difficult for mathematics educators to grasp that the 'social' transcends inter-individual interaction for the construction of both mathematics and school

mathematical learning (Restivo, 1998). Interaction among people in the construction of knowledge can well be conceived as talk and dialogue happening in a *vacuum.* In this case the 'social' interaction resembles 'mental' interaction —in a traditional psychological sense— because it gets emptied of all its contextual foundation. The 'social' encompasses the people, their interactions, their activities in particular social spaces and historical times, the traditions and rituals of entering into those spaces and the overall structures in which all the former take place. Second, it is difficult for researchers in mathematics education to get rid of the shadow of professional mathematics to comprehend school mathematics as yet another social construction that, given the particular settings in which it is constituted, obeys the rules of practice that are not those of the professional community of mathematicians.

As an example of what the latter criticism means let me examine the recontextualisation that van Oers (1996) undertakes of Vygotskyan cultural-historical psychology to produce some knowledge about school mathematics. Van Oers starts by defining mathematical learning as 'a process of making sense of mathematics as it is brought to us by cultural history' (p. 92). This process involves negotiation of the culturally-created meanings of mathematics. A theory of learning should then give an account of how those meanings, which are in the first place negotiated by mathematicians in their socio-cultural activity, are formed by children in the context of the school. The centre of a theory of learning that intends to provide an account of the meaning creation resides in how the teacher —a more competent participant in the process of interaction and negotiation of meaning, that is, a representative of the mathematical culture— can in fact support and direct the students' acquisition of meaning in desired directions. It is interesting to note how, in van Oers' formulations, there seems to be a decontextualisation of the social space of the classroom and of, consequently, mathematical learning. The situatedness of the processes of learning seems to blur once it has been stated that the core of 'culture' and 'society' resides both in the intrinsic nature of mathematics and in the close interaction between students and teachers in the classroom. Once this decontextualisation happens, then the essence of the social and cultural dimension of learning —what would differentiate van Oers' formulations from, for example, those of a constructivist— is the maintenance of a necessary resemblance between the process of producing mathematics —by mathematicians— and coming to know school mathematics —by students in the classroom. In other terms, it seems that the socio-cultural pillars of this approach to mathematical learning have diluted.

But van Oer's approach is one among many viewpoints that could be identified as being part of the social turn. As a representative example of this variety, the reader can take a look at Steffe, Nesher, Cobb, Goldin and Greer (1996). In this book there is a section called *'Sociological and anthropological perspectives on mathematics learning'*. In this section there are ten chapters, authored by recognised researchers, in which the main assumptions of their perspectives are presented. The books edited by Atweh, Forgasz and Nebres (2001), Boaler (2000) and Burton (1999) also include some chapters that represent other (related) approaches. Some approaches are closer to cultural psychology —e.g., van Oers (1996)— while other have adopted situated cognition theories —e.g., Forman (1996). Some of these views could be seen as complimentary to the traditional psychological perspectives

—such as the work of Cobb and collaborators (e.g., Cobb, 2000) who argue for the coordination of individual and social analysis of learning in the classroom—, while some argue in favour of the alternative nature of their theoretical constructions (see Cobb, Jaworski & Presmeg, 1996). Thus, as mentioned in the editorial comment to the introductory section in this book, there are multiple definitions of the 'social', even within the social turn.

Within this multiplicity, I see that a socio-political trend, much less represented in terms of published research, is in consolidation within the social perspectives. This trend deserves to be examined and characterised as a line of thought within the social turn.

SPOTTING THE SOCIO-POLITICAL TREND IN MATHEMATICS EDUCATION RESEARCH

In his account about the origins of the social turn, based mainly on the consideration of English-language publications, Lerman (2000, p. 24) noted that 'the receptivity of the mathematics education community to social theories was due more to *political concerns* that inequalities in society were reinforced and reproduced by differential success in school mathematics, than social theories of learning' [my emphasis]. Lerman suggests here that some researchers in mathematics education started focusing on the fact that there seemed to be a systematic exclusion of some students from the possibility of engaging in the learning of mathematics. These researchers, in search for understandings of this fact, found support in social theories. It is clear that the 'political concern' of some of these people resonated with the new tools that developments in sociology, anthropology and critical education could offer.

This route had already started to be explored by people in other non-English-speaking research traditions. *The politics of mathematics education* (Mellin-Olsen, 1987) was probably the first book in mathematics education in which the term 'politics' appeared in the title. Power was central to Mellin-Olsen's multidisciplinary view of mathematics education. Behind his book and approach there was a tradition of work in Norway and Denmark in which the 'political dimension' of mathematics education had been explored and discussed (e.g., Mellin-Olsen, 1977; Skovsmose, 1980, 1981a, 1981b). This Scandinavian tradition, rooted on Action Theory and Critical Theory and drawing theoretical tools from a variety of social sciences, can be considered as a key element when tracing the emergence of the socio-political trend.

Some key notions

It is important to examine more carefully how the political concerns of some scholars developed into what could be called a 'socio-political' trend in mathematics education research. In other words, it seems relevant to bring some elucidation to the meaning of the terms 'social' and 'political' in this expression. The 'social' component of this term is more or less defined according to the clarifications and

essential points that were noted above as being the difference between the social turn and mainstream research in mathematics education. The 'political' component, however, has not been clearly defined yet. I would like to contend that even if the social rum derived from a political concern of some researchers, not all the theories and approaches developed as part of the social shift have in reality incorporated an analysis of *power* in association with mathematics education.

Let me start by a simple definition of the term 'political' as awareness of the existence of *power.* Now let me reiterate, for example, van Oers' socio-cultural psychology of mathematics learning. Where does a consideration of power appear in van Oers' theory? A similar question can be asked for all the ten socio-cultural chapters in Steffe at al. (1996). It is possible to argue that 'power' appears in association with statements of the type: Since mathematics is a *powerful* knowledge in our society, then it is important to improve the access of as many students as possible to a quality mathematics education. Such an assertion implies, in other terms, that mathematics and mathematics education *empower.* That there is empowerment associated with mathematics and mathematics education is sometimes an explicit assumption —see e.g., Cobb & McClain (in press) when justifying the relevance of students' statistical understanding as a part of their competence as citizens—, but most of the times it remains tacit. It is interesting to highlight that this type of political concern, however, does not differ substantially from the also tacit political concern of psychology-oriented research in mathematics education. Skovsmose and Valero (2001) have argued that much research work seems to adhere to the idea that mathematics is in itself an indispensable, good and desired knowledge in our current (Westernised) world, and that mathematics education has the positive role of enculturating the new generations into that knowledge and all its related values. The unquestioned intrinsic goodness of both mathematics and mathematics education represent the core of its 'political' value: If students and citizens come to learn a considerable amount of mathematics properly, they will become *per se* better people and better citizens; that is, mathematics and its education *empower* or have the capacity of giving power to people. In other words, there is an *intrinsic resonance* between mathematics, mathematics education and power. The quotation from Malloy (2002) above illustrates this type of argument.

The problem with this kind of assumption is that there is no necessity for a further examination neither of mathematics as a knowledge and of mathematics education as practices, nor of power. Power, in that form and definition, is taken for granted, and whenever it appears as part of the research discourse or the public discourse about mathematics and mathematics education, then good mathematics and mathematics education practices get vested with a veil of sanctity and redemption of humanity. The acceptance of ideas related to the intrinsic resonance of mathematics, mathematics education and power helps sustaining the alchemy of mathematics as a school subject that can be used very efficiently in the administration of the child —to put in it Popkewitz' (this volume) terms. Furthermore, there is a fundamental problem with the attribution of power to mathematics in this way. Saying that mathematics is powerful means that mathematics in itself can exert power, what implies that mathematics is given the status of a social agent. Mathematics is given a life of its own that it does not have.

It is people, in their activity, who use mathematics as a tool of power. Saying that mathematics is powerful, therefore, leads us to a new kind of Platonism (Valero, 2002a). In this way, the field becomes trapped again in the 'Platonist illusion' that the sociologists of mathematics have so fiercely criticised.

What power means in association with mathematics education needs to be carefully examined (see Skovsmose & Valero, 2002; Valero, 2002a). Therefore, it is important to examine the naïve definition of power presented above, and to bring forward a more complex notion of 'political' and of 'power'. Let me adopt a view of power rooted in the Marxist tradition. From such a perspective, power is the capacity of some —the owners of resources or a dominant class— to mould the living conditions of others —the dispossessed— by alienating them from the produce of their work activity. Such a capacity, rooted in basic forms of production, is reinforced by a whole ideological superstructure, which supports and feeds the maintenance of class divisions. Power, then, is a capacity of some people —or groups of people— to keep others in their condition of excluded. Although this definition, so formulated, may misrepresent the depth of its theoretical lineage, it is important to highlight that its essence is a division and a struggle between those who are 'included' and those who are 'excluded'. This struggle represents a relation in which some tend to win —although there are always spaces for contestation and resistance on the side of the excluded (as Apple, 2000, reminds us).

This conception of power has taken different shapes in mathematics education. A significant representative of this view is the political challenge posed by ethnomathematics to the reign of Western, white mathematics. A fundamental critique by D'Ambrosio (1993, p. 10) is the uncontested imposition of mathematics as 'a form of logic and rational thinking that became the characteristic feature of the human species' (my translation). Because of this privileged status in the cultural construction of the Western world —a particular, but universalised rationality—, mathematics 'is positioned as a promoter of a certain model of exercising power through knowledge' (p. 24, my translation). In the historical development of the Western world, which has impacted the transformation of the rest of other peoples, mathematics 'brings the memory of the conqueror, the slave-owner, in other words, the dominator; it also refers to a form of knowledge that was built by him, the dominator, and that he used and still uses to exercise his dominance' (D'Ambrosio, 1996, p. 114, my translation). Powell (2002, p. 17) also highlights that ethnomathematics departs from forms of thought that privilege 'European, male, heterosexual, racist, and capitalistic interests and values'. This critique of mathematics as a tool of power is incorporated into research and into the pedagogical proposals derived from it —see Knijnik (this volume). Bauchspies (1998, 2000, forthcoming) also illustrates the role of mathematics and science education in relation to power structures in classrooms in West Africa.

There are numerous studies approaching the issues of equity in mathematics education on the grounds of students' race, class, gender and language, among others, which adhere to this definition of power. A significant example is the work of Frankenstein (e.g., 1987, 1995) and her understanding of mathematics education as a critical process in which students realise their conditions within a system of class division through their mathematics education experience. The point being

proffered by this branch of mathematics education is not only adopting a critical position towards the contents and the processes of learning, but also towards the role that mathematics and mathematics education play in the very same social conditions of students, as well as in the possibilities of transformation of those conditions.

One element that clearly emerges from this type of definition of power —in association with the use of Critical Theory— is the necessity of questioning both mathematics and mathematics education practices. Such a questioning leads to an interrogation of ideas such as the intrinsic goodness of mathematical knowledge —e.g. Is it possible to assume that mathematics is a knowledge associated exclusively with progress and the well being of humanity? Or do we need to consider the involvement of that knowledge in the creations of both wonders and horrors in our current technological society? (D'Ambrosio, 1994; Skovsmose, 1994)—, and the intrinsic 'empowering' nature of mathematics education —e.g., Can we really trust Malloy (2002) in her view that good mathematics education will in fact make good citizens? Or should we consider the ways in which textbooks, policy makers and even mathematics education researchers build the myth of participation around mathematics education (Dowling, 1998)? In the case of the ethnomathematical program it is clear that any reformulation of mathematics education as social and cultural practices includes an examination of the goods and evils of the uses of mathematics within the social structures in which they emerge. The 'uses of mathematics' here do not only refer to the concrete applications of mathematics in the development of technological devices —as Skovsmose (1994) emphasises— but also the 'functionality' that people give to it in the construction of social relations and culture. It is also clear that educational practices get constructed around a constant interplay between different types of knowledge —e.g., school mathematics and practice-bound mathematics— and around discussions regarding the legitimacy and strength of each type of knowledge to address a particular situation (see Knijnik, 1996). To summarize, a notion of power rooted in Marxist and Critical traditions highlights the necessity of incorporating *critique* as an essential element of a socio-political approach. The examination of power requires critique as a means to offer a counterpart to naïve views about the way in which mathematics and mathematics education are implicated in the creation and maintenance of particular social structures.

A third possibility to define power, which moves away from the shortcomings of the two previous definitions, is power as a relational capacity of social actors to position themselves in different situations and through the use of various resources of power. This definition is clearly exposed in Cotton and Hardy (this volume). This definition implies that power is not an intrinsic and permanent characteristic of social actors —as the two previous kinds of definitions may entail—, but power is situational, relational and in constant transformation. This transformation does not happen directly as a consequence of open struggle and resistance, but through the participation of actors in the construction of discourses. In this sense power is subtle; and precisely because of this subtlety it becomes 'more powerful'. When power is defined in these terms, it becomes possible to perform a very fine grained analysis of how mathematics and mathematics education are used by people in particular

discourses and of the effects of those discourses on social practices and, consequently, on people's lives.

This last way of defining power has not been so popular among mathematics education researchers, probably due to the fact that it is related to more recent postmodern and poststructuralist conceptions of power (e.g., Foucault, 1986). However, this type of definition could stimulate new prolific production in mathematics education research. The reason for this is that such a definition of power finds resonance not only with the advancement of postmodern ideas in education (e.g., Popkewitz & Brennan, 1998) and in mathematics education —as suggested by Ernest (this volume)— but also with new possibilities of reinterpreting many of the theories that have been at the centre of the social turn. Consider, for example, the ideas of situated cognition and learning as increasing participation in communities of practice in relation to this political viewpoint. It becomes possible to study how power is an essential constituent of mathematics education practices in educational institutions.

Let me exemplify this possibility. Many of the recontextualisations of, for example, Lave's (1988) ideas in mathematics education research have been a substantial part of the social turn. Viewing learning in terms of participation and belonging to communities of practice is an alternative to a conception of learning as individual mental processes. However, I find that many of these recontextualisations have done away with the socio-political depth of Lave's notion of situated cognition. One of the central points of Lave's argument is that the triad person-in-activity, activity and setting —the relationship in which cognition takes place— is dialectically constituted in a multiplicity of contexts that provide meaning to the relationships in the triad. Lave emphasises the fact that that triad —located in what she calls the 'experienced, lived-in world'— is dialectically constituted in relation with what she calls the 'constitutive order' —'the mutual entailment of culture, conceived as semiotic systems, and organisational principles of the material and social universe (of political economy and social structure)' (Lave, 1988, p. 178). This multi-contextuality and deep political nature of cognition is lost when researchers in mathematics education reduce the notion of setting to, normally, a mathematics classroom. This is also lost when researchers declare the classroom, the students and the teacher to be 'social beings', while building a whole discourse throughout the research process, which in fact isolates the classroom from both the social arenas in which it is immersed and from larger contexts. The result is the creation of objects which are given the label 'socio-cultural', but which in reality are conceived as objects of analysis that exist in a vacuum. The effect of this is, again, a new kind of 'Platonisation' of the social and political practices of mathematics education. A way of highlighting power in mathematics education within Lave's framework could be to pursue the question that she herself posed to the dominance of traditional cognitive science on the value of decontextualised knowledge: What are the systems of values, that take part in the historical frames in which cognitive science developed, which made such a conception of knowledge 'dominant' —at the expense of contextualised, derived-from-practice knowing? What is it that makes particular kinds of school mathematics education practices develop in ways that are valued as the 'right' way of teaching and learning mathematics? What are the

discourses, at different levels, which give teachers and students particular positions in those practices? How do students and teachers change —and in which direction— their participation in those practices, and to the benefit of whose positioning do those changes happen? These new questions could guide us into investigations that reveal the fact that 'learning mathematics' is a highly political and social act that needs to be understood in full connection within the multiple contexts in which that activity and practice unfolds.

Finally, one element that emerges strongly when examining the definition of power in terms of positioning is the notion of *context.* Some critics have qualified mathematics education research as being internalistic, and one of the issues that they cite is its blindness towards the context of learning (e.g., Apple, 1995). Some trends in mathematics education research have paid significant attention to context when identified with the mathematical or real-world association that may trigger cognitive processes —e.g., within constructivist theories of learning. Special thought has also been given to the interaction context —the chances of communicative exchange between teacher and student and among students— in the classroom. However, definitions of context in terms of the situational context —to use Wedege's (1999) formulation— or even the socio-political context —to use Valero's (2002a) and Vithal and Valero's (2003) definition— have been comparatively more limited, but certainly present in studies that could be labelled as socio-political. The socio-political context can be defined as the macro-sociological space that has an influence on the more focalised interactions of mathematical teaching and learning in micro-contexts such as the classroom (Valero, 2002a). The notion of socio-political context invites mathematics education research to regard the possibility of analysing mathematics education practices in the interplay between 'the lived-in-world' and the 'constitutive order' —to use Lave's terms. Paying attention to this context and incorporating it as an essential element of analysis in mathematics education is a way of breaking the internalism of the field of study and its analysis.

It is interesting that the recognition of the role of context in mathematics education has been limited to the description of the 'context' of a situation under analysis. This is the case in many research reports where one finds statements such as, for example, 'the context of this research is a primary school in a low-class area of London'. Such a description seems to support the idea that being aware of the context implies simply mentioning it and, later on, forgetting its existence and its significance for the understanding of the analysis in question. Rather, the central point is finding ways of knitting together the micro contexts on which mathematics education researchers normally concentrate —such as a community of learners in the classroom— with the multiple layers of contexts in which that micro context is inserted, with the aim of finding significant revelations about the social and political essence of the educational practices of mathematics. All chapters in this book provide examples of this. As one illustration I will mention Chronaki's (this volume) analysis of her experience in studying mathematics education in a cultural setting which is not her own. Her noticing of school norms, uniforms, ways of talking and behaving that may not normally be part of the typical focalised observations of a mathematics education researcher —interested in the *learning* of *mathematics*— acquires relevance in the enterprise of the researcher giving meaning to mathematics

education in that social setting. Vithal (this volume) also highlights that part of the significance of a pedagogy of dialogue and conflict in a country such as South Africa resides in recognising and including in the analysis the unstable and conflictive nature of social relations in that country. Without these types of considerations, any analysis of mathematics education as a socio-political activity gets in fact depoliticised. The constitutive relation between micro and macro context, then, is a salient feature of a socio-political approach in mathematics education research.

Summarising, in this section I have examined three possible definitions of *power* in mathematics education, and I have also pointed to *critique* and *context* as associated notions that go hand in hand with power. I also attempted to argue that both critical definitions of power in relation to the role of mathematics and mathematics education in society, and the incorporation of context in mathematics education analysis are some of the most salient characteristics of research within the socio-political trend in mathematics education. In other words, socio-political approaches to research mathematics education are characterized by sensitivity towards and a serious incorporation of power, critique and context as relevant concepts to understand the practices of mathematics teaching and learning. This does not mean, however, that these are the only characteristics. Neither does it mean that there is unity among the research projects adopting a socio-political approach. My claim here is that it is possible to recognise some common concerns that may appear across the multiple particular perspectives that can be labelled socio-political.

Some methodological remarks

A consideration of research methodology is missing in this discussion. One obvious answer to the question 'What makes a piece of research 'socio-political'?' is that both, the theories and the methodologies on which a research relies and develops further, have to be socio-political. I think, however, that this connection is not immediate and that it is not a necessary and sufficient condition for research in mathematics education to be socio-political. Vithal (this volume) draws attention to the necessity of encompassing the theories to illuminate mathematics education practices, with the theories to illuminate methodology, that is, the theoretical foundations of the research process itself. It can well be that a study emerges from a socio-political concern —such as the unequal access of women to significant mathematics education— but the researcher ends up generating a research process that diverges completely from the initial concern because the researcher's ways of acting along the process and of substantiating her claims fall within the standards of rationality of the most traditional of mainstream research —e.g., some of the studies on gender reported in Keitel (1998)[2].

It is my contention here that socio-political research in mathematics education also has the task of constructing alternative discourses about the research process itself. I would like to briefly discuss the issue of visibility of the researcher and the

[2] See a critical review of Keitel (1998) in Valero (2001).

revelation of his/her 'self' (Krieger, 1991) as a strategy to break the neutrality of traditional academic discourse, and to contribute to the creation of new ways of communicating research.

Knijnik (this volume) states that

> the genre of my writing is to a certain extent different from that of the usual mainstream research texts [...] For me, the personalising of the text represents a very political act because it highlights the fact that knowledge production is not a neutral activity. This personalisation reveals the subjectivity of the researcher, her/his political stance, and the ways of interpreting the world; all of which imprint the topics and the methodologies that the researcher chooses within the research process.

A similar revelation of the self is found in Chronaki (this volume) where she invites us to see how she made sense of the culture of others through a reflection on her own school (mathematical) experiences. Cotton and Hardy (this volume) make clear the multi-vocality of their writing by combining their voices but also separating what each one of them has contributed with in the writing of a common chapter. Meaney (this volume) also makes transparent to the reader the process of negotiation that she lived when working with a Maori community. Gorgorió, Planas and Bishop (this volume) also evidence the multiple tensions that the researchers faced in working with immigrant students in Catalonia.

The revelation of who we are and what we stand for as researchers constitutes a transgression of the established norms of traditional academic discourse. This discourse, based on the idea that knowledge production and research are technical processes in which the 'knower' is separated from the 'known', and that their main goal is to produce 'objective' descriptions, explanations or interpretations that leave the 'known' untouched by the 'knower', is limited in recognising the role that the knower in fact pays in constructing the known while interacting with it in the process of research. Normally this pretension of objectivity is masked behind the generation of cold, distant, objective formulations behind which the researchers —with all their subjectivity and social groundedness— hide (Valero & Matos, 2000). In making the researcher visible —in ways that are more significant than a politically correct use of a personification of the discourse through the use of the pronouns 'we' and 'I' — socio-political research may open to the critical examination of the reader the products of the research process, the intentionality of the researcher, and the paths that the researcher decided to take when meeting the people with whom she engaged in the research.

This visibility is not a matter of the transparency of the methods and of the correctness of their application —as suggested in different criteria of research quality such as those described in, for example, Schoenfeld (2002). It is a matter of evidencing, in the way we express ourselves —our written and oral discourses—, that we recognise the dialogical, political and social nature of our task as researchers in mathematics education, and that we are implicated in constructing part of the practices of mathematics education in educational institutions when we act in those spaces as researchers. In other words, it is being consistent with what Restivo (1999) would call the 'social construction conjecture' at the level of our own research

endeavour. This characteristic provides an additional answer to the question posed at the beginning of this subsection. Adopting a socio-political approach is not only a matter of choosing a particular set of theories and methodologies. It is an 'attitude' that seeks for consistency between the former and our activity as researcher. This attitude also shows that the researcher is in search of appropriate ways for communicating the interpretations of her or his activity.

A PERSPECTIVE IN CONSOLIDATION

In the field of study called mathematics education there have been multiple trends delineated not only by their topic and object of study but also by their theoretical and methodological principles. Although a great deal of research has been done within what may be called a 'psychology-mathematics oriented' trend, an increasing amount of research has emerged as part of a 'social turn'. Within this social turn, there has appeared research adopting a 'socio-political' viewpoint.

If it is true that personal 'political concerns' of some researchers have been at the root of the development of this type of studies, such political concern has evolved in a more systematic examination of the ways in which power is a defining element of both mathematics education practices and research. Socio-political perspectives in mathematics education research are under consolidation. Researchers adopting these perspectives engage in the critical endeavour of examining not only the nature of the 'objects' of mathematics education research, but also the process of doing research, and proposing alternative —and complementary— forms of interpreting, explaining and understanding mathematics education practices.

ACNOWLEDGEMENTS

I want to thank Mahesh Bhatia, Wenda Bauchspies, Sal Restivo, Ole Skovsmose and Robyn Zevenbergen for their comments to previous versions of this chapter.

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FIRST DIALOGIC UNIT

CHALLENGING TRADITIONAL RESEARCH AREAS FROM SOCIO-POLITICAL PERSPECTIVES

Research on mathematics education from socio-political viewpoints establishes a critique of the conceptualisations and results that dominant research has produced in central areas of the discipline. At the same time, it provides alternative and complementary ways of understanding the phenomena of mathematics education. This dialogic unit brings together two chapters in which we find a search for alternatives in understanding classroom interactions and assessment. These two chapters reveal a conception of the 'social' as trespassing the boundaries of social interaction and becoming situated in broader social structures, with consequences that also enter larger spaces than the classroom. It is also evident in the chapters that, through mathematics education practices, certain students are positioned as low achievers in mathematics. Such positioning does not depend —in most cases— on students' actual mathematical ability, but on the interpretations made by teachers of students' participation in classroom interactions and in assessments routines. Both the language game in schooling and the assessment classification systems contribute to the generation of a disadvantaged position of students with respect to the access that a better mathematical-related positioning may offer students in contexts such as the Australian and British educational system.

Stephen Lerman and Robyn Zevenbergen critically examine the idea that a student's mathematical success is due to some innate or biological characteristic — most often called 'ability'. They take issue with the context within which learning mathematics is located and with the practices within those contexts. That some students may be more successful than others is conceptualised and analysed through the understanding of the ways in which classroom practices may hinder or facilitate students' success. More precisely, they undertake an analysis of classroom talk using the theoretical tools offered by Bernstein. They reveal how the language of schools and curriculum of a particular register is replete with cultural assumptions built into it. When students come into classrooms, mathematics included, they are immersed in a language environment that is more or less familiar to them depending on their own backgrounds. School (mathematics) practices can marginalise students whose home language is different from dominant school registers. Thus, success in the mathematics classroom is related to the students' capacity to master the discursive rules of recognition and realisation that regulate classroom forms of

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talking and interacting. Some students, particularly middle and upper class students can recognise those rules and manage to master them —and as a result master school mathematics—, while working class students, socialised in very different discursive practices, may never come to grasp the rules that direct school (mathematical) language. The possibilities for them to gain powerful positions thus become limited. With this analysis, it is possible to recognise the impact of the social milieu of classrooms and practices within those classrooms and how they can exert influence over who is seen to be 'mathematically able' and who is not. The critique of the assumption of individual ability as the essential marker of high performance opens a space to understand how mathematical learning is constituted in classroom practices which embody some of the mechanism of social control and differentiation that appear more evidently in other social contexts.

The chapter by Dylan Wiliam, Hannah Bartholomew and Diane Reay examines the practices of assessment in mathematics education. Their argument starts with a careful scrutiny of the assumption of objectivity that underlies the construction of assessment systems: as social constructions obeying to particular values and goals, an understanding of assessment techniques can never be separated from the historical and social context in which they are created and used. The belief on 'neutral' assessment is completely unacceptable if the whole issue of what it means 'to assess' is considered from a socio-political viewpoint. If assessment as a whole should be examined in relation to the ideologies that they purport, the it is also indispensable to understand how the implementation of particular kinds of assessment in mathematics shape the realities of mathematics classrooms. In other words, breaking the assumption of objectivity in relation to assessment tools invites a questioning of the consequences of assessment in mathematics education practices. Some research work has already started to point out the catastrophic effects of assessment in students' perception of themselves as learners. The authors continue this type of exploration and, using data from classes where students have been ability-grouped according to assessment results, they argue that students build images of themselves not only as learners but also as persons out of their interpretations of the mathematical assessment in the school. The consequences of mathematical education go beyond classroom settings and, in association with public images of 'success' or 'lack of success', contribute to the constitution of quite a disadvantaged positioning for these students. The analysis confirms that, on the one hand, different levels of context are implicated in the constitution of mathematics education practices, and that, on the other hand, those practices are an arena of power where participants are constantly struggling with their own positioning in them.

Both chapters raise issues about how the social practices of school mathematics impact students' participation, about how students are judged by teachers and assessment schedules, and how such assessments shape the students' positioning as learners of mathematics. Both chapters show the subtle ways in which taken-for-granted practices —classroom interactions and assessment practices— considerably influence the way in which students come to see themselves as learners of mathematics.

STEPHEN LERMAN AND ROBYN ZEVENBERGEN

THE SOCIO-POLITICAL CONTEXT OF THE MATHEMATICS CLASSROOM

Using Bernstein's Theoretical Framework to Understand Classroom Communications

One of the perennial problems faced by mathematics educators is why so many students appear to fail mathematics. This is particularly evident in these new times when economic rationalism is entrenched in the educational reforms nationally and internationally. Numeracy is a key feature of the reforms and hence serious considerations are made of what it is to be numerate, but with little consideration of the social context within which judgements about levels of numeracy are being made. Most of the literature tends to examine the problem from an individualistic level, particularly from a psychological base, and through seeing mathematics as central to the problem. From such perspectives, the 'blame' for failure is often placed with the victims, and engenders a deficit model of thinking. In contrast, we examine critically the social context within which mathematics learning occurs in order to identify and understand aspects of the classroom milieu and how they impact on students' performance.

Central to our work are the theoretical tools provided by Bernstein (2000). His work is extensive in examining the sociolinguistic aspects of schooling in order to reveal how social advantage and disadvantage are reproduced through the practices of schooling. It is not our intention to apply Bernstein's ideas in novel ways in our chapter: this we have done elsewhere (Zevenbergen & Lerman, 2001; Morgan, Tsatsaroni & Lerman, in press). Rather it is our intention to demonstrate how Bernstein's ideas can help us within the community of mathematics education researchers to identify practices in the classroom and how they constitute particular forms of working and being. As a consequence of being able or not to participate in these practices, students are more or less likely to be constructed as learners of mathematics. These practices often remain transparent to teachers and educators; they have little to do with mathematics or cognition but are intrinsically related to the social context of mathematics classrooms. The ways in which this occurs have been identified as being related to the structuring practices of school mathematics — that is, it is bound to the social and political context of learning. Bernstein's thesis, with which we concur, is that the practices of classrooms are cultural representations that are more or less accessible to students based on their cultural and social backgrounds. Being able, or not, to participate in the social milieu of the

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mathematics classroom demands particular knowledge of the unspoken —and untaught— rules of classroom culture. Those students who are able to identify and participate within the dominant, yet invisible, practices of school mathematics are more likely to be constituted as effective learners of mathematics than those who are not.

There is a small but growing body of work in mathematics education that specifically addresses this issue. In the UK, Dowling (1994, 1998) examined the ways in which textbooks for different abilities position students; Brown (1999) studied parental involvement in their children's learning of mathematics; Cooper and Dunne (2000) explored the national system of testing in Britain in Key Stage 3[1] mathematics, and more specifically the way different items are structured — differentiation in classification and framing values. The results of this latter study showed that certain categories of children do not recognise the context of the question and therefore their answers draw on everyday resources rather than the specialised resources of mathematics (see also Holland, 1981). In Australia, Singh (1993, 1995) examined student communication in computer settings in primary school classrooms. In South Africa, Ensor (1999) examined teacher trainees' recontextualisation of their teacher training courses to the school site. Recontextualisation might be rendered simply as the subordination of the practices of one activity, for example the intellectual activity of producing mathematical knowledge, to the principles of another, such as school mathematical activity (Bernstein, 1990; see also Dowling, 1998). Ensor's use of this term to describe practices in, on the one hand, teacher training departments and, on the other, schools, is a further development and testing of Bernstein's model. Lubienski (2000) and Zevenbergen (1998, 2000a, 2000b) have addressed aspects of the social context of mathematics classrooms and Lerman and Tsatsaroni (1998) have discussed why children fail in mathematics. These authors have indicated ways in which practices in mathematics are biased towards the middle-classes.

In terms of this paper, we will use a number of examples and draw on students' responses to mathematics questions in order to highlight aspects of Bernstein's work that we find most useful for analysing social and political aspects of mathematics education. As such, the theoretical model offered by Bernstein provides grounding for understanding aspects of classroom talk. A Bernsteinian perspective has as central the notion that the practices within schooling 'work selectively on how the student is controlled in the classroom' (Davies, 1995, p. 147) particularly in relation to social class, gender, ability and ethnicity.

TOOLS FOR ANALYSIS

Bernstein (1981) uses the notions of 'classification' to refer to the underlying principle of the social division of labour, and 'framing' to refer to the interrelationships of its own discipline. As a discipline, mathematics is usually strongly classified —that is, there are clear demarcations in schools between mathematics and other disciplines. School timetables reflect the demarcation, and

[1] This stage is from year 7 to year 9, the key stage test is at age 14.

teachers' and students' identities are constituted by it. Strong framing occurs where there is a clear division of labour between the students and the teacher, with the teacher assuming a dominant position of authority in the class. Traditionally, school mathematics is seen to be strongly classified and framed as it is often taught as a discipline quite distinct from others, and taught in a way where there is an emphasis on specialised skills.

Bernstein (1996) is detailed in explaining how power and control are translated into different pedagogies; the implications are that if students are to be successful they need to recognise the unspoken, or invisible, aspects of some pedagogies, particularly reform ones, as we discuss later. Two important considerations need to be made: one is how tasks are framed for students —the issue of contextualisation and recontextualisation—, and the second is how they are answered by the students —the issue of recognition and realisation rules.

Contextualisation and recontextualisation

Mathematics problems have been investigated by Dowling (1998) through the UK School Mathematics Project (SMP) series of textbooks, and by Cooper and Dunne (1999) through UK examinations. These authors, using Bernstein's theoretical model, have argued that mathematics is constituted by esoteric knowledge that is then recontextualised into problem form. Following on from Bernstein's fundamental thesis that students from working-class backgrounds generally do not move from one discourse to another when asked to reclassify objects into different systems of classification, this work suggests that some students have difficulty in identifying mathematics problems as essentially mathematical when they are posed in everyday contexts. This thesis raises questions about the posing of tasks into everyday contexts as serving as distractions from the main mathematical underpinnings of the task. For example, let us consider the problem:

> Steve has bought 4 planks of 2,5 m. each. How many planks of 1 m. can he saw out of these planks? (Verschaffel, De Corte & Lasure, 1994, p. 275).

In this problem, there is an expectation that students will treat the task as a school mathematics task and identify the assumptions built into such a task. In contrast, if students see the task as a 'real' one, they can translate the problem according to the demands of the everyday context. Such an interpretation may involve consideration of waste material and a decision that minimal waste is desirable. Therefore one should cut lengths of greater than one metre to ensure limited waste, lengths slightly under one metre so that 3 planks can be sawn from each larger plank —making 12 in total— or assume that there is some mechanism for gluing the leftover pieces together to form two extra planks —to make 10 in total. From a mathematical standpoint, there is only one correct response, that of eight 1-metre planks. However, if the student fails to reinterpret the task as a mathematical one rather than an everyday task, there is substantial scope for considering the everyday demands of such a task, thus potentially producing an incorrect response.

In tasks such as this, or similar tasks —such as the number of buses needed for a certain number of people or how many cans of paint are needed to paint a room—, students need to conduct the arithmetic of the tasks and then to round the answer to the next highest number. Students who round up remainders are constructed as knowing the mathematics of the task. In contrast, other students may construct responses that draw on their practical knowledge —of carpentry, bus hire or painting— so that their responses are invalid in the mathematics context. In a mathematical sense such responses are deemed incorrect as the mathematical discourse invalidates the practical discourse. Some students do not recognise that the tenets of the task are mathematical rather than practical —that is, they fail to recontextualise the everyday task into a mathematical task, instead offering an —incorrect— response to the question.

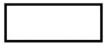
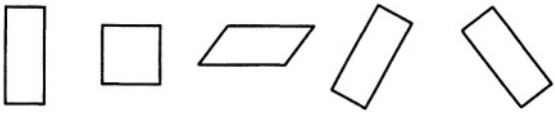
In their extensive work on UK testing regimes, Cooper and Dunne (1999) have appropriated Bernstein's work to demonstrate the effects of social class on performance and report that where questions are embedded in clearly recognisable mathematical contexts, students from working-class and middle-class backgrounds are likely to respond in similar ways. That is, there is little difference in performance on such tasks —tasks that they refer to as 'esoteric'. Such tasks are what would traditionally be described as 'pure mathematics'. What is concerning is that where tasks are embedded in 'realistic' contexts, differences emerge in performance. They argue that the embedding of problems in realistic contexts distracts working-class students from the mathematical demands of the tasks such that they are more likely to read the task as an everyday problem. In contrast, middle-class students are more likely to identify the mathematical discourse and so respond appropriately. That is, they are able to realise legitimate responses to the tasks posed. Cooper and Dunne (1998) challenge a commonly-held assumption that working-class students were more likely to be concrete thinkers due to their perceived slower cognitive development, and hence more likely to perform better on concrete tasks. Their analysis has shown that working-class students may perform equally as well —as a group— as their middle-class peers on esoteric tasks —mathematical ones— but perform less well than their middle-class peers on realistic —or contextualised— tasks due to what Bernstein (1996) identifies as recognition and realisation rules. They conclude that working-class students have been socialised within non-school practices that predispose them to see and respond to the world in different ways from their middle-class colleagues whose experiences aid in the appropriation of school practices. When students fail to identify the recognition rule, they are unable to respond appropriately.

Recognition and realisation rules

One aspect of pedagogy is the rules through which students come to participate in interactions —with the teachers, texts and peers. As we have said, Bernstein (1996) refers to such rules as recognition and realisation rules. These rules occur at the level of the individual. Recognition rules are the means by 'which individuals are able to recognise the specialty of the context that they are in' (p. 31), whereas realisation

rules allow the student to make what are seen as legitimate responses within a particular context. Students may be able to recognise the power relations in which they are involved and their position in them, that is, they may have the recognition rules, but 'if they do not possess the *realisation* rule, they cannot speak the legitimate text' (p. 32, italics in original). For example, with the context of the classroom and an interview situation, students recognise that the teacher has power and that they should conform to expectations. However, when the teacher asks questions or has particular expectations of the students, students must be able to respond in a manner that is seen as appropriate in the classroom. In the mathematics classroom, students must be able to recognise that when a teacher asks a question, there is an expectation of a response. Furthermore, when a mathematics question is posed, they should recognise that there are particular responses that are desired. Common tasks used in mathematics often relate to contextualised questions —such as: 'There are 289 students at the school sports. If a bus carries 45 students, how many buses are there needed to transport the students back to school?' The task requires the students to recognise that it is mathematical, and not about hiring buses. By knowing this, students are able to identify that the response required is one that rounds the answer up to the nearest whole number. By undertaking this deconstruction of the task, students are able to recognise it as mathematical and realise their response within the school mathematical discourse.

The following task was taken from a series of interviews with students from a range of schools and year levels (Zevenbergen, 1991). The students were interviewed individually and asked to respond to the tasks.

> Suppose you had a garden this shape and you were in a helicopter right above your garden looking down on it.
>
> 
>
> Which of the following shapes would be like yours?
>
> 

While many of the students were able to respond to the task correctly, it was more likely that, when incorrect responses were offered, they were from students from working-class backgrounds. Typically, incorrect responses involved answering the question as if it were a task involving identification of the shape of their gardens at home. For example:

Robyn:	Why did you take that shape [the square]?
Girl:	Because it looks like the shape of my garden.
R:	Is your garden at home like that?
Girl:	Yes.

Boy:	None of those.
R:	Why aren't any of them the same?
Boy:	My garden goes like that [draws a semi-circle in the air].

These two quotes indicate that the students selected different shapes from the correct ones in the problem based on their out-of-school knowledge. Walkerdine (1982) argued that students often select the 'wrong' discourse for the selection of responses, based on the use of key words. In this task, the use of 'your' can work as a distracter for the students, so that it cued them to identify the task as being personal rather than mathematical. However, Bernstein's theoretical position offers a richer interpretation of these types of responses where the notion of recontextualisation in concert with recognition and realisation rules need to be considered, particularly bearing in mind the social backgrounds of students.

In the first instance, the task of recognising a shape has been recontextualised into a realistic context. However, questions need to be asked about how 'real' the context is given that many students would not have flown in a helicopter. If the task were to be posed as an esoteric mathematical one where the problem posed referred explicitly to the mathematical demands —such as 'which of the following shapes is the same as this one?'— then the work of Cooper and Dunne (1999) would suggest that students may be able to solve the task since the mathematics was apparent, and the demands of the task were also apparent. Instead, the recontexualising of the task into a realistic context serves as a distracter for some students due to the recontextualising process. Lubienski (2000) reported similar findings where her students from working-class backgrounds preferred esoteric tasks and the middle-class students were comfortable with problem-based tasks which they needed to unpack in order to find the mathematics.

In the second instance the students failed to recognise the context of the question —the question is not asking about their personal gardens, but rather some abstract garden that has nothing to do with them personally. Students need to recognise that mathematics education is rarely a personalised game, but something that is often abstracted from the personal domain. Where questions may be embedded in discourses that suggest, or even encourage, a personification of mathematics, this may not be the case. Indeed, mathematics increasingly becomes depersonalised as the students move through to higher levels of content. For these students —and others—, the incorrect responses indicated a mis-recognition of the context of the problem.

SOCIAL CONTEXT OF CLASSROOMS

Students bring to school very different discursive rules (Heath, 1983) that influence how they act and how actions are interpreted. Lubienski (2000) indicates clear preferences for styles of working and mathematics in her study of a mathematics classroom. In concert with the different backgrounds and discourses that are brought to school, students experience very different classrooms, often based on the teacher's perceptions about students' learning styles. Research suggests that students from lower classes are often exposed to imperative control (Davies, 1995), rote

instruction and low-level exercises with low levels of expectations from teachers (Means & Knapp, 1991). They were more likely to receive rote instruction whereas middle-class peers experienced problem solving (Anyon, 1981). In considering the different discursive backgrounds of students, teachers' perceptions of their students' learning styles —that frequently correlate with the social background of the students—, and the ways in which classrooms and curricula are organised for students depending on their backgrounds, it is also important to take into account the interactions within a classroom.

Questioning is an integral part of classroom practice. Sullivan and Clarke (1991) have been strong advocates of the use of good questions in mathematics classrooms, arguing that some forms of questions are better than others. Posing questions that encourage deep and analytical thinking is seen as preferable for developing desirable dispositions towards mathematics than questions that are shallow and rely on rote learning. Yet, the literature on social class and questioning indicate that there are particular forms of questioning dominant in different classrooms. While some of this is due to teacher preference, Lubienski (2000) argues that students, in her study, cite preferences for different forms of questions: Working-class students prefer the more traditional forms of questions whereas middle-class students are happy with both traditional and problem-orientated questions. Zevenbergen (2000b) has raised similar issues in relation to the ways in which questioning influences how students are able to access knowledge and interact in mathematics classrooms. When this literature is coupled with the research of Heath (1982, 1983), where there are clear differences in the ways in which questions are posed at home than at school, it becomes essential to consider how interactions and questions in the school differentially impact on learners and learning in mathematics.

Bernstein (2000) uses the term 'pedagogic discourse' to refer to the ways in which knowledge is transmitted in the school arena. By this he refers to the ways in which knowledge and cultural values are relayed to students. Rather than considering the two, knowledge and values, as separate, he argues that they are inseparable since it is not possible to convey knowledge without values. When moving mathematics from the mathematicians to the school, the practice of mathematics is recontextualised into school mathematics. This is achieved through the pedagogic discourse. Similarly, as the teacher recontextualises mathematical practices into school mathematics, the pedagogic discourse reconstitutes the practice into a new form. Within the pedagogic discourse, two discourses operate. The *instructional discourse* relays knowledge and more visible forms of values to students. In contrast, the *regulative discourse,* which remains invisible, manifests the criteria by which students are judged as complying or not with the cultural order. Bernstein (2000, p. 34) argues that there is 'no instructional discourse which is not regulated by the regulative discourse'. It is through the regulative discourse that the instructional discourse gains its internal logic —how the interactions and content are framed, sequenced, and delivered. For any curriculum, including mathematics, the regulative discourse contains ideological elements which are often unknown or unrecognised by the participants.

When interacting with students, teachers are more likely to take on the value system of the middle-class, posing pseudo-questions in terms of the regulative

discourse by means of which behaviour is controlled —e.g., 'Would someone like to read it out for me please?' or 'You can do this in small groups'. Such statements or questions are not politically neutral, but are a preferred position within the language system. They convey particular views about what is seen as appropriate teaching practice and what are seen to be appropriate ways of behaving in a classroom. Ethnographic studies (Heath, 1983; Zevenbergen, 1995) have shown that students from working-class backgrounds often appear to 'misbehave' in classrooms but argue that this may not be due to 'bad behaviour' from the students but to a misrecognition of the implicit rules for interacting with the pedagogic discourse. That is, students do not comply with —or do not recognise— the regulative discourse within the pedagogic discourse and, hence, appear to be violating classroom norms. In understanding mathematical interactions within this framework, it becomes possible to analyse classroom interactions quite differently. In our observations of classrooms posing the question 'would someone like to read it out for me?' is often seen as not a good helpful question because it does not meet with the same reaction from different classrooms' clientele. Bernstein's theoretical position allows us to theorise that posing questions in this way is likely to be unproblematic for middle-class students who are able to recognise the implicit assumptions in the question, whereas working-class students are potentially likely to interpret the question literally. Posing this type of question may be seen as a representation of the value system of the middle-classes and as such, the regulative discourse is apparent —the type of questioning is imbued with middle class values. Students who do not respond 'appropriately' may be misinterpreting the regulative discourse by using the literal translation of the question. Posing this type of question may be 'bad' form, but it is a middle-class representation and as such, when imposed in a working-class classroom, represents symbolic violence rather than bad questioning

Within the regulative discourse, Bernstein (1996) proposes that behaviours are controlled. Students from middle-class backgrounds have experience with the form of command often posed in mathematics classrooms and are able to negotiate its implicit meaning —e.g., 'read out the instructions', 'would you like to open your maths books?'. In contrast students from working-class backgrounds are at risk of not interpreting the cultural demands embedded in the direction-posed-as-a-question and may see that there is an option. It is not uncommon in such cases for the student to be interpreted as a 'behavioural problem' and asked to leave the room when they respond in the negative. In our work with teachers (Zevenbergen, 1995), they often report that even the inclusion of 'please' can be interpreted as conveying an option to participate in the interaction. Many teachers working with disadvantaged students have indicated that they are reluctant to include 'please' within their pedagogic discourse for fear of the potential consequences. Many of these teachers also report that they feel uncomfortable with this deletion from their speech as it is incongruent with their ways of being —a well-mannered (middle-class) teacher.

Consider the extract below taken from a classroom where the teacher is working with students from working-class backgrounds. He introduces each mathematics lesson with a problem and the students are able to solve the problem by any means —within limits.

T: We are going to work in small groups on this problem. Would someone like to read it out for me please?

S: [reads the problem from the chalkboard] A length of rope is cut in half, and one half is used. The one third of the other half is cut off and used. If the remaining piece of the rope is 10 meters, how long was the original rope
[students interjecting]

T: Please be quiet as Steven reads it
[finishes reading problem]

T: OK.... So you've got work out how long the original rope was by following all those instructions. No, you don't tell me now. You can do this in groups. You can talk about it, you can write little notes down on your maths book if you like, or a piece of paper, on your note pad, and well try, but some people think they know the answer straight away, but talk about it with the people and see if they agree with you that that's the length. Alright, now can you see if you can find at least one other person to work with, put your hand up if you can't.
[students begin working in small groups]

(Zevenbergen, 1995, p. 88)

In considering the instructional discourse, the teacher is posing a question and asking students to respond in order to ensure clarification of the task and the demands. This is a common practice and is not unusual. However, from the point of view of the regulative discourse, other issues need to be considered. Questions about whose cultural values are being expressed, represented and valued through the regulative discourse become important. Often the questions being posed are not neutral but particular representations of a cultural system. Through the regulative discourse, teachers are able to validate particular cultural systems while marginalising others without necessarily being conscious of their actions.

Consider the statement 'you can write little notes down on your maths book if you like'. One should consider the differences in the instructional discourse where the teacher is instructing the students to make notes but from the perspective of the regulative discourse, the comments also —through the use of 'if you like'— appear to be placing a softening on the demand. For students unfamiliar with posing options, the option of 'if you like' may be interpreted literally and if they do not like, they may not do it. However, the regulative discourse, through representing the moral order, is softening the language to impose a directive through the posing of a pseudo question.

In contrast, the instructional discourse seeks to control the content of the mathematics lesson. In this example, the teacher states 'So you've got to work out how long the original rope was by following all those instructions'. Here, the problem is stated —find the length of the rope— and how to solve it —follow the instructions. However, the instructions are not evident and from the mathematical standpoint need to be extrapolated from the given information. In this case, the teacher is hinting at a 'working backwards' strategy, although not clearly stating it. The students need to unpack the appropriate interpretation of the teacher's discourse.

As we discussed above, often the mathematics is embedded in a context and students need to recognise the contextual or language cues as well as the mathematical cues.

In considering the ways in which the teacher conveys to students the mathematical content, it is clear that there are a number of issues to be taken into account. In the first instance, there is a need to recognise that within the pedagogic discourse, aspects of the moral —or cultural— order are being conveyed and that this is a dominant feature of the interaction in spite of it being invisible. Second, the practice of teaching mathematics is, in fact, a recontextualising process whereby the mathematics that is often used by mathematicians is unlike that which is undertaken in schools. If school mathematics is identified unproblematically with academic mathematics, often seen as the reification of perceived truths, its authority and status within the curriculum is secure. What is more rather the learning of decontextualised school mathematics is, again unproblematically, seen as simply transferable to situations outside of schooling to be mathematised. The recognition that school mathematics is a recontextualisation of academic mathematics, a process that always carries with it values and ideology, should demystify mathematics within the school curriculum and should suggest that 'transfer' must take place across a boundary whose strength must be noted. Finally, Bernstein refers to the three 'message systems' of curriculum, pedagogy and assessment through which identities of both teachers and students are produced in different pedagogic modes. We have examined teachers' questioning, which is an element of pedagogy, and the content of the curriculum through the garden shapes problem. Wiliam, Bartholomew and Reay (this volume) discuss how different assessment practices emerge as a result of social and political beliefs. Although these authors do not use Bernstein's work to theorise assessment, the framework they use resonates with our arguments here in that we focus on the ways that message systems produce identities in mathematics classrooms rather than drawing on other, perhaps psychological, explanations.

REFORM AND MATHEMATICS EDUCATION

The implications of the issues that we have discussed in this paper bear significantly on considerations of how to improve the teaching and learning of school mathematics. As calls from politicians and mathematics educators increase around the world to make school mathematics more meaningful and relevant to students, we would suggest that some consideration be made of these reforms in light of Bernstein's theoretical model. In proposing reforms that are more 'meaningful' and 'relevant', critical questions need to be posed. For example, critical educators have asked questions about whose knowledge and whose mathematics are represented in the curriculum; that is, to whom is this knowledge relevant? Within mathematics education, Lubienski's (2000) USA research is conducted at a time when inquiry-based mathematics classrooms are being seen by 'reformers' as the desired form of pedagogy for all students. Lubienski questions that assumption and indicates, as Bernstein's work would predict and the many studies reported here support, that students from working-class environments and some minority ethnic groups appear to be disadvantaged. Whilst we would not want to suggest that, as a result of studies

such as Lubienski's, working-class or minority students should receive a traditional diet, the research does imply that some work needs to be done, both theoretically and practically to mitigate the effects of invisible pedagogies —such as through modifying the strength of framing (Morais, Neves & Fontinhas, 1999).

A reform along the lines of that in the USA could not take place in the UK because of the dominance of the official pedagogical field (Bernstein, 2000). In the UK, Government controls what takes place within each classroom, particularly in mathematics classrooms[2]. There is almost no space for resistance or initiative by teachers in the UK whereas the reform in the USA was initiated and driven by the National Council of Teachers of Mathematics. In earlier decades the situation in the UK was different and one can see, for example in the introduction of the New Mathematics in the early 1960s or of problem solving and investigations in the 1970s, the initiatives of mathematics teachers and researchers. The political structure of education in the USA is quite different to that in the UK, being decentralised to individual States and beyond, to school districts. Hence, despite the 'math wars', efforts by the mathematics education community to propagate a particular approach to teaching and learning mathematics continue and indeed develop. A further element in the difference between the UK and the USA is the status of academics active in educational research. Whilst there are critics of that research in the USA, in the UK there are moves from within the Government to take control of the funding for educational research and concentrate its use into 'relevant' research. In Australia the Government already has considerable control of educational research funding. 'Relevance' as a criterion is presented as commonsense and populist, whereas it in fact represents an opportunity for the implementation of the Government's programme. There are a number of other factors, but this is not the place to develop these further. Suffice to say that the concatenation of a range of social and political circumstances have resulted in these differences.

Most classrooms in the USA are still dominated by a traditional performance model (Bernstein, 2000) whereby what is emphasised in classrooms is the production of appropriate texts —in our case mathematical texts— through explicit (visible) pedagogical means. Assessment looks for what is missing in the student's production (for a deep analysis of assessment in mathematics applying Bernstein's theories see Morgan et al., in press). This mode is supported by behaviourist psychology. There are clear demarcations between mathematics and other subjects on the school curriculum —strong classification— and the classroom is strongly controlled by the teacher —strong framing— in terms of what is legitimate as mathematics. The teacher is positioned within a hierarchical discourse where she or he represents the view that the mathematics curriculum is designed in such a way so that students learn the mathematics that they will need for further studies when they grow older. The teacher knows best. In fact the teacher is not as free as this sounds

[2] Since September 1999 a National Numeracy Strategy has led to the implementation of a daily mathematics lesson in every primary (elementary) classroom with content and teaching style prescribed. Whilst it is not legally required, all schools are inspected for achievement of the aims of the Strategy and it is a brave and rare school that will continue to use its own curriculum and teaching styles. An extension of the Strategy is also being implemented in the first 3 years of secondary schools.

and is herself or himself controlled by the hierarchical discourse. This discourse positions the teacher as traditional, with a dualist view of mathematics and of pedagogy, and as teacher-centred rather than student-centred, or using conventional lecture-oriented teaching.

The 'reform' is a quite different model, a competence model, where one looks for the student demonstrating mathematical competences. We should make it clear that we are using Bernstein's terms when referring to 'performance' models and 'competence' models, which are not the everyday sense of these terms. 'Performance' emphasises students' productions, particularly textual, as legitimate text, whereas 'competence' refers to more hidden criteria for legitimacy of their produced texts. Competences are much harder to identify than a student's performance since partial understandings, what a student says rather than writes and so on, might still demonstrate a degree of competence. In general, the criteria for the production of legitimate texts in the competence model are not explicit —invisible— whereas they are explicit —visible— in performance models. Hence teachers require much greater training for this approach, which is, of course, much more expensive. One might conjecture that educational systems that move back from competence models to performance models may be driven at least partly by cost[3]. The particular version of competence model that is represented by the reform is a liberal/progressive mode (Bernstein, 2000). One can identify other modes within the competence model: a populist mode (anti-elitist) such as an ethnomathematical approach, and an emancipatory mode —concerned with indicating sources of oppression— such as critical mathematics (Lerman & Tsatsaroni, 1998). Other modes are certainly possible. The liberal/progressive mode emphasises what is present in the student's text and is supported by a Piagetian, constructivist psychology. Whilst there is still a strong distinction between mathematics and other subjects in the curriculum, that is strong classification, the framing is much weaker, that is control and authority are more diffuse. The control of the production of what are thought of as appropriate mathematical texts appears to be much more open to negotiation and interpretation, especially from an everyday mathematics perspective although it is not so open in actual fact —invisible pedagogy. Students can approve other students' productions and, within limits, what is legitimate mathematically can be classroom specific. The mathematical authority is shared within the classroom, again within limits.

Our point here is that the reform approach arises from an ideological position, just as any other approach to education. There are consequences for differential success and failure that should be examined impartially by researchers. Bernstein's approach enables a systematic analysis of the classroom consequences of ideological positions, bringing together macro-issues and micro-issues.

[3] Bernstein (2000) described the current directions in the UK as a new form of performance model, quite separate from the traditional and supported by Vygotskian psychological theories. See also Merrtens and Wood (2000).

SOME METHODOLOGICAL REMARKS

Readers may have found some of the terms and meanings used in this chapter quite difficult. Bernstein's theory constitutes a *strong grammar,* that is to say the terms have specific meanings and references, and in order to use his ideas, one must do the work required to see how these terms are used in order to develop and apply them. There is an internal coherence to his theory and a researcher must be cognisant of the interrelationships between all his concepts in order to use the theory appropriately. As such, there is an apprenticeship involved in coming to understand how to interpret and apply his ideas. In order to illustrate these issues of methodology, we will refer back to the example we gave earlier, regarding the problem of garden shapes. Bernstein points to the difference between everyday discourses, in which we —as mathematics teachers— might recognise mathematical ideas, and the esoteric, hierarchical, specialised discourses of mathematics —which are recontextualised into school mathematics. Students need to learn to read problems set in the mathematics classroom in the desired way if they are to produce legitimate texts, i.e. 'correct' responses. Students are required to read a question about gardens in an esoteric mode, in other words, in a mode that is expected in the (school) mathematics context, rather than in a realistic mode, as in the responses of the two students to the problem of the shapes of 'their' garden. There are times when it will be appropriate for students to work within the realistic setting, in a classroom investigation of a 'real' problem. However, as Cooper (2001, p. 247) argues, in the situation typified by the gardens problem, students need to 'bracket out considerations that would be relevant in the case of a real "realistic" problem'. We can now apply Bernstein's theory, as in Cooper and Dunne's book (1999), and set up rules so that esoteric and realistic problems can be recognised by the researchers —and readers— and subsequently used to analyse students' responses across a range of questions.

In other situations we might wish to set up a two-way —or more complex— structure, drawing on Bernstein's theory, in order to analyse information collected for some research. For example, in Morgan et al. (in press) we re-examine some interviews with teachers, first carried out by Morgan (1998). The teachers were commenting on students' writing of classroom investigations for a high stakes national examination. The teachers' responses were analysed according to their use of the official criteria distributed by the examination board, as against other unofficial criteria of their own —from a range of possible resources. At the same time the responses were analysed according to whether the teachers were looking for what the students had left out of their answers, from a mathematical point of view, as against looking for what understanding the students had demonstrated in what was present in their writing. This provides a two-dimensional model for analysis.

In these brief comments we have tried to encapsulate the depth and complexity of Bernstein's theory but also to highlight the methodological work that needs to be undertaken to benefit from his framework. Sociological theories, and Bernstein's work in particular, have been relatively under-utilised in the field of mathematics education research. Working with his theory requires immersing oneself within it, developing a model for analysis of a set of observations —turning them into data—

and of recognition rules for identifying how to classify a datum as a particular object, and revisiting and modifying the model through the conversation between the theory and the data (see Brown & Dowling, 1998). We have insisted on explaining the systematic approach of research in Bernsteinian mode here in part because we are concerned that the mathematics education community is often not careful enough about methodological considerations: too frequently researchers move directly into issues of research methods.

CONCLUSION

Unlike other theories in mathematics education where the interpretation of incorrect responses may be based on the adoption of psychological models of learning —such as Piagetian notions of cognitive development where the students are caught in the concrete/abstract divide— Bernstein's theory offers considerable potential to understand the socio-political basis to such differences. Bernstein (1996) found that middle-class students, as young as seven years, are able to privilege official pedagogic codes over local or home pedagogic codes. Holland's (1981) research used the example of classifying foods. She found that middle-class students were more likely to classify them according to food groups —a school-based classification system— whereas working-class students were more likely to offer local classification systems, such as what would be offered as Sunday lunch. Moreover, she noted that middle-class students were able to switch between codes when asked to offer different classifications, whereas this was not the case with the working-class students who tended to rely on local pedagogic codes. Within a language framework, what becomes critical when working with students is to recognise whether or not they are able to read the mathematical context from the 'realistic' contexts in which many problems are set. Put another way, teachers need to understand that part of being able to answer questions correctly is about the students' learning of realisation and recognition rules and not 'simply' mathematical knowledge. The examples provided here give some insight into how the discursive practices in school mathematics can be restrictive for students, depending on their social background.

Students need to be able to recognise that the teacher is embedding mathematical tasks in particular discourses and that these discourses may or may not be relevant to the task. Whereas this is often undertaken under the guise of making mathematics more meaningful and relevant to the life experiences of the students, it often creates another—invisible—layer of disadvantage to some groups of students.

Finally, it should be clear that we are not opposed to one form of mathematical pedagogy or another. In general we align ourselves with those whose orientation is to developing mathematical thinking amongst all students. Taking a sociological view, however, requires a symmetrical analysis of all forms of recontextualisation and an indication of the links between political positions and the regulative discourse. It is only through revealing these links, and their effects on classification and framing, that one can change these latter to enable more students to succeed in school mathematics and thus gain access to symbolic power.

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DYLAN WILIAM, HANNAH BARTHOLOMEW
AND DIANE REAY

ASSESSMENT, LEARNING AND IDENTITY

For most of the last century, educational assessment derived its principal research paradigm from psychology, and while there was some acknowledgement of the differences in emphasis between psychological and educational measurement —as they were termed—, there appeared to be reasonably broad consensus that the theoretical resources developed in psychology were appropriate for addressing problems and issues within education.

Within this paradigm, the creation of tests and other forms of assessment has been regarded as an essentially technical and objective undertaking although there has, during the last quarter-century, been an increasing acceptance that educational assessments have social consequences —people change what they do because of those assessments. In particular, the work of Samuel Messick has shown that any analysis of the validity of assessments that ignores the social context in which the assessments are used is necessarily impoverished. However, most analyses have tended to regard the social consequences of the use of assessments as separable from the technical issues involved in their construction, and even the analyses of assessments that do take into account the social consequences of educational assessments have tended to look at large-scale assessments, and at large-scale effects.

In this chapter, we will argue that there are no such things as 'objective' assessments because their design is governed by considerations of how they will be used. We then use Messick's theoretical framework for validity argument to show that the meanings of educational assessments cannot be separated from their social consequences, and because of the high-stakes contexts in which they are used, assessments frequently come to shape the constructs they are designed to measure. By drawing on empirical work we have undertaken, we then illustrate the power of educational assessments to provide both constraints on, and affordances for, the way that students develop their identities in classrooms. In this way we show that assessment, learning and identity are inextricably linked.

PSYCHOLOGICAL AND EDUCATIONAL ASSESSMENT

The development of educational and psychological assessments have, by and large, been driven by the best of motives. The first systematic assessments were apparently conducted in China, in order to regulate access to the civil service. There was a

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 43—61.

concern that entrants to the civil service were almost exclusively drawn from the ruling classes, and formal testing was introduced in order to find ways of selecting the most talented applicants from all classes of society. Similarly, in the 1930s in the USA, the president of Harvard, J.B. Conant, was concerned that Harvard's students were predominantly the sons of those who had themselves been to elite universities (Lemann, 1999) and wanted to find a way of selecting students on the basis of their abilities.

Thus, although some authors have argued that the development of tests of this kind was linked to the eugenics movement, the concerns of the two projects — eugenics and aptitude testing— were in fact diametrically opposed. The eugenics movement was predicated on an assumption that ability is inherited (see Selden, 1999, for an excellent account). With such a view, aptitude testing is unnecessary, because one can select the most talented students by reference to their parents' achievements. In contrast, the desire for a system of aptitude testing is implicitly founded on a belief that ability is only weakly inherited —if at all— and is therefore, to all intents and purposes, randomly distributed throughout the population, irrespective of ethnicity, sex and social class. The problem is, of course, that 'ability' becomes rapidly conflated with access to educational opportunities, which makes its identification within a population extraordinarily difficult. For those who wish to use an assessment to identify talented individuals, the key concern is that the assessment does, indeed, identify ability, rather than irrelevant features such as the quality of education received.

In the USA, for example, with its highly devolved education system, setting a test that would fairly assess the scholastic achievements of students across the whole country would be impossible since each school was free to set its own goals. Furthermore, a measure of scholastic achievement would be an impure measure. While success in school depends on the capability of the student and on their perseverance, it is also dependent on the quality of teaching and specifically the opportunity to learn (see Bursten, 1992). In a system where it is believed that education should be funded locally, as is the case in the USA, then students from affluent areas are likely to be at an advantage. The response, therefore, was the creation of the Scholastic Aptitude Test —now termed just SAT— which was intended to measure aptitude for higher education irrespective of the quality of schooling experienced. The extent to which such a test does this, is, of course, a question of *validity.*

VALIDITY AND RELIABILITY

In the earliest days of educational and psychological testing, the validity of an assessment was defined as the extent to which it assesses what it purports to assess (Garrett, 1937). Initially, this was simply a requirement for content validity. In other words, the test should assess a relevant and representative sample of the content of interest. This was generally investigated through the use of a panel of experts who would be asked to look at each item on the test and rate its relevance and also, then, to comment on the overall balance of the items in the test. The important point here

is that even in these earliest days, validity was an essentially social construct, depending on the consensus of a panel of experts.

However, we also want tests to be reliable. After all, here is no point in having a 'good' test —that is, one which assesses all the relevant content— if the result of an individual depends as much on chance as on her or his skills and abilities. The reliability of a test can be thought of as a kind of 'signal' to 'noise' ratio, and if we want to maximise the reliability of an assessment, we can do this either by reducing the 'noise' or by *increasing the 'signal'*.

In the context of educational assessment, 'increasing the signal' entails creating a test that maximises the differences between individuals in the same way that in communications engineering, for example, increasing the signal would correspond to maximising the potential difference between presence and absence of signal in a wire. The fact that tests tend to distribute scores across the whole mark range is therefore in no sense 'natural'. It is the result of a decision to improve the reliability of a test not through reducing the error in scores but instead attempting to mask the errors by making the differences between students as great as possible. Nor is the fact that scores on tests tend to produce a 'bell-shaped' pattern in any way natural. It is, rather, the result of decisions about the kinds of items to include. By replacing items of moderate difficulty with harder and easier items, a test with a rectangular distribution of scores can easily be produced.

This process of test construction therefore *requires* the production of tests that maximise the differences between individuals. This, in turn, requires tests to place less emphasis on what is common between students, such as their experience of schooling, because this common experience would tend to reduce the differences between students. After all, if one sets a test in which students were asked to describe the activities in which they engaged during a school day, it is likely that everyone who went to school would pass, and those that did not, would not. The fact that our educational assessments find large differences between students in what they can do is therefore not natural at all, but a direct and immediate consequence of the need for reliability.

For example, many studies have found that schools have comparatively little effect on educational achievement (e.g., Rutter, Maughan, Mortimore & Ouston, 1979). How has this been established? By measuring educational achievement with a test that was designed to maximise differences between *individuals.* Items that assess the common experiences of all students are not used because they do not discriminate, so we should not be surprised to find that the differences between students at the same school —sometimes called the within-school variance— is greater than differences between schools —between school variance. Such tests *create,* and *reify* the constructs they purport to assess.

By this, we do not mean to suggest that constructs such as 'mathematics' are completely capricious. Clearly there are limits to how far a construct can be distorted in the search for reliable means of assessment. But we hope from the technical arguments above that it is clear that a range of social factors intrude into the design of apparently 'objective' assessment instruments.

As an illustration of how ingrained these ideas are, imagine that we wanted to measure differences in students' achievement in schools using different mathematics

curricula. Instead of using conventional tests we could modify a standard test for our purpose by excluding all the items that show little difference between schools, and adapting existing items to place greater emphasis on aspects which differ systematically from school to school. To many people, this 'feels' wrong —it feels as if we are fixing the test to give us the result we want, and that somehow there must be a 'natural' test. But there is not. There is no neutral ground on which we can stand. As Cherryholmes (1989, p. 115) remarked, 'Constructs, measurements, discourses and practices are objects of history'. The meanings that we can legitimately attach to test results are also products of their history, and assessments cannot be understood outside the social context in which they are used, and do not make sense unless the history of that social context is also understood.

Although many, if not most, tests claim to be retrospective in that they purport to indicate the extent to which a student has acquired a certain body of knowledge, they are almost always also used prospectively —for example to select individuals for employment or further educational opportunities. This means that what a test 'purports' to assess is only part of the picture. The information from the test, once in the public domain, can be used in all kinds of ways not foreseen by the constructor of the test. Therefore, if it is to be at all useful, the concept of validity cannot be a property of a test, nor even of the results of a test. To be useful, validity must be, rather, a property of the inferences made on the basis of test results. As Cronbach (1971, p. 447, emphasis in original) noted, 'One validates, not a test, but an *interpretation of data arising from a specified procedure'*.

For example, those who wish to defend the use of an aptitude test such as the SAT for admission to higher education would compare the SAT scores of individuals with their grades in college or university —typically the grades achieved at the end of the first year. A strong correlation between these two measures —often called the *predictor* and the *criterion* respectively— is taken as evidence of the utility of the predictor for selection. A difficulty with this kind of study is that we are getting only half the picture. We do have evidence about how well those actually admitted to higher education did, but we know nothing about how well those not admitted would have fared had they been admitted, and to address this simply by admitting more students into higher education, suspecting that many of them would fail, would be ethically questionable.

Furthermore, because all tests of achievement are, as noted above, impure measures, the relationship between predictor and criterion is likely to be different for different groups. Yet any differences of this kind will be masked if only the overall correlation between predictor and criterion is considered. For example, in the USA, it has been found that while many minority ethnic groups score less highly than whites on the SAT, for a given SAT score the grades achieved by minority ethnic students at the end of the first year of college are *higher* than those of whites. In general it appears that the differences in *criterion* scores between minority ethnic groups are only half as great as those in the *predictor* scores (Hakel, 1997). In other words, using such a predictor for selection would systematically under-represent the potential for success in minority ethnic students. It is for this reason that many of the elite universities in the USA have replaced requirements for specific SAT scores with the requirement that a student is in the upper quartile of their age-cohort at their

school. Of course, this approach, too, is fraught with difficulties. In general, no test can give us access to an 'untainted' evaluation of an individual's capabilities.

Nonetheless, the requirement for a predictor to correlate well with a criterion seems unexceptionable. After all, if the correlation is not good, then one can hardly use the predictor as a predictor! Some have gone so far as to say that, for predictive validity, correlation is the only thing that is important:

> the information about validity is in the correlation coefficients [...] The nature of the measurement is not what is important to this statement. The important fact being reported is that these variables can be used to predict job performance within the limits of accuracy defined by the correlation coefficient (Guion, 1974, p. 288).

This approach to validity certainly seems more objective than simply asking panels of experts to comment on the balance and appropriateness of items in a test. Yet this definition of what makes a 'good test' has further unexpected consequences.

A key requirement to achieving a high correlation between predictor and criterion is that each of the items in the test must *discriminate* well. In other words, we want to make sure that each item on the test is more likely to be answered correctly by those who are good at whatever the test is measuring than by those who are not. Of course, the problem is that we do not yet have a measure of what the test is meant to be measuring —that is, why we are developing the test— so what we do is then to see how well each item correlates with the scores obtained on the other items in the test. Items with poor correlations are then removed, thus increasing the difference between students, so increasing both the reliability, and the predictive validity of the test.

This process, is, of course, the reverse of what is usually imagined as happening. Rather than taking a well-defined domain and devising a test to assess an individual's competence on that domain, the development of the test is driven by the requirements laid down by psychometricians for what makes a good test. When these tests are used in high-stakes settings, students are coached to produce the best possible results, and so the test comes to stand for the entire subject. In this way, tests come to define what they are supposed to measure (Hanson, 1993).

While social and political factors are crucial, if rarely acknowledged, influences on the apparently objective processes of developing both educational and psychological tests, there is a crucial distinction between educational and psychological tests that renders the former even more open to social influences. Most psychological tests are restricted. They are, for the most part, available only to specialists, who must generally receive specific training in their administration before they can be purchased. In contrast, educational tests are widely available, and even when they are restricted, such as is the case with large-scale 'aptitude' tests such as the SAT, they are used in high-stakes contexts and as a result a large number of publications giving students practice in similar items has been produced.

It was this realisation that the quality of assessments could not be understood independently of the social situations in which they are used that prompted Messick (1980) to argue that the ethical and value concerns implicit in educational tests, and the social consequences of their use, should also be considered as part of the key

concept of validity. This was encapsulated in a model of validity as the crossing of two facets: the function, and the basis of test interpretation (see Figure 1).

	Result interpretation	Result use
Evidential basis	Construct validity **A**	Construct validity and relevance/utility **B**
Consequential basis	Value implications **C**	Social consequences **D**

Figure 1: Messick's framework for validity enquiry

The two cells in the upper row (A and B) deal with traditional conceptions of validity. The evidential basis of result interpretation (cell A), encompasses those aspects of construct validity concerned with how well the assessment represents the domain being assessed —often called content validity—, while predictive and concurrent validity —often grouped together as criterion-related validity— can be regarded as aspects of the evidential basis of result *use* (cell B). Construct validity was originally used as a kind of 'leftovers' box for the validation of tests where no agreed definition of test content existed and where there was no widely agreed criterion variable against which to validate the test —e.g., for a test of 'math anxiety'. Over the last forty years, however, there has been increasing agreement that construct validity is 'the whole of validity from a scientific point of view' (Loevinger, 1957, p. 636). Put simply, construct validation is an enquiry into the evidence supporting inferences made on the basis of assessment outcomes.

Messick's contribution was to show that construct validity focused on the *evidential* basis of result interpretation and use, and that an understanding of how tests and other assessments actually function in society requires an investigation of the *consequential* basis as well. Furthermore, these are not separated activities, since the consequential basis of result interpretation and use can affect the evidential basis. For example, those who argue for the use of multiple choice tests in the assessment of mathematical performance use evidence from empirical studies to show that while such tests cannot assess all aspects of mathematics, the correlation between scores on extended-response and multiple choice tests in mathematics is very high. Therefore, they maintain, although multiple-choice tests do not assess all aspects of mathematics, the scores from such tests can be used as a good proxy for those aspects of mathematics not tested.

However, such a claim requires that the relationship of student performance on the tested and untested material remains unchanged. While this might be true in low-stakes settings, such as for the large-scale light-sampling testing such as the United States National Assessment of Educational Progress (NAEP) and international comparison studies such as the Third International Mathematics and Science Study (TIMSS), for those tests where life-affecting consequences accrue to students or

teachers, it is likely that the tests —or more precisely, the use made of information from the tests— will change the behaviour of those involved.

The use of multiple-choice tests as the sole or predominant method of assessment sends the message that the skills assessed in such tests are the ones that really matter. In other words, the tests embody *value implications* about the subject that come to define the subject (cell C). In this way tests that have already defined what they were designed to assess reinforce the idea that this definition of the subject is the one that matters.

These value messages will then influence the actions of teachers and students to place greater emphasis on multiple-choice items, with the social consequence that the kinds of mathematics assessed in constructed-response assessments are neglected (cell D). The strong correlation between the multiple-choice and constructed-response items, which meant that one could be used as a proxy for the other, is now weakened, so that scores on multiple-choice tests are no longer a good guide to performance on other items. The social consequence of relying on a multiple-choice test does not therefore impact just the consequential basis of the assessment's validity. It also fundamentally changes what the test is measuring, and its relationship with other assessments.

The political and social dimensions are important, therefore, not just to understand 'how we got here' in terms of assessment, but also to understanding 'where we are'. The political and social dimensions are immanent in our current assessment practices, not just in what they do but in what they are.

> Belief systems concerning the individual should not be construed as inhabiting a diffuse field of 'culture', but as embodied in institutional and technical practices—through which forms of individuality are specified and governed. The history of the self should be written at this 'technological' level, in terms of the techniques and evaluations for developing, evaluating, perfecting, managing the self, the way it is rendered into words, made visible, inspected, judged and reformed. (Rose, 1989, p. 218)

Objective assessment is therefore not just difficult to achieve, but by definition impossible. Our search must therefore be not to strive to free our assessments from their subjectivities —this can never be done— but rather to understand the origins of those subjectivities, how they arose, what purposes they serve, and perhaps most importantly, who benefits and who does not.

There are several good accounts of how these issues play out on a large scale (e.g., Broadfoot, 1995; Hanson, 1993). However, there are far fewer accounts of the role that assessment plays in shaping the identities of individual students in schools. Therefore, for the remainder of this chapter, we will illustrate some of the themes raised above by reference to the role that assessment plays in shaping the day-to-day reality of classrooms.

In many countries both in the 'North' and the 'South', greater and greater emphasis is being placed on the assessment of students; they are being tested more often, and their performance has important implications not only for the students themselves, but also for their teachers and for the schools they attend. The publication of 'league tables' of school performance and the widening of parental

choice combine to exert considerable pressure on schools to maximise the performance of their students on state-mandated tests and examinations. This pressure is felt by students, and has a major impact on what happens in classrooms. In the remainder of this chapter we will draw on data from two studies to examine the ways in which students' perceptions of themselves as learners are affected by the assessments to which they are subjected. Drawing first on data from a study into the impact of ability grouping in secondary mathematics classes, we consider some of the ways in which dominant images of mathematics as remote, abstract and very difficult are reinforced by assessment procedures which have profound repercussions in schools and dominate the mathematical identities that students are able to develop. However, the impact of assessment regimes is not limited to mathematical identities. In the subsequent section, we illustrate how the pressure to perform well in the national tests for 11-year-olds in one elementary school in England had profound implications for students beyond the subjects being assessed, extending to their potential careers and even raising questions about their moral worth. In this way, assessments influence what is to be learnt, how it is to be learnt, and even what it means to be a learner. Ultimately assessments even shape who you can be.

ABILITY-GROUPING AND ASSESSMENT IN SECONDARY MATHEMATICS

There is a widely-held concern, supported by a significant body of research, that grouping students by ability is divisive, and results in severely limited opportunities for many students, particularly those from working class and some minority ethnic backgrounds. Despite this, in Britain during the last ten years large numbers of schools have reintroduced or widened their use of ability grouping (Boaler, 1997c). The primary reason for this appears to be a desire to boost a school's standing in the 'league tables' of school performance by making the school attractive to middle-class parents, who tend to prefer a high-degree of ability grouping within schools so that the education of their children is not disrupted by less motivated students (Gewirtz, Ball & Bowe, 1995). High stakes assessments therefore have an impact on what takes place in schools, in terms of how grouping is structured and how resources are allocated. Our research suggests that the impact on individual students is also significant.

While in many school subjects, such intensive use of ability grouping is a relatively recent phenomenon, mathematics has long been widely considered to be particularly unsuited to mixed ability teaching. Twenty years ago, when heterogeneous ability grouping was at its most popular, a government report found that 80% of mathematics teachers in England, compared to only 3% of teachers of English, thought that mixed-ability groups were inappropriate for teaching their subject (Her Majesty's Inspectors of Schools, 1979). The most recent data collected by school inspectors suggest that 94% of students in England are taught mathematics in homogenous ability groups in the upper secondary years.

In order to investigate the impact of ability grouping on students' achievement in, and attitudes to, mathematics, we have followed a cohort of approximately 1000 students in six secondary schools as they moved from being taught in mixed-ability groups in their seventh and eighth years of compulsory schooling (Grades 6 and 7) to homogenous ability groups or 'sets' in years 9, 10 and 11 (see Boaler, Wiliam & Brown, 2000 for a fuller account of the study).

While the current pressure to prioritise examination performance is clearly being felt in mathematics classrooms, the reasons for mathematics teachers' near wholesale rejection of mixed ability teaching also relate to the dominant model of mathematics as highly abstract and very difficult. Mathematical ability is seen to be a rare commodity, and most students are assumed to be incapable of making much progress in the subject. Such perceptions were evident among the students we interviewed, and clearly have an impact on their sense of themselves as learners of mathematics.

Interviewer: You don't think you're very good at [maths]?
Dean: No I'm not, I don't really have a natural gift for it I don't think.
I: But you're in the top set.
Dean: I think the only reason I'm in there is because in the first year we had Mr Williams and he said he wanted to push me. He didn't really think I was up to the standard but with a little push I could.

(Dean, set 1, Alder School[1])

Fathima: Also people find maths very hard. There is always a psychological thing in your mind that maths is hard. No matter what, everyone thinks maths is hard. So when you're trying to concentrate you're thinking, no, maths is hard, I don't want to do it.
I: So where do you think that comes from?
Fathima: I don't know, people all around. People —you don't see mathematicians being a normal person— they have to be really big and brainy

(Fathima, set 1, Cedar School)

These perceptions both feed off, and feed into, the prevailing model of mathematics education in British secondary schools, and they are reinforced by assessment practices which emphasise differences between students. The notion of 'ability' is seen to be particularly salient in relation to mathematics, and the gulf between those who do and those who do not possess ability to be enormous —although as we saw earlier, this gulf is in no sense natural but a product of the way that success is defined in mathematics, and the need for 'good' assessments to discriminate between individuals. Whereas many school subjects are taught in such a way that the same activities may be tackled by students in a range of different ways, and at different levels, mathematics is more often taught according to a hierarchical model

[1] The names of students and schools are, of course, pseudonyms.

whereby new learning depends on what has previously been learnt, and it is much more usual for students of different abilities to be set completely different work, thus resulting in curriculum polarisation and restriction of the opportunity to learn.

Furthermore, most mathematics assessments require students to answer closed questions in predictable forms, and so mathematics teaching tends to focus on teaching a range of 'standard' procedures. The emphasis is on learning a series of steps and becoming fluent at applying them so as to obtain correct answers to closed questions.

As discussed above, the influences of these sorts of pressures serve to define the subject as it is taught in schools, and fundamentally change students' experiences of mathematics. In particular, they enshrine particular models of what it means to be successful in the subject, with the result that it is very much easier for some students than for others to regard themselves as being good at maths. As Broadfoot (1995, p. 68) argues:

> In education, as in other areas of contemporary social life, the advent of 'normalizing judgement' makes possible the idea of fixed definitions of competence. This normalizing judgement combines with the idea of 'hierarchical observation' to provide the 'rational authority' for competition and selection. [...] This Benthamite notion of 'panoptic' surveillance, in which individuals learn to judge themselves as if some external eye was constantly monitoring their performance, encourages the internalisation of the evaluative criteria of those in power, and hence provides a new basis for social control.

Consistent with Boaler's earlier study (Boaler, 1997a), our work in the six schools suggests that while these factors operate in all mathematics classrooms, they are particularly salient in the group containing the highest-achieving students. This 'top set culture' tends to marginalise many of the girls who are put in these groups (Bartholomew, 2001). This culture both draws on, and reinforces, notions of the elusive nature of 'mathematical brilliance', and of there being a clear hierarchy of mathematical ability among students, and is fuelled by the emphasis on the speed of working typical of top sets (Boaler, Wiliam & Brown, 2000). The top set environment lends itself to easy competition between students, but the climate is one in which success, and therefore notions of 'ability', is determined by a students' capacity to generate large numbers of correct answers quickly. This reinforces the idea that the students with 'real talent' in mathematics are those who can perform at a high level in lessons without appearing to have to work very hard and, in a reversal of the usual association of bad behaviour with low ability, in top set groups it is often the students who 'muck about' to some extent in lessons who are regarded as being the students with most ability in the subject (Bartholomew, 2001). This resonates with Walkerdine's finding that, while 'hard-working' is seen as a positive trait by teachers in working-class schools, it is viewed negatively in middle-class schools, where academic success is expected of all, and students who have to work hard to achieve that success are regarded as lacking ability. Walkerdine also found that, whereas boys are frequently seen to have mathematical 'flair' regardless of their actual attainment, high achieving girls routinely have their success dismissed as

the product of plodding hard work (Walkerdine & Girls and Mathematics Unit, 1989).

In many schools at which students are grouped according to their ability, the composition of the different groups is sharply polarised along social class lines, with middle-class students concentrated in the higher sets and working-class students in the lower sets (Gillborn & Youdell, 2000; Hallam & Toutounji, 1996; Harlen & Malcolm, 1997; Sukhnandan & Lee, 1998). In mathematics, it is also the case that boys are frequently over-represented in top set classes and this was certainly the case in our six schools. Although within this study, the composition of the top set groups varies considerably from school to school, they are all places where the collection of values promoted speaks to a particular middle-class masculinity. The rationality of mathematics, the image of the 'great mathematician' and the possibility of being regarded as particularly clever if you can do well in mathematics without being seen to take your work too seriously, seem to have a particular potency for middle-class boys. In most of the top-set classes we have observed during the course of this study, the students who are regarded as being the 'best' in the class are those who display most confidence in lessons, who are quickest to find answers, and who make sure everyone else in the group knows that they got there first —often a group of middle-class boys.

Yet these conceptions of success are riddled with contradictions, and it is important to recognise that the 'pecking orders' established in top set groups do not represent an absolute hierarchy of mathematical ability. Rather the 'recognition rules' (see Lerman and Zevenbergen, this volume) focus on particular aspects of mathematical competence, such as speed, memorisation of facts, etc. which are not regarded as central by professional mathematicians (Buxton, 1981). The students who cannot, or for whatever reason do not wish to, respond with appropriate realisations are denied access to the highest status positions.

Those who are seen to have 'real ability' are generally those who can respond with appropriate realisations, but in practice this is often likely to be those students who are best at suspending their disbelief for long enough to apply the realisation rules (see Lerman & Zevenbergen, this volume) they have been taught. Boaler found that, while both girls and boys feel that highly procedural top set lessons limit their opportunities for understanding mathematics, in general boys are more inclined than girls to 'play the game' (Boaler, 1997b). Thus, while many boys appear able to work through a set of exercises without questioning too much, and to derive some meaning and motivation from competing with their classmates, many girls —unable or unwilling to compete on these terms— withdraw in lessons and are perceived by their teachers —and often their peers— to be lacking ability (Bartholomew, 2001; Boaler, 1997d).

As Lerman and Zevenbergen (this volume) point out, these recognition and realisation rules are conveyed to students through the pedagogic discourse, but they have their origin in the assessments that, through the processes described above, come to define what counts as mathematics.

We are not, of course, proposing that the students who are widely regarded as being good at mathematics are in fact less good at the subject than those who are seen to be struggling in lessons. Rather we are suggesting that the culture of top-set

mathematics groups, and of mathematics more generally, reinforced by the assessments that are used, makes it very much easier for some students to believe themselves to be good at the subject than for others. The case of one particular 'top-set' student, Tania, who was able to change significantly her perception of herself as a learner of mathematics illustrates the extent to which the responses of individual students are bound up with the wider context that defines, and constantly reinforces, what it means to be good at maths. In a questionnaire that she completed at the end of Year 10 (grade 9), in response to the question 'what do you think are the bad things about your maths lessons?' she wrote:

> We go though the topics very quickly, without having enough time on one. A lot of the people in the class are naturally very clever, and it is embarrassing to get something wrong in front of them. (Tania, set 1, Willow School)

This reponse was typical of many of the 'top-set' students in this study. However, when she was interviewed in Year 11, she began by stating that her approach to mathematics had changed completely since the previous year. It is interesting to think about the ways in which she was able to change her perceptions of herself as a learner of mathematics:

> I: So something must have changed.
> Tania: My attitude. More thinking about myself than what other people know. That instead of what other people know and what I don't know, it's more what I know. Now I'm concentrating on that.
>
> (Tania, set 1, Willow School)

This sounds like a small step to take. She realised that by focusing on her own progress, rather than worrying that she is performing less well than others, she could concentrate on the areas where she needed to improve. Yet this awareness demanded considerable changes in her understanding about mathematics, and how success at the subject is achieved.

Boaler, Wiliam and Zevenbergen (2000, p. 201) argue that useful insights into the nature of mathematics education could be gained from shifting the focus away from the question of 'ability' and rather think in terms of students 'belonging' to the community of practice (Wenger, 1998) of those who are successful at mathematics. They continue:

> Changing the emphasis from 'ability' to 'belonging' [...] demythologises the special status of mathematics. The idea of 'belonging' immediately raises the question of 'belonging to what?' allowing the possibility of multiple communities of practice, rather than a single monolithic edifice.

It is exactly this 'single monolithic edifice' that leads so many students to believe that there is only one way to do mathematics. The fact that they are unable to compete with the quickest in terms of producing realisations is evidence that they lack that special talent. The dominant image of success is one from which many students feel excluded. In order to incorporate being successful at mathematics as an aspect of her own identity Tania first had to reconceptualise what it meant to be successful —in other words to change the recognition and realisation rules. This

involved dismantling the hegemonic male-dominated model of the brilliant mathematician, and the belief —so deeply ingrained in many of the students we interviewed— that mathematics is something you either can or can't do. In recognising that different students approach mathematics in different ways, she was able to demystify the performance of some of the other students in her class, and, at the same time, to begin to feel more confident in her own abilities:

Tania: There were a couple of lessons where it really sort of hit me as like I was really working hard and I really changed my attitude in maths. I found that the people I thought were so clever —I was getting better marks than them and I was more ahead of them in class, while they were just like chatting. So well I thought, you know...

[...]

Tania: I think that with some people, like the people in my class — the ones people feel threatened by— those kind of people, I find that they'll just stick to it like this is it, this is how you have to do it and you always have to do it like this. Whereas me, I can't do it like that. That's why I bring in old work because I won't be able to answer the question like how they do it. So I'll try and bring in everything I know and try and find an answer.

[...]

I: So, what do you think it is that they do?

Tania: It's like —imagine we're doing an equation or something and we're trying to find a solution to it, they'll say "Here's the formula, this is what you do." Where I would probably go "If I look back at this topic, I can use that to solve this bit" and then I'll do that and then I'll get an answer like that.

(Tania, set 1, Willow School)

NATIONAL TESTING IN PRIMARY SCHOOLS

All students in England and Wales are tested at the age of 11 in English, mathematics and science. Although these assessments are relatively 'low-stakes' for students, in that little in terms of their individual futures is contingent on the results, the stakes are very high for their teachers. Schools whose students are judged to be gaining insufficiently good scores in these tests are subject to 'special measures' involving visits by government inspectors as often as every three months, and if subsequent improvements are deemed too slow, the school can be closed down, and all the teachers lose their jobs. These tests therefore create huge pressure on teachers to improve their results. In our study of a class of 11-year-olds (Reay & Wiliam, 1999) at Windermere School, we found that the teacher in turn placed huge pressure on the students to improve their performance:

> I was appalled by how most of you did on the science test. You don't know anything. I want to say that you are judged at the end of the day by what you get in the SATs[2] and some of you won't even get level two[3].

Some of the students understood that it was the teachers, rather than the students, who were really being assessed in the tests:

I:	So what are the SATs for?
Jackie:	To see if the teachers have taught us anything.
Terry:	If we don't know nothing then the teacher will get all the blame.
Jackie:	Yeah. It's the teacher's fault.
Tunde:	Yeah. They get blamed.
[...]	
Mary:	SATs are about how good the teachers have been teaching you and if everybody gets really low marks they think the teachers haven't been teaching you properly.

However, some felt that although the tests were really assessing the quality of teaching, they could nevertheless impact upon their own lives:

I:	So are they important, SATs?
Lily:	Depends
Tunde:	Yes
Terry:	No, definitely not.
Lewis:	It does affect your life
Ayse:	Yeah, it does affect your life
Terry:	No, as if it means you know I do badly then that means I'm gonna be a road sweeper.
I:	You mean, you think that if you do badly in SATs then you won't be able to do well or get good jobs?
Jackie:	Yeah, 'cause that's what David [the class's teacher] is saying.
I:	What is he saying?
Jackie:	He's saying if we don't like, get good things, in our SATs, when we grow up we are not gonna get good jobs and...
Terry:	Be plumbers and road-sweepers...
Tunde:	But what if you wanted to do that?
I:	Instead of what?
Terry:	Footballers, singers, vets, archaeologists. We ain't gonna be nothing like that if we don't get high levels.
I:	And does that worry you about your future?
Jackie:	Yeah.

[2] National curriculum assessment at the ages of 7, 11 and 14 —the end of each of the first three 'key stages' of compulsory education— consists of two components: a series of judgements made by the school about a student's performance over the key stage, generally called 'teacher assessment', and an externally set standardised assessment. Originally, these were to take the form of a series of tasks, called 'standard assessment tasks' (SATs) rather than traditional tests. However, by the time the system was fully implemented in 1994, the government had replaced the tasks with formal timed written tests. The external components of national curriculum assessment for 11 year olds have therefore never been called 'SATs', but teachers, students and parents continue to refer to them in this way, and so, for simplicity of presentation, we have followed this usage.

[3] Level 2 is the average level of achievement of 7-year-olds.

Lewis: Yeah.
Ayse: Yeah it worries me a lot
Terry: No, because he [referring to the teacher] is telling fibs.

For some, not only were the tests seen as critical filters, affording or denying admission to key occupations, but also as having predictive powers that extended into the moral sphere:

Sharon: I think I'll get a two, only Stuart will get a six[4]
I: So if Stuart gets a six what will that say about him?
Sharon: He's heading for a good job and a good life and it shows he's not gonna be living on the streets and stuff like that.
I: And if you get a level two what will that say about you?
Sharon: Um, I might not have a good life in front of me and I might grow up and do something naughty or something like that.

Now for some students, particularly older ones, a natural response to such a regime might be to resist and to absent oneself from the entire assessment process. As Foucault (1977) has observed, being documented was once the prerogative only of society's elite and even for most of the last century, assessments were used primarily for the minority, for example for entrance to higher education. In such a climate, not to have been assessed was unremarkable, and so such resistance might be a sensible strategy. However, where assessment is universal, then not to be assessed is to be marked. Those who are not assessed are not just lacking in some desired attributes. They are beyond the pale.

This appears to have been realised by some of the students. The tests for 11-year-olds in mathematics were 'tiered' in order to improve their reliability, so that each tier gave access to only a restricted number of the available levels. In order to ensure that students were entered for an appropriate tier, students scoring below the minimum threshold for a particular tier would not be awarded a lower grade but would instead not be awarded a level at all. This was forcefully communicated to students in this school, as the following interview extracts makes clear:

Hannah: I'm really scared about the SATs. Ms. O'Brien [a teacher at the school] came and talked to us about our spelling and I'm no good at spelling and David [the class teacher] is giving us times tables tests every morning and I'm hopeless at times tables so I'm frightened I'll do the SATs and I'll be a nothing.
I: I don't understand Hannah. You can't be a nothing.
Hannah: Yes, you can 'cause you have to get a level like a level 4 or a level 5 and if you're no good at spellings and times tables you don't get those levels and so you're a nothing.
I: I'm sure that's not right.
Hannah: Yes it is 'cause that's what Ms. O'Brien was saying.

I: Norma, why are you worried about SATS now?
Norma: Well, it seems like I'll get no points or I won't be able to do it, too hard or something.

[4] Level 6 is a standard equivalent to that achieved by above-average 14-year-olds.

I:	What would it mean to get no points?
Norma:	Well instead of being level three I'll be a nothing and do badly—very badly
I:	What makes you think that? Have you been practising?
Norma:	No, like I analyse ... I know I worry about loads of things.

These extracts, and others from other students in the class, show that a metonymic shift took place over the year leading up to the tests. From thinking of themselves as students who might *get* a particular level, the students changed to talking about themselves as *being* a level three, four, five or six. The causes of this shift are, of course, complex, but there can be little doubt that a major influence was the culture of the school which had embraced the need to improve its test scores irrespective of the consequence for the students' achievement in wider terms. Students were increasingly valued not for their personal qualities, but rather for what they could contribute to the targets set for the school by the school district. For many of the students in the class, the results of these assessments came to be bound up with not just what kinds of careers might be open to them, but who they were now, who they could be, and even their moral worth. Resistance to this process was not considered an option even by those students who had some insight into the nature and purpose of the assessment, while for others, the prospect of not being given a level at all was clearly worse than getting a level of some kind, no matter how low. This is particularly interesting in that the tier of entry is the decision of the school, rather than the student, and yet, the responsibility for failure in this regard has been effectively passed to the student. Thus despite their insights into this situation, the students accept that this is the way things have to be, and even though they know that the purpose of the tests is to assess the quality of the teaching they receive, failure is taken to be the responsibility of the student.

CONCLUSIONS

In this chapter, we have tried to show how apparently neutral assessments are not objective at all, but rather are 'objects of history' —created to fulfil particular social functions, which have shaped the assessments in particular directions that are not readily apparent. The seemingly innocuous requirement for the results of a test to be reliable requires that the test disperses individuals along a continuum so having the effect of placing a magnifying glass over a very small aspect of human performance, and this is particularly marked in mathematics. It represents a process of 'making difference' where little difference existed before. These hidden biases become especially important when the assessments are used as outcome measures for schooling processes, since the processes used in their development have inbuilt tendencies to maximise the differences between individuals.

At the same time, this process of maximising difference does so in a uni-dimensional way. Rather than maximising difference in terms of the various ways in which students differ, one particular variable is elevated to the exclusion of others. This is then exacerbated even further if only limited forms of assessment —for

example, multiple choice tests— are used, presenting a stark realisation of the Macnamara fallacy[5]:

> The first step is to measure whatever can be easily measured. This is OK as far as it goes. The second step is to disregard that which can't easily be measured or to give it an arbitrary quantitative value. This is artificial and misleading. The third step is to presume that what can't be measured easily really isn't important. This is blindness. The fourth step is to say that what can't be easily measured really doesn't exist. This is suicide. (Handy, 1994, p. 219)

In our study of six secondary schools the huge difference perceived by students and their teachers between students who are successful in mathematics and those who are not is not natural, but again the result of historical forces. Different definitions of mathematics would lead to different assessments that might not distinguish so sharply between students, but might distinguish students who had followed different curricula instead, thus making success a multi-dimensional, rather than a uni-dimensional construct.

Wenger (1988) shows how learning to become a medical claims processor involves adopting the practices of the community of claims processors:

> They learn how not to learn and keep their shoulders bent and their fingers busy, to follow the rules and ignore the rules. They learn how to engage and disengage, accept and resist, as well as how to keep a sense of themselves in spite of the status of their occupation. They learn how to weave together their work and private lives. They learn how to find little joys and how to deal with being depressed. What they learn and don't learn makes sense only as part of an identity, which is as big as the world and as small as their computer screens, and which subsumes the skills they acquire and gives them meaning. They *become* claims processors. (pp. 40-41, emphasis in original)

In the same way, for most students in mathematics classrooms, there is only one way to become successful as mathematics students, and that is to take on the role, the *identity,* of mathematician that is laid out for them by the school. At Willow School, the nature and importance of mathematics assessments promoted —or at least was entirely consistent with— a highly procedural pedagogic discourse, and discouraged approaches which would lead to a more critical view of the nature of mathematical knowledge. For four out of her five years of secondary schooling, Tania believed that she could not be successful at mathematics, that she could not become a participant in this community of practice, because she could not identify with the hegemonic masculine image of mathematics that was communicated to her through the assessments to which she was subjected.

Fortunately for her, she was able to carve out for herself a distinctive mathematical identity, but for a variety of reasons, other students will not have her agency. For them, the choice is either to take on the one particular version of the role

[5] Robert Macnamara was the USA Secretary of Defense during the Vietnam war who argued that the ratio of Viet Cong/North Vietnamese Army losses to USA/Army of the Republic of Vietnam losses was an important measure of military effectiveness: 'Things you can count, you ought to count. Loss of life is one.'

of mathematician that the school lays out, or to disengage and identify themselves as being unable or unwilling to do mathematics. Ironically, these may be exactly the people who would make good mathematicians because of their desire for deep understanding.

At Windermere School, although some students —Terry for example— were able to resist the simple equation of test success with career success, most of the students were not. They genuinely believed that their test results would determine not just the quality of their future lives, but also, in some cases —Sharon for example—, their moral worth. The magnification of difference, along a small component of the whole make-up of an individual, led to a labelling of students so that they came to *be* their levels of attainment.

In each of the two examples presented in this chapter, the assessments used have had a very powerful influence on the identity of students. Assessments serve as the message systems of communities of practice, informing the students about the extent of their participation in the community, but assessments also have an indirect effect on identity through their impact on learning:

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming—to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity. (Wenger, 1998, p. 215)

Assessment, learning and identity are therefore inextricably related. Although they are often taken as unexceptionable, assessments come to define fields of enquiry, and yet apparently innocuous requirements for reliability and validity have profound consequences. Those who end up as 'winners' and 'losers' are in large measure the result of the choices made in creating these assessments. To understand what these assessments do, to understand who can, and cannot be successful, and what that means for them, one needs to investigate the historical and social forces that have shaped those assessments. When 'researching the social', nothing can be taken for granted.

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SECOND DIALOGIC UNIT

FINDING ALTERNATIVE THEORETICAL TOOLS

Researching mathematics education from socio-political perspectives implies recognising that mathematics education is a social and political phenomenon. Such recognition is associated with the choice of theoretical tools based on the assumption that mathematics education consists of socially constructed practices forming power relations among their participants. As it has been argued before (see Valero, this volume), theoretical tools rooted in disciplinary fields such as sociology and critical education have provided inspiration to mathematics education researchers in their endeavour to tackle their gaze of study from socio-political perspectives. This dialogic unit offers a conversation between two chapters drawing on postmodern and poststructural thinking. The chapters invite the reader to reflect about the possibilities that those theoretical frameworks bring for analysing and interpreting the practices of mathematics education in relation to previously existing frames of reference.

The theoretical models that have dominated traditional mathematics education have been limited in their potential to grapple with issues of power, marginalisation and success. The advance of socio-political approaches has demanded different theoretical tools and, as a result, researchers have turned to wider fields of social research in order to appropriate, modify and extend tools, approaches and understandings of issues in mathematics education. Particular —though divergent— theories have been used. The work of critical sociology in the 1970s spawned serious challenges to the broad field of education. Those challenges have been taken, on a limited scale, in the field of mathematics education during the 1990's. Critical theorists have challenged the curriculum and pedagogies used in mathematics education as being hegemonic and reinforcing particular views of the world, creating the marginalisation of other possible worldviews. Poststructuralist and postmodernist theories have also filtered into the ways in which mathematics education is being scrutinised nowadays. As a challenge to critical theories, these more recent trends have suggested that the notions of power and agency have stagnated in the realm of classical Marxist theory, as the world has moved into new economic, social, political and cultural forms. New notions of power, agency and practice have entered the debate and have offered alternatives for understanding mathematics education.

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 63—64.

This debate is encapsulated in the two chapters in this section. Paul Ernest presents a reflection on the significance of postmodern ideas in the field of mathematics education research. He discusses the rejection of the metanarratives that dominated the scientific discourse, but that now are being replaced by a multiplicity of accounts which are also related to social groups and their political agendas. He also points to a fundamental change in the conceptions of practice and knowledge, especially in how the latter cannot be separated from the former when giving an account of the processes of knowledge production and their historical and geographic situatedness. This discussion leads to a careful consideration of context. In agreement with the idea that has already been contended in previous chapters, the inseparability between 'text' and 'context' is reinforced here with a consideration of the actions of 'fragmented, multiple selves-in-context'. Ernest argues that the rejection of a conception of human beings as unitary, rational selves challenges simplified analysis of, for example, mathematics students and teachers. Finally, the adoption of postmodern ideas also includes an examination of the narratives that mathematics education, the field of academic research, constructs. Ernest's discussion touches on one of the points by Popkewitz: the theoretical frames and constructs that emerge out of research are also bounded to particular communities, their ideologies and their way of positioning their discourse as valuable and 'powerful' tools to understand practice. In other words, postmodernism in mathematics education research invites to take the step of studying research in itself in search of a clarification of the networks of power associated to this knowledge.

In contrast to the theoretical discussion in Ernest's chapter, Tony Cotton and Tansy Hardy show the way in which their different postmodern, theoretical 'toolboxes' allow them to read the same piece of text related to an episode of practice differently. In some sense, the exercise that they engaged in exemplifies some of the theoretical points raised by Ernest. Cotton and Hardy start by addressing the debate between critical theory and poststructuralism, and signal the relevance of the notion of 'discursive practice' as a key to alternative conceptualisations of culture and power. The two authors engage in a dialogue among themselves and their use of different theoretical positions —one inspired mainly on Rawls' concept of social justice and the other based on Foucault's techniques of power— to understand the ways in which each manifests different interpretations of text and power. Their chapter highlights the constraints and affordances of the theoretical approaches used in order to interpret the processes of schooling in mathematics. Similar to many other chapters in this book, Cotton and Hardy close their reflection with a discussion of the role of the researcher in the generation of 'images' about classrooms, students and teachers, as well as about the very same process of doing research in mathematics education.

In these chapters, the authors demonstrate the value of theory in analysing practices and discourses within mathematics education. Central to both chapters is the important contribution of these theories in analysing, understanding and theorising notions of power within mathematics education practices, as well as in revealing the positioning of the researcher and his/her activity as part of the knowledge-production —discourse generation— enterprise.

PAUL ERNEST

POSTMODERNITY AND SOCIAL RESEARCH IN MATHEMATICS EDUCATION

Postmodernism is a brash new style, which, in the mouths of some authors, claims to supersede what came before. In my view much of its claimed originality is overhyped, and as a cultural —as opposed to philosophical— discourse it celebrates the hybrid, pastiche, and ironic. These exhibit double —or more— codings as multiple meanings collide. Thus hypocrisy, insincerity and double standards are all potential dangers for postmodernism. If I adopt, for the moment, the standpoint of a cynic, postmodernity can be read as an excuse for inauthenticity and ethical expediency. However, simultaneously, the philosophical discourse of postmodernity, much of which I support, celebrates authenticity, being-here-now (Heidegger's Dasein). These are attitudes that acknowledge our multiple existence in the linked but disparate worlds of our experience: the bodily, mundane, discursive, political, professional, institutional and cultural realms. Thus the irony is that the postmodern celebrates the transgressive —crossing the traditional boundaries of knowledge, values, economics— as well as those of good taste, decency, and morality. And this leaves me feeling split —both a supporter and a critic of the various stances attributed to post-modernism.

This is all heady stuff. But it takes me away from the more down to earth and dare I say it —serious— aims of this chapter. Habermas (1990) distinguishes the philosophical and cultural discourses of postmodernity —and modernity. Most of the above musings are to be found in the cultural discourse. In the present context it is the philosophical discourse I shall draw upon. For the purposes of this chapter I will pick and choose those elements of postmodernity relevant to my project, namely developing the conceptualisation of socio-political and socio-cultural research in mathematics education. In doing so I will not take the space to offer a definitive account of postmodernism —not that I could necessarily supply this. Rather I will devote my space to three themes which, although varying in support from the pantheon of postmodernist and poststructuralist authors cited here, are undoubtedly important in mathematics education.

The three elements or themes I have chosen to focus on are the rejection of metanarratives, a focus on conversation and language, and the knowledge / power / economics nexus.

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 65—84.

REJECTION OF METANARRATIVES

Descartes' (1637) seminal contributions to epistemology heralded the beginnings of modernism. He imagined a logical master plan that could provide indubitable foundations for all knowledge, beginning with a small basis of clear and distinct ideas. This plan was modeled on the axiomatic geometry of Euclid, which Hobbes (1651) called the only true science bestowed on humankind. Descartes' powerful rational vision was to dominate many areas of knowledge, including philosophy, physics, and mathematics. Even Crowley's tract on black magic adopted the axiomatic, modernist style. Thus ironically, irrationality adopts the guise of rationality in modernism.

Modernism sought to recast knowledge using the logical structure of axiomatic geometry as a model, thereby to demonstrate the indubitability of the knowledge claims involved. This model is central to what Lyotard (1984) terms the metanarrative of modernism: the style of narrative of the overarching philosophical discourse used to legitimate scientific and all 'rational' knowledge.

Early twentieth century philosophy of mathematics also reapplied this model to mathematics itself, in the quest for absolutely certain foundations for mathematical knowledge. However the failure to achieve this certainty by the logicist, intuitionist and formalist philosophies of mathematics is well documented (Ernest, 1991, 1998; Kline, 1980; Tiles, 1991). Technical results in mathematics —Indefinability and Incompleteness theorems and Complexity theory— have called into question the modernist metanarrative of certainty. Nevertheless, the legacy of the modernist project in the philosophy of mathematics, and the modernist image of mathematics in general, have lived on throughout the twentieth century and beyond in the guise of professional and popular theories and beliefs of mathematics as an unquestionably certain body of knowledge. Furthermore incompleteness and other limiting results have simply been subsumed into the body of mathematical content without causing too much indigestion, just as irrational and imaginary numbers were absorbed in the past. Within mathematics and philosophy only a vocal minority oppose this received view (Davis & Hersh, 1980; Hersh, 1997; Kitcher, 1984; Lakatos, 1976; Tymoczko, 1986), although in mathematics education and the sociology of knowledge such opposition is more widespread (Bishop, 1988; Bloor, 1991; Ernest, 1991; Restivo, 1992; Restivo, van Bendegem & Fischer,1993; Skovsmose, 1994).

Applications of the rationalist legacy of modernism in the physical, social and management sciences have continued unabated despite technical setbacks like those mentioned above. Science seeks to build unified abstract theories to explain the phenomena of the world and to predict future regularities and outcomes. The absolute space-time framework of Newtonian mechanics may have given way to relativistic and quantum models of the universe, but the laws of science continue to be used to make powerful and widespread predictions, and the metanarratives of certainty and control still dominate. Throughout the 20^{th} century the raw modernist perspective dominated other subjects, such as positivistic philosophy and behaviorist psychology. In modified form it gave birth to Structuralism, that seductive and productive set of theories embodying the structural metanarrative. This included

economic and political theory (Marx), linguistics (de Saussure, Jakobson, Chomsky), developmental psychology (Piaget), psychology (Freud), anthropology (Levi-Strauss), not to mention mathematics (Bourbaki). It also impacted on research in mathematics education both through the human sciences and directly through the influence of mathematics itself. The rational planning model of management, itself a development of modernism, continues to be applied in the government of education, despite radical critiques of its verisimilitude, efficacy and underlying value assumptions (Stenhouse, 1975).

Modernism has thus embodied not only the notion of foundation but also that of rational structure and form. In addition, many applications have included a further idea: that of progress and development. Foundationalism presupposes the logical development of knowledge and structure from an axiomatic foundation. But other theories too, e.g., biological (Darwin), social (Compte, Marx, Spengler), educational and psychological (Piaget), assume that ontogenesis and phylogenesis unfold in a sequence of stages, and that later phases are in some measure more complete or superior than earlier phases of this development. Thus in its various guises modernism embodies foundation, progress, and structure. Coupled with these underpinning metaphors are metanarratives of certainty, determinism, control, uniqueness and boundary maintenance. These metanarratives —or perhaps it is just a single overarching metanarrative— assert not only their own legitimacy but also their superiority and unique access to truth and knowledge. The arrogation of these properties to itself gives modernist metanarratives a political edge. For in the hands of a hard core within professional groups metanarratives maintain themselves and their boundaries through delegitimating and pathologising alternate narratives in their domain of discourse, including such examples as those listed in Table 1.

Mainstream professional group	**Alternate narratives delegitimated/pathologised by metanarrative of mainstream professional group**
Scientists (active in the 'science wars')	Sociology of Science and Studies of Science, Technology and Society
Mathematicians	Fallibilist, quasi-empiricist and social constructivist accounts of mathematics
Medical profession	Alternative and complementary medicine
Traditional Anglo-American philosophers	Postmodernist, relativist and radical feminist philosophy
Positivist educational researchers	Interpretative and alternative paradigm research

Table 1: Alternate narratives and mainstream professional group that reject them

Thus metanarratives assert not only their own legitimacy but also their superiority and unique access to truth and knowledge. Table 1 only represents a limited selection of the sources —fields of study— of alternative narratives. In addition, it is based on an implicit and simplified dual classification of professionals in the fields into mainstream professionals —maintainers of the traditional metanarrative and

pathologisers of the alternative narrative and practitioners— and the radical or peripheral practitioners —proposers of the alternative narratives. As in any historical social setting such binary oppositions are oversimplified because not everyone can be meaningfully classified in this way, and such memberships, even where they hold, vary with time. Thus in the case of educational researchers, interpretative and alternative paradigm researchers, insofar as such a group can be identified, were once marginalised, delegitimated and perhaps pathologised, but in the past decade or so have become part of the body of mainstream educational researchers which is no longer dominated by the metanarrative of positivism.

In the case of scholars proposing fallibilist and social constructivist accounts of mathematics, such narratives have become accepted, at least in part, by a minority of mathematicians (Hersh, 1997; Tymoczko, 1986), although many others still vociferously oppose and pathologise them. However, in this instance where there is conflict, it is often between philosophers and commentators on mathematics, on the one hand, and university mathematicians on the other. Thus fallibilist commentators may be perceived by mathematicians as critics who parasitic on the primary creative activities of the doers —creative mathematicians—, just as artists, writers, composers often see their critics as parasitic on the creative endeavours in their respective fields.

Similarly, postmodernist philosophers are no longer so vociferously excluded from mainstream Anglo-American philosophy. The arguments of Rorty, Derrida and others now being taken seriously enough by mainstream philosophers to be critiqued (e.g., Haack, 1993) and accepted (e.g., Everitt & Fisher, 1995).

In this section I have illustrated how the epistemological stance of metanarratives in asserting their unique access to truth and knowledge is not only challenged by alternative narratives, but also that this logical opposition can be seen in terms of conflicting social groups. Thus logic and epistemology slide into politics.

The postmodern rejection of metanarratives is not, however, just the revolt of a younger generation against the strictures of an older generation. As such it would have no epistemological validity. It stems from a reappraisal of the older metanarratives that are found to be deficient. In the philosophy of mathematics the metanarrative project failed to find secure foundations for mathematics. But this raises the question, are such foundations necessary when the practice of mathematics is alive and well?

> What does mathematics need a foundation for? It no more needs one, I believe, than propositions about physical objects - or about sense impressions [...] The *mathematical* problems of what is called foundations are no more the foundation of mathematics for us than the painted rock is the support of a painted tower. (Wittgenstein, 1978, p. 378, original emphasis)

The seed of destruction of the structuralist metanarratives are already present within them, for when de Saussure distinguished the synchronic and diachronic aspects of language he has already made a place for an evolving historical conception of language alongside as a structural grammatical view. Both structuralist and behavioral theories in the human sciences have an embedded lacuna (Henriques, Holloway, Urwin, Venn & Walkerdine, 1984). This is the space that needs to be

filled by human agency and mind. For example, in traditional positivist educational research, there is no space for the idiosyncratic or spontaneous meanings of the persons investigated; their shifting identities, desires, fantasies, and so on. Thus, although the shift from modernist to postmodernist perspectives can be seen, in some sense, as an epistemological rupture (Bachelard, 1934) or a paradigm revolution (Kuhn, 1970), it is also possible to see them as developments arising from antinomies —seeds that are already present.

In this section I have argued that there is no ultimate secure foundation for epistemology and knowledge. In Lyotard's (1984) terms, there is no overarching metanarrative to guarantee secure knowledge, and educational research must reflect this realization. Indeed what I have shown is that looking at putative metanarratives in action brings us ineluctably to groups of persons in conflict: contesting the legitimacy of ways of understanding knowledge, reality and professional practices. However, what this argument does not dispel is the sense that we do all share an underlying reality, or at least a part of one. Although it may at first seem that this contradicts my rejection of metanarratives, my claims about this shared reality are ontological not epistemological. I claim that, in some unchallengeable pre-scientific and pre-philosophical sense human beings all have the experience of living together on the Earth. As a common species, we share to a greater or lesser extent bodily functions and experiences that make our sense of being who we are and of daily life commensurable. The postmodernist view is bottom-up, in the sense that the given, 'thrown' (Heidegger, 1962), preconceptualised experience of being an embodied person living in some sort of society or 'form-of-life' (Wittgenstein, 1953) in the world is taken for granted. This provides the grounds on which all knowing and philosophy begins, although no essential knowledge or interpretation of the basal lived reality is either assumed or possible. This perspective contrasts with the top-down position of modernist metanarratives in which a legitimating rational discourse and the 'gaze' of a reasoning Cartesian subject is assumed to precede all knowledge and philosophy.

Likewise, postmodernism rejects essentialism: the notion that subjects, among other entities —both human individuals and disciplines or school subjects— have an enduring unified and defining set of properties that characterize the 'essential nature' of the subject. Essentialism fails because of inescapable diversity on both the synchronic and diachronic planes. The synchronic plane is characterized by multiple and contradictory elements —whether they be different social practices or different elements of the self— all contributing necessary aspects whose composite sum sits in an uneasy and perhaps unstable equilibrium under the static label of the subject.[1]

The diachronic plane is characterized by multiple and distinct elements with trajectories that emerge and vary over time propelled by complex interactions and historical contingencies, and not purely by some internal or even evolutionary logic. Thus persons, disciplines and school subjects, such as mathematics, all grow and change as inner forces and developments and external circumstances intertwine and

[1] Recognition of this coexistence of contradictory elements —like the opposite and repulsively charged particles held together by the strong nuclear force of the atom in the nucleus— is part of what Derrida (1976) terms deconstruction.

cross over the shifting boundaries of the subject —flowing one way and ebbing the other.

THE MULTIPLICITY OF PERSPECTIVES/PRACTICES

A central implication of postmodernity is the acceptance of both unity and diversity in knowledge and the social processes of knowledge formation. The unity stems from the fact that all knowledge derives from the experience and actions of persons in social practices. Thus the categorization and division of all knowledge —by the rejected rationalist metanarrative— into rational vs irrational, knowledge vs values, and into the discrete areas of arts, humanities, social sciences, sciences, technology, breaks down. Similarly the division of knowledge into the discrete disciplines and areas of study that include mathematics, physics, education, sociology, psychology, economics, English, French, etc. is revealed as arbitrary and inessential. First and foremost, all areas of human knowing are a unity of interpenetrating, overlapping and shifting knowledges. They may be periodically redefined, reconceptualised, and recontextualised, but they are all founded on the powers of understanding of the human subject, thus connecting all realms of knowledge.

> The various forms of knowledge can be seen in low-level developments within the common area of our knowledge of the everyday world. From this there branch out the developed forms which, taking certain elements in our common knowledge as a basis, have grown in distinctive ways. (Hirst, 1977, cited in Brown, Fauvel & Finnegan, 1981, p. 230)

The assertion of this unity is not the imposition of a new master-narrative. Instead it is the view that human knowing is in the first instance amorphous, undifferentiated and shifting to which the metanarratives of modernity have presumed to impose an unquestioned —and unquestionable— classifying framework. Once this is stripped away we can question the separation of knowledge and values; cognition and affect; fact, theory and practice; discovery and justification; and following, Foucault (1970), knowledge, power and economics. In mathematics we can question the artificial and enforced separation of applied mathematics, university research mathematics, school mathematics, ethnomathematics, accountancy, and so forth, at the same time as we can question whether 'mathematics' names any unified and identifiable central area of knowledge or social practice.

> Subjects [e.g., mathematics] will be regarded not as monoliths, that is as groups of individuals sharing a consensus both on cognitive norms and on perceived interests, but rather as constantly shifting coalitions of individuals and variously sized groups whose members may have, at any specific moment, different and possibly conflicting missions and interests. These groups may, nevertheless, in some arenas, all successfully claim allegiance to a common name, such as 'mathematics'. (Cooper, 1985, p. 10)

At the same time, and perhaps even more dramatically, as this quotation shows there is the implication of diversity throughout knowledge, and indissolubly linked to it, the social processes of knowledge formation —not to mention the diversity of the human subject her/himself, to be returned to below. For knowledge is produced in

social practices and there is great diversity in multi-centred socially situated practices across time, space and institutional locations. Taking this further, the postmodernist view of knowledge is not as some ethereal 'substance' that exists beyond the mundane world in some Ideal space of Platonic forms or Popper's (1979) World 3. Instead, knowledge, like everything else, is grounded in the material world and is located in discursive practices. Another way to describe this is, as Lyotard (1984) does, in terms of Wittgensteinian (1953) language games embedded in human forms of life. Thus knowledge comprises texts —in the broad sense encompassing oral utterances and many other modes of representation and semiosis—, made and received by persons within social/institutional settings with their rules, norms, expectations, interpretations, etc. Since knowledge, i.e. text, is indissolubly linked to its context, it is not something that can be sent by one person to another as a orphan, standing on its own and bearing all its own meaning. The meanings of text —for there is always a wide range of interpretations possible— are shaped and delimited by the contexts of making or utterance and those of interpretation —which need not be the same.

These reflections have immediate implications for education —both the processes of education and educational research. For part of education is concerned with the transmission and communication of knowledge. But knowledge is not fixed, determined and ready-made. For the always-necessary interpretation of text —i.e., the ascription of meaning to it— is always linked to its context of utterance and use. The school maths text, the transcribed teacher talk, written pupil work, interview protocols, etc., all require considerable interpretation by the learner or researcher and no reading can ever be final. Even if there were a 'correct' reading of the meaning of one of these texts, and postmodemity denies this, there would be no means of knowing who had achieved it. Instead what we have is a process of negotiation between participants which may move towards a consensus of interpretation, never forgetting that there is also a background ideology which holds in place a notion of correctness underpinning agreement, which is itself a function of the discursive practice with its own historical trajectory[2].

In the classroom such an ideology holds in place the myth that there are unique meanings and answers and that the teacher has privileged access to them —with occasional lapses. In the discipline of mathematics this ideology maintains the myth that there are self-subsistent entities named by mathematical texts and determinate and true relationships that hold between them —this is the myth of Mathematical Realism or Platonism. In educational research several ideologies subscribe to the myth that theoretical terms —e.g., attitude, belief, intelligence, skill, understanding, giftedness, mathematical ability, etc.— name tangible entities that subsist in individuals —entities that research can uncover and reveal.

The reason that postmodernism perceives and identifies these ideologies and rejects these myths is because of its commitment to the irreducible multiplicity of both practices and perspectives. First of all, taking the variety of social practices as a basic given there is no **a priori** reason to believe that even those that are

[2] Cotton and Hardy (this volume) provide a broader discussion of the role ideology plays in constituting power relations and perceptions of normality in the classroom.

conventionally labelled the same way —e.g., mathematical research groups, mathematics classrooms, educational research projects, to name the three types of category pertinent to the present enquiry— are essentially the same. It is rather that the observed shared characteristics —the emergent family resemblance, to use Wittgenstein's (1953) term, that defines the term in each of these three cases. Thus this first line of argument is ontological —what exists from this viewpoint are the range of lived experiences of social realities, not some rational order embodied in a descriptive framework, i.e., imposed by a metanarrative.

Second, given that for postmodernism meanings depend on the tripartite relationship between text, context and knowers, the multiplicity of interpretation or perspectives is an inescapable feature of all knowing and knowledge. Thus this second line of argument is epistemological and concerns the multiplicity of interpretations and knowledges that humans construct. There is no final touchstone for what is true, valid, or justified knowledge. Various justifications provide stronger and weaker warrants within their contexts of application —some strongly persuasive for a given group in a given epoch— but no justifications are transcendent or superhuman.

The irrevocable multiplicity of practices and interpretations has a powerful impact on research in mathematics education. It means that all practices, whether group —i.e., classroom—, individual subject, or even researcher practices, are all unique, but also indissolubly connected with adjacent practices. This suggests in the first instance that the interpretative research paradigm is the only one that can be used because of its focus on unique individuals and cases. However, this conclusion is unwarranted, even though the ontological and epistemological assumptions normally associated with this paradigm sit more comfortably with the postmodernist stance of relativism. For this consonance does not make the scientific research paradigm unusable to postmodernists. The scientific paradigm can be used *instrumentally* as a means to investigate operational and measurable constructs, i.e., from a hypothetically circumscribed perspective. Thus levels of mathematical achievement, hierarchical theories of understanding or measures of attitudes to mathematics can legitimately be utilized in research provided it is understood that the conceptions are social constructions of limited meaning and of circumscribed validity and not dimensions of some human or superhuman reality. Such an instrumentalist approach to theories and theoretical constructs has long been legitimate in philosophy of science and mathematics (Losee, 1980). Indeed instrumentalism is a central tenet of Dewey's (1920) pragmatist epistemology, so it has a respectable lineage. Within postmodernism, the tentative acceptance of such tools and perspectives as part of a larger repertoire is always legitimate provided that such usages are always ironic or at least perpetually self-critical and self-aware of the very real dangers and limitations involved.

The growing importance and centrality of research methods and methodologies in mathematics education signals another development: inter- and trans-disciplinarity and the dissolution of hard boundaries between fields of knowledge. The range of different research methodologies now in play in mathematics education are drawn from and shared with human sciences —psychology, sociology, anthropology, management studies, cognitive science—, humanities —history,

philosophy, linguistics, Hermeneutics, literary analysis— and draw upon elements of the methodologies of the physical sciences —biology, physics, mathematics. This illustrates how one part of modernism's legacy, the rigid barriers between adjacent fields of inquiry, is dissolving, and an increasing number of interdisciplinary fields of study are developing.

Part inspired by the groundbreaking work in sociology and studies of Science, Technology and Society (STS) much of contemporary thought regards knowledge as historically and culturally situated. Included in this is the recognition that knowledge, money and power do not circulate in different and non-intersecting realms, thus challenging the Cartesian dualism of mind and body. Instead, there is a growing acceptance amongst some, at least, that knowledge, money and power are all materially embodied and that all form an interconnected part of the human world we inhabit. This insight is part of the emerging perspective of postmodernism according to which a number of knowledge fields are being simultaneously reconceptualized as distributed and concretely-based practices. These fields include the following:

1. Philosophical postmodernism (e.g., Lyotard, 1984; Rorty, 1979): Grand logical —'top-down'— metanarratives, like Descartes' rationalism, are being replaced by locally distributed —'bottom-up'— knowledge practices, in which knowledge is produced, shared and warranted in 'local', institutionally grounded linguistic practices. The academic community of mathematics educators may be regarded as one or more 'local' community of this type.
2. Wittgensteinian (1953) epistemology: Meaning and knowledge are regarded as situated in habitually conducted and changing 'language games' embedded in social 'forms of life'.
3. Tacit and personal knowledge (e.g., Ryle, 1949; Polanyi, 1958): These forms of knowledge play a central role in human and scientific knowing, but are not expressible in explicit prepositional form, contrary to the ideals of logical, rational, scientific knowledge. Instead individuals and communities build, use and share this knowledge in their practices and practical activities.
4. Sociology and philosophy of science: There is a new emphasis on the historical, laboratory and rhetorical practices of scientists instead of on overarching theories of method (Feyerabend, 1975; Kuhn, 1970; Woolgar, 1988; Simons, 1989). Thus much of the knowledge of scientific communities is embodied in their communities of practice and discursive practices.
5. Philosophy of mathematics: This embodies a shift of emphasis onto the methodologies and practices of mathematicians away from logical theories of mathematical knowledge and truth (Lakatos, 1976; Kitcher, 1984).
6. Social epistemology (Fuller, 1988; Toulmin, 1972), semiotics (Eco, 1977), and feminist epistemology (Harding, 1991): Parallel 'bottom-up' developments in epistemology have been taking place in these domains, recognising that knowledge is both created and warranted in local communities of practice.
7. Mathematics education: A wide variety of developments hinge on the centrality of implicit knowledge and beliefs at the personal and group level among learners (Glasersfeld, 1995; Hiebert, 1988; Schoenfeld, 1985), teachers (Ernest, 1989a; Jaworski, 1994; Shulman, 1986) and researchers (Ernest, 1998, 1999a). However,

more directly postmodernist perspectives on mathematics education can be found among the work of people who have developed and applied socio-cultural theories and in doing so have recognised some of the key foundation elements of postmodernism. These include, from the learners' perspective Lerman and Zevenbergen (in this volume) and Cotton and Hardy (in this volume), from that of teachers Adler (2001), and for researchers the work of people such as Ernest (1998) and Vithal (in this volume).

These examples illustrate the shared epistemological shift in which different knowledge fields are reconceptualized in parallel ways as comprising multicentered human practices. Within these decentred practices, knowing cannot be divorced from the concrete particulars known. These range from exemplary problem solutions and knowledge of laboratory practice in science (Kuhn, 1970; Woolgar, 1988), through knowledge of particular linguistic practices and speech acts (Austin, 1962). This means that the nature of knowledge is being reconceptualized in parallel ways in many fields; a development that is very important for education in which the selection, recontextualization, and communication of knowledge, as well as the assessment of its acquisition, are central activities.

Given the increase of inter- and cross-disciplinary links within knowledge and practices, this suggests that parallel developments in what are traditionally regarded as disparate fields may reveal and build stronger ties than the old nominal identities that held 'subjects' together. Coupled with further developments such as the shared use of research methods and methodologies across different disciplinary areas this further accelerates the breaking down of the old map of knowledge with its strong borders between different subject 'countries'.

FRAGMENTED POSTMODERN SELF

Just as the disciplinary subject is deconstructed by the gaze of postmodernity, so too is the personal subject. Part of the postmodernist 'revolution' has been to reject the traditional model of the unitary rational self in favour of a multiply fragmented postmodern set of selves-in-context. There are two aspects to this reconceptualisation. First, there is the rejection of an essential unified self. This has been anticipated by a number of seminal thinkers. Hume (1739) rejected the existence of a coherent self-identical and essential self in favour of a stream of mental impressions and events. Freud theorized a multiple self, incorporating three domains, translated for the Anglophone world as Ego, Superego and Id. More recently, cognitive scientists have theorized mind as modular, with local knowledges, skills and agencies in place of single controlling intelligence (e.g., Minsky, 1986; Gardner, 1983, 1987). Since the groundbreaking work of philosophers such as Ryle (1949), Polanyi (1958) and others on tacit and personal knowledge mentioned above, it is increasingly recognised that these forms play an essential role in human and scientific knowing and activity, but are not expressible in explicit prepositional form, contrary to the ideals of logical rationalism. More recently this has lead to the embodied and enactivist insights that individuals have

skills and know-how of which they may not be fully aware and that are manifested only in activities in appropriate contexts and situations. In addition to the deep theoretical and epistemological implications of this shift, it also has important implications for educational assessment. For it entails not only that demands for the explicit statement of educational knowledge are inappropriate tests of learning, but also that both task presentations and the social contexts of testing will impact decisively on how the knowledge mastery is manifested by learners (see Lerman & Zevenbergen, and Wiliam, Bartholomew & Reay, this volume).

Second, there is the growing acceptance of a social view of self. This may be said to originate with Mead, Bloomer and the symbolic interactionists, on the one hand, and the Vygotskian school on the other.[3] Currently, new emphases on situated learning either prioritize context over individual minds, or give it at least equal status (Lave & Wenger, 1991; Wenger, 1998). Wenger (1998) theorizes the development of personal identity in terms of experiences in social practices, and activity in communities. Since individuals are normally active in several social practices this involves multiple aspects of identity. Several different theorisations underpin such views. Wittgensteinian (1953) epistemology regards meaning and knowledge as situated in habitually conducted but organically changing 'language games' embedded in social 'forms of life' —purposive, context-bound social activities. Building on the insights of Wittgenstein and Vygotsky, the Social Constructionists stress the formative import of discourse in the construction of selves (Gergen, 1985; Harré, 1979; Shotter, 1993), and indeed have termed their new approach to the social psychology of the self Discursive Psychology (Harré & Gillett, 1994). An even more radical approach is that of poststructuralism, which theorizes self as distributed over a number of different discursive practices (Foucault, 1972; Henriques et al., 1984). This perspective regards persons as having multiple selves that are elicited and evidenced in different social contexts. Clearly there is a single material human being underpinning these different selves. However, distinct identities are constructed in different discursive practices by the different positionings of the individual through the linguistic and social arrangements in place.

For example, in researching the attitudes of primary school teachers towards mathematics (Ernest, 1989b) I found an interesting contrast between two types of attitudes. Primary teachers in my sample tended to have a negative attitude to mathematics but a positive attitude to the teaching of mathematics. My interpretation of this is that for these teachers, being questioned about mathematics elicited their not entirely successful or happy classroom memories of being positioned as struggling learners of mathematics. In this discourse they would be searching for certainties in a discourse beyond their control, with agendas set by others, and in the case of my sample, without the strong associations of success. In contrast, the second set of questions tap into their positioning in the discourse and practice of teaching mathematics, where they are powerful, authoritative and setting the agenda themselves. The fact that both discourses nominally concern mathematics is

[3] Anticipations of the social origin of mind as internalised conversation can be found as least as far back as Plato (1961, p. 127): 'when the mind is thinking, it is simply conversing with itself'.

secondary. What matters, according to my interpretation, is the different selves constructed through their positionings in the discourses. One is autonomous, powerful and in control and is associated with positive attitudes —the 'mathematics teacher'. The other is weak, subjected and not in control and is associated with negative attitudes to mathematics—the 'mathematics learner'.

This example illustrates how the one and the same individual can manifest different selves in different discursive practices according to their positioning. Associated with these different selves may be different attitudes, abilities and intellectual resources, as Evans (2000) has shown for adult students of mathematics. This raises particular difficulties for the traditional problem of the transfer of learning. For, from this perspective, the knowledge and skills learnt in one context or practice —e.g., school— that are intended to be applied in another context —e.g., work— cannot simply be carried across if the individual and her/his intellectual resources are reconstituted differently. Thus even if two tasks in two different contexts appear to the mathematical gaze to be one and the same task, it cannot be presumed that a given individual will be equally capable with either. For a given individual may well be positioned and hence constituted differently in the two contexts and thus may have access to different resources and skills. It is necessary to build 'bridges' between the two contexts.[4]

In mathematics education the term 'context' can have two distinct meanings: task context and social context. Task context concerns the mode of representation of a task, the way a problem is set, including any external situation to which it apparently refers. Applied problems in the mathematics classroom typically utilize 'context' in this sense, as do numerical situations in a nursery school 'play shop'. To leam to solve mathematical problems incorporating different contexts in this sense is an important part of schooling. It involves interpreting different modes of task presentation, and isolating the mathematical task from the contextual 'noise'. While this may also be part of the second sense of 'context', this is not the most important feature. For the social meaning of context concerns the distinctiveness of social practices with different locations, actors, resources, and purposes. There are two main things a person takes with them into a new social practice. First, their psychological make up —not in some unalterable essential sense— but with a formed if evolving psyche and a still open history of emotional sensitivities and responses. Second, a person also takes with them knowledge of signifiers and some of the associated skills. However different meanings and significations may be activated in the second social practice. A person in a new social context is in some sense the same person, with some corresponding emotional make-up and signifier resources, but open to the development of new facets of the self through new positionings, relationships, activities and meanings.

Thus a postmodernist conception of the fragmented, multiple self has strong implications for the outcomes of learning and the problematic issue of 'transfer' of learning from one social context to another, and in particular from the practice of school learning to other communities of practice.

[4] What this metaphor might mean remains an open question.

A critical reconsideration of the learning subject, i.e., the learner in school and in mathematics lessons, thus involves acknowledging the development of a fragmented postmodern self —multiple selves in different social contexts— that change over time. In the developed world schooling became mandatory in the twentieth century from the junior years —5-7 years of age— to the junior high school —15 years of age or older. This is relatively recent and illustrates how the development of the self —in all its multiplicity— is a contingent historical construction dependent on social developments. Already within this school career multiple identities will be developed, and the 'self as learner of mathematics' identity, important as it may be to the community of mathematics education researchers, is by no means the dominant identity. In addition to 'self as learner of mathematics' —which may be distinguished by either success and positive attitudes or perceived failure and negative attitudes— is the 'self as pupil', 'selves as learners of other school subjects', as well as 'self as boy or emerging man', 'self as girl or emerging woman', as well as identities associated with social groups, deviancy, etc. These last four types of selves have strong presences outside as well as inside school. According to the communities of practice the person belongs to other new 'selves' may be spun out and developed. Thus a consideration of the development of a mathematical identity in learners requires acknowledgement of all of the interconnected complexity of being a person, with both multiple and simultaneous selves as well as the complex trajectories over time.

These considerations raise the question: How is a person's mathematical identity, i.e., their 'self as learner of mathematics', developed? What are the key features of the experiences and communities of practice within the mathematics classroom that aid this construction? This is not to exclude out-of-school experiences as they contribute, but evidently some particular experiences within school are central to this development. The answer I wish to propose is that the central feature is the engagement with a particular form of discourse. Learning mathematics is about engaging with texts in a formal conversational practice employing a full range of semiotic devices, including enactive representations, symbolic apparatus —of the tangible material kind—, spoken and written language, icons, abstract symbols, etc., all within a heavily regulated social context. For example, Wiliam, Bartholomew and Reay (in this volume) show how students' identities get formed when students participate in the practices of school mathematics assessment. This brings in the third key feature of postmodernity, the foregrounding of conversation, language and semiosis.

FOREGROUNDING LANGUAGE, CONVERSATION AND SEMIOSIS

The third element of postmodernity I wish to focus on is its foregrounding of language, semiosis and conversation, with language understood broadly enough to include all forms of semiotic representation.

Although self-awareness in the use of language and other representational media is a feature of late modernism, postmodernity goes beyond this to assert that all knowledge, knowing and communication is bounded by text. Thus the slogan 'all is

text' is a legitimate characterization of postmodernism's epistemological claims. This claim falls into disrepute when it is wrongly attributed to the ontological realm and seems to deny the grounded material nature of being.

In the present context the immediate issue is what postmodernity's foregrounding of conversation, language and semiosis implies for mathematics education. This can be addressed on two interconnected planes, separated by the ambiguity in the term 'mathematics education'. The term signifies both a practice — or rather a set of practices— and a field of knowledge. Mathematics education describes both the teaching and learning of mathematics, to school children for example, and a field of study and professional knowledge: an academic specialism with its own Ph.D.s, conferences, journals and books.

Mathematics education as a discipline or area of knowledge, i.e., in this second sense, is constituted by a cluster of communities of practice in which the productions of text and their communication —conversation— are central. A social perspective on mathematics education research must focus not only on its products —texts, knowledge— and their uses, but also on its producers and users, although neither is a unified and monolithic bloc. Producers of educational research comprise different institutions and communities of practice with complex power relations both within and without their communities. Their most common location is in universities, where typically the concern is with the promotion of research culture. University researchers are dedicated to the production of educational texts with primary attention to their internally defined quality, including meeting standards of presentation, providing theoretical justification, careful use and reporting of methodology, and cautious conclusions avoiding over-generalization. From an insider's perspective such emphasis can be seen as reflecting integrity and a dedication to maintaining quality and pushing back the frontiers of knowledge. To an outsider it can be read as inwardness, utilization of resources to benefit only university researchers, and thus following self-serving values. The nature and value of educational research and its products are thus contested.

External users of educational research include politicians, administrators and teachers and their legitimate primary concern is with the immediate applicability of educational research to help shape educational policies or practical activities in schools. The interrelations between producers and users is complex, and include the exercise of power by commissioning research or criticizing its products —by users— and critiquing or proposing changes to educational practice —by producers. What emerges is far from a simple production followed by use model, such as might be found in industrial production. Instead there are complex contests and interactions in different discourses with distinct underlying ideologies. The products of these discourses are themselves texts and constitute part of these interactions, being discursively saturated and imbued with values and ideologies. An analysis of the different communities involved in mathematics education research and their discourses, ideologies, power relations, contests and the role of texts in all this would be a valuable and enlightening area of study in itself. Recent examples of such contests and conflicts are the 'Math Wars' in the USA, and controversy in the UK over the introduction of the National Curriculum in mathematics (Dowling &

Noss, 1990; Ernest, 1991) and the regulation of primary mathematics teacher education (Ernest, 1999b).

In the teaching and learning of mathematics a central place is occupied by texts, both the discursive representations constructed and utilized by teachers to communicate to learners, and those inscribed by learners themselves to communicate to teachers —or peers and others also involved in the teaching/learning process—, or simply as practice. Teacher 'texts' can include the following:

- The 'live' discourse and representations constructed by teachers in talking, using body language, inscribing on chalkboards, overhead transparencies, computer screens.
- The arrangements of tangible material object such as apparatus, models, displays and including the furniture layout of a classroom.
- Preprepared books, worksheets, computer software, tests and examinations distributed directly to the learners to utilize.

These are distinguished here only for ease of description and the categories interpenetrate each other and the three 'modes' are often all utilized together.

In the teaching and learning of mathematics we can also focus on conversation, the social, context-bound process of exchanging texts or communicating meaning. Mathematics, like any other area of knowledge, is learned through individuals — learners— participating in language games embedded in forms of life. Personal competence in mathematics is acquired through prolonged participation in many socially-situated conversations in different contexts with different persons. Initially, the forms of life are domestic and out of school, and these provide an essential set of capabilities for young persons to enter into the novel, formalized learning settings in schools and other educational institutions. Schools, of course, only represent one cluster of contexts and social practices into which young learners enter into and learn from. These are planned teaching and learning situations in which the teaching of mathematics is deliberate. In the context of such intentional forms of mathematics education —in or out of formal institutional settings— certain individuals — teachers— structure mathematical conversations on the basis of their own knowledge, and texts, as indicated above, in order to offer mathematical experiences to learners, with the overt aim of developing their mathematical competences. They direct, structure and control mathematics-learning conversations to develop the skills of reading and writing mathematical texts both for teaching and assessment purposes. These two functions are irrevocably intertwined, except in their extreme forms where they are temporarily and conventionally separated —e.g. expository lecturing and marking external assessments.

The learning conversation extends beyond the immediate teacher-pupil interaction. In school contexts, there are attenuated conversations including learner-textually presented answer interactions, learner-computer presented answer interactions, learner-peer interactions. In out-of-school contexts there are in addition to the above, learner-parent and learner-significant other interactions.

The public representation of mathematical knowledge as texts within a teaching-learning conversation —including these attenuated textual variants— may be necessary but of course is not sufficient for such knowledge to become personally appropriated by individual learners. Sustained two-way participation in such conversations is also necessary to generate, test, shape and validate mathematical performances. Teacher-pupil dialogue, which is asymmetric in classroom forms with the teacher dominant, typically takes place in at least two levels: spoken and written. In written 'dialogue' pupils submit texts —written work on set tasks— to the teacher, who responds in a stylized way to its content and form —ticks and crosses, marks awarded represented as fractions, crossings out, brief written comments, etc. The overt objective of such conversation is that of ensuring that the learner is making the intended interpretation in reading mathematical texts —tasks— and applying the intended procedures in writing texts —responses, answers. The underpinning aim is that the learner will successfully appropriate collective mathematical knowledge and competences, and not some partial or distorted version. Appropriated mathematical knowledge —insofar as the pupil's skills in reading/writing mathematical text can legitimately be described in this way— is potentially unique and idiosyncratic, because of human creativity in sense-making and the inescapable multiplicity of textual interpretations. The possibility of unintended interpretations is further multiplied because school mathematical knowledge is not something that emerges out of the shared meaning and purpose of a pre-given form of life. Instead it is a set of artificially contrived symbolic practices whose meaning is not already given, but deferred until the future.

Although this formal and structural account makes conversation in teaching and learning sound mechanical, so that perhaps one of the partners, i.e., the teacher, might be replaced by a machine, this is far from the case. Despite its common pathological deformation in educational practice, conversation is fundamentally a moral form, and is not just about exchanging information or giving commands. For it entails engaging with a speaker or listener as another human being, not just as a source or end-user of information, nor as a compliant subject. Thus in education the use of the conversational metaphor in the teaching and learning of mathematics ideally should entail a number of things. For a start:

- Mutual respect and trust between teacher and learner.
- Listening to learners; showing —and feeling— an interest in their views, in their conceptions, and in their sense-making.
- Making teaching into participative and responsive conversation, into a dialogue where there is respect for the learner's intelligence and where there is space for learner initiative too.
- Treating real subjects and content of mutual interest and of mutual benefit.

The overt purpose of the teaching and learning of mathematics is the development of mathematical powers in the pupil. To put it another way, this is the development of the mathematical subject, which is achieved through employing text and engaging in mathematical conversation in which the symbolic comes to dominate. This is achieved through a historical process with pupils working through mathematics texts

over the years of schooling, mostly engaging in repetitive activity in performing and completing tasks. I estimate that between the ages of 5 and 16, the years of statutory schooling, an average British child works on 10.000 to 200.000 written tasks.[5] Thus the mathematics learning career of a child encompasses many thousands of tasks, and working through these texts and conversations —including exposure to demonstrations, exposition, examples, comments, feedback and discussions with teachers, peers, parents and others— results in the development of different capabilities, capacities and an important dimension of the learner's subjectivity. The pupil learns to interpret and respond to, i.e., read and write, mathematical text, not as to a narrative, but as a series of textual tasks. Such tasks require inscription using restricted technical language incorporating new symbols and figures, using regulated standard notations and methods of computation and textual transformation, and the use of minimal forms of expression including the avoidance of deixis involving pronouns and spatio-temporal locators (Ernest, 1998).

In learning to maintain a depersonalized, objectified and standardized discursive style in mathematical writing, the learner is also subjectifying her/himself, i.e., constructing a limited and new self-identity as a mathematical subject. Social regulation in the mathematics classroom provides the context for this discursive regulation, and here emotions, attitudes and affect in general play a decisive part. From the perspective of subjectivity, how the learners feel about their interaction with mathematical text and their conversational engagement with the teacher is of paramount importance, and crucially affects the development of mathematical — textual— capacities. However such outcomes are not determined purely individualistically. Social background and the cultural resources that children bring with them, as well as the social expectations of others, have a major impact on the identities that children develop in schools.

Adopting a postmodern perspective on the teaching and learning of mathematics with its focus on texts and conversation foregrounds new features. It enables a break away from the Idealistic view of knowledge transmission and construction presupposed by the individualistic psychological discourses of cognitivism and constructivism.[6] Instead of seeing learning and knowledge as something that is acquired, it focuses on subjectivity as emerging from long-term engagement with text. Instead of seeing understanding as the goal of education, it focuses on the emergence of agency in the mastery of a symbolic, textual system. Meaning making is of course one aspect of this, but ironically only in mathematics does effective functioning also require the deliberate laying aside of meaning in the traditional sense. As King (1982, p. 244) observed, '[o]nly in mathematics could words be left

[5] This estimate is based on the assumption that children each attempt 5 to 50 tasks per day, and have a mathematics class every day of their school career.

[6] These perspectives presuppose an underlying metaphor of the mind as a container separated off from the material world. Transmission perspective see knowledge items as sent or copied into the container. Constructivism views knowledge as self-constructed within this container. Nevertheless both remain wedded to an Idealistic dual ontology in which knowledge is viewed as non-material, made of a different substance than bodies and books.

meaningless'. The relationship between learners and texts, and how each inscribes the other, is an area that is as yet inadequately explored.[7]

CONCLUSION

Adopting the perspective of postmodernity, even if only temporarily, forces a reconceptualisation of knowledge, learning and mathematics education on us. It requires the relinquishing of the certainties that the metanarratives of rationality provided in mathematics, psychology and educational research methodology. Instead all knowledge, text and education can only be accounted for by multiple and contestable narratives, with different social footprints and justificatory discourses (Lyotard, 1984). The traditional division of disciplines and separation of realms of human thought and action are eroded, and multiple maps of knowledge can be drawn. Likewise many traditional concepts are revealed to be shifting in meanings and multiply interpreted over the passage of time.

The foregrounding of conversation and text as central to knowledge, education and the production of subjectivity raises new questions for mathematics education research to consider. However multi-perspectival views of mathematics, learning and research paradigms will not lead to a consensus. For mathematics education is a covert battleground in which the discourses of different practitioner and professional groups compete for dominance.

The lenses that postmodernity provides reveals mathematics education as an ineluctably social field of study and practice. The production and warranting of text and knowledge is a function of communities of practice and their discourses. The redrawing of the map of knowledge is fundamentally tied to the outlines and interactions of these social communities of practice. Text and conversation, so central to education and all of knowledge production, are social tools and practices. Even the production of subjectivity, long held to be the sole province of individual psychology, turns out to be a social process in which text and conversation are deeply implicated. Thus a postmodern view of mathematics education is above all, a social view.

Such a view, however, raises a number of reflexive questions. Does adopting the perspective of postmodernity risk imposing a new metanarrative on the discourse of mathematics education? To what extent does this perspective offer new insights not available from a more directly social view, e.g., from sociology? Given its relativism, does this view retreat from values and fail to address such issues in the deeply value-laden area of education?

There is the danger of postmodernism becoming an orthodoxy with its own characteristic style of discourse, just as over-zealous and inflexible concerns with anti-sexism and anti-racism in extreme cases can lead to a humorless and inflexible Political Correctness. Such postmodernist strategies as deconstruction, critique of essentialism in concepts, identifying lacunae and the unsaid in arguments, and so on,

[7] The same questions can be addressed concerning the texts and subjectivities of researchers in mathematics education. Chapters such as Chronaki (this volume) and Knijnik (this volume) represent a contribution in that direction.

can appear to make up a new style, in effect a metanarrative which assumes a privileged standpoint from which to critique all outside texts. However such appearances would be false. For a postmodernist text to make arrogant claims and expect them to stand would be self contradictory. All that such texts can say is: 'here is a potentially interesting way to see this issue' or 'here are some areas of concern in an argument or narrative'. Are such potentially interesting ways to see issues uniquely in the gift of postmodernism? Probably not, for there are currently so many interesting developments in the social and human sciences that none of the insights or new problematiques suggested above cannot be found elsewhere. But such a judgement should be based on the fruitfulness of different perspectives in suggesting particular areas worthy of attention, not their claims of unique and superior insights.

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TONY COTTON AND TANSY HARDY

PROBLEMATISING CULTURE AND DISCOURSE FOR MATHEMATICS EDUCATION RESEARCH[1]

Defining the Issues; Tools for Research

Currently there is an international focus on problems within mathematics education. There is a political demand for a mathematically literate citizenship who can compete in developing national economies in a global market. For many researching socio-political dimensions of mathematics education, issues of equity and inclusion are to the fore. We argue that for those involved in the activities of mathematics education to address these problems we must acknowledge that all research is deeply contextualised and cultural. In this chapter we set out defining issues for this and discuss what this might mean through a practical example from our own research. Here we try out particular theoretical 'toolkits' to reveal the cultural practices of school mathematics education and how these practices become licensed. We also consider the effects that these practices have on both learners and teachers.

OUR INTENTIONS AND OUR APPROACH

Much mathematics education research has moved to appeal to notions of the 'social' and the 'cultural' to account for the failure of mathematics education to produce mathematically literate citizens. Studies of the social and cultural contexts of mathematics education are gaining prominence in both national and international arenas. However we are concerned that the complexity of working within social and cultural contexts is not fully acknowledged. Superficial applications of social theories give 'flat' accounts of the classroom, free from power and affection, and do not account for a persistent failure to help certain groups of learners. In moving into the realm of the social, society is encountered as organised, ordered communities where knowledge is produced through social practices, and requires a further shift to work with the fields of social and cultural studies as ways of theorising the functioning of any endeavour of human society. This clearly embraces the endeavour that is learning and teaching mathematics.

[1] A previous version of this paper has appeared in Hardy and Cotton (2000).

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 85—103.

In this chapter we argue that those of us involved in the activities of mathematics education[2] need to acknowledge that our work is deeply contextualised and cultural in order to start to address such problems. We examine what this implies for mathematics education by outlining and defining key issues and discussing conceptualisations that we have found valid.

There is a growing body of maths educators in the UK and internationally (e.g., Dowling, 1998; Klein, 1998; Lerman, 1998; Zevenbergen, 1996) who are striving to think differently about mathematics education research, the nature of the objects of its study, its concepts, and its methods of study. This requires the acknowledgement that there is an inevitable privileging and denial in any theory that might be used in research. For example, working with the idea of individual difference can promote the effects of normalisation and produce a deficiency model —using terms such as 'average (levels of attainment)' and 'raising standards' defines what is 'normal' and generates the 'abnormal' ('below average' and 'failing to meet the required standard'). A rethinking might, then, be started by shifting from talking of individual difference to talk of diversity. Such a move can be seen in the work of Walkerdine (1988, 1994) and others who work within critical psychology where they see the individual in the intersection of 'overlapping language games'.

We seek then, to problematise 'the cultural' as manifested in mathematics education research, and to outline a re-examination of concepts and ways of working that this requires. Such a re-examination requires us to make a reflexive move. Where we use tactics that foreground a view of diversity we must look for the tract of this inevitable privileging in the interpretations that we offered and consider the impact that this might have on our meaning making and conclusions.

The specific 'experiment' for this chapter is to draw out particular theoretical notions from the ideas of contemporary cultural theorists and, from these, identify tools with which we can research teachers' and children's practices. These 'tools'[3] orient our examination of how teachers and pupils talk about the positions they take in mathematics classrooms. From this we try to expose the ways in which both are involved in culture creation through classroom practices; and the way particular cultural practices become validated.

This strategy uses extracts from journals and transcribed interviews where children and teachers describe their practices, together with the theoretical notions of 'social justice' and 'regulation of knowledge'. We use two theorised 'toolkits': one drawn from notions of social justice and the other from the conceptions of power and the production of knowledge. We intend to show how these theories can intersect to expose the nature of the cultures created in mathematics classrooms. To do this, the two 'readings' are juxtaposed to consider the multipositioning of both

[2] We try to maintain our multiple positionings within mathematics education in this text and to permit these to be heard in the readings and arguments that we offer here. This gives an intended multivocal nature to this chapter. We are inserted in the text as individual authors and researchers, as well as collaborating authors of this chapter. We relate to this work as teachers and teacher educators of mathematics. This chapter is itself an example of the research practices that we seek to re-examine and our own research activity is forever implicated in the conclusions that we attempt.

[3] The role of theoretical tools in practitioner research is more fully discussed in Hardy and Hanley (1997) and Hardy (1996).

teachers and pupils, to consider who is marginalised and how, and to consider in what senses they are not the 'norm'. This gives some indication of why teachers and mathematics educators are not helping many groups of learners to learn maths.

We also use this strategy to consider valid ways of working as maths education researchers. We have found that maths education research has to engage seriously with post-modern approaches to educational discourse (Ernest, this volume); to think differently about the nature of the objects of its study, its methods, its concepts, in order to tackle issues of mathematical literacy, equity and exclusion. We argue that we must identify ways in which we can become aware of the effects of our practices. Dowling (1991, p. 2) takes our attention to this concern when he refers to Foucault's work:

> People know what they do; they frequently know why they do what they do; but what they don't know is what what they do does. (Foucault & Deleuze, 1972, p. 208.)

It has been suggested (e.g., Bourdieu & Passeron, 1977) that schools serve to reproduce the existing injustices in society through practices seen as common sense in school, but which are based on the class structure present in society. It is this trick of power to masquerade as 'common sense' that leaves participants unaware of the effects of their practices (see Lerman & Zevenbergen, this volume). Similarly researchers in mathematics education must become aware of the effects of their practices, of the effects of how they describe classrooms, children, teachers and mathematics and of how and where they discuss their work —of 'what what they do does'.

Ernest (this volume) looks at the validity of frames that we use. He sets out the theoretical consequences for research deriving from the social practices that are mathematics education and articulates clearly the obligations that the use of postmodern tools places on the concepts of mathematics subject knowledge, learners, teachers and classroom practices. This offers support for our view of the limitations of individualising and monovocal approaches. We build on this theoretical examination to argue the urgency of rethinking and exemplify a strategy for becoming aware of the effects of our teaching practices. With this task in mind, we discuss in the next section how our theoretical perspective and particular strategy can help develop key methodological tactics for mathematics education research interested in issues of inclusion and social justice. For this we consider through what process these practices gain their power and start to think through what we might do differently.

CULTURE, DISCOURSE AND DISCURSIVE PRACTICES

The terms 'culture' and 'discursive practices' are central to the two theoretical frames with which we work. Here we explore these terms and their interconnection briefly.

Culture is an extremely difficult term to pin down. It is particularly difficult; as learners in our schools are at a point of cultural transition, and in many ways operate

at the intersection of several cultures, and indeed, move between cultures depending on the context in which they find themselves (Cotton, 1999). Perhaps rather than work with a single idea of culture we can view learners as belonging to a shifting series of subcultures.

Some views of culture perpetuate the 'naturalness' of certain constructs and will hide the effects of educational practices. For any social endeavour that is researched, the structure that is selected will effect what comes to the attention of the researcher. It will stress some aspects and ignores others. This in turn influences the interpretation made of that site. Structural metaphors then must be chosen with care. For example, a view of 'dominant' culture with satellite (secondary) cultures, offers only one way to gain access to the dominant culture: that is by a complete shift from the subculture to take on all the values of the dominant culture. That structure would be one of exclusivity, dominance, and disconnectedness.

Any structural metaphor also outlines a view of identity. Pupils in schools, as well as their teachers, are engaged in culture creation and through this process identity creation (Wiliam, Bartholomew & Reay, this volume). In our own work we see this as continually shifting and changing and so we seek metaphors that reflect this shifting nature.

Discourse is a highly significant term for our approach. We start from the premise that education practices are discursive in nature; that is, that they 'work' through the language. Hence as we learn to speak in a particular cultural setting we pick up the basic knowledge and rules of that culture at the same time. Discourse in its broadest senses means anything written or said or communicated using signs, including actions in the classroom, resources used, and arrangements of the furniture. Foucault (1972, p. 213) extends this to consider how knowledge is actually produced and induces power through what he calls 'discursive practices' in society[4]. Foucault argues that these discursive practices have profoundly shaped the structure of our society. He shows that all human disciplines define human beings at the same time as they describe them, and that these organised forms of knowledge working together with their associated institutions, have significant effects on people particularly in terms of positioning them in these practices and determining their possible actions.

> The modifier 'discursive' puts conceptual stress on the ways in which all practices are bound up in systems of knowledge. Institutional sites are studied in terms of these rule bound sign systems (discourses) that infuse everyday activities, and that differentiate people in relation to cultural norms that constitute self regulatory ways of knowing. (Zipin, 1998, p. 316)

Consideration of the discursive nature of mathematics education practices foregrounds language and texts. In this we are marking an equivalent shift to that indicated by Ernest (this volume) in his highlighting of conversation. This is a shift away from seeing culturally accepted norms and knowledge as formed and perpetuated through traditions and a shift towards seeing knowledge production

[4] A discursive practice is constituted by actions of the members, their interactions with each other and texts, communications and artefacts from that practice.

coming about through a process of describing and ordering things in particular ways. The constitution of the UK National (School) Curriculum for Mathematics for 1995 (DFE, 1995) is an example of mathematics described in categories: 4 areas of mathematics, each ordered and described for 8 levels of attainment —a 'constructed naturalness'.[5] The nature of mathematics is somehow changed as a result; this categorisation becomes a cultural norm, regulated through descriptions that come to be taken as natural and obvious. It is forgotten that these were constructed (arbitrary) categories. The way that school mathematics is constructed, the determination of children's mathematics and mathematics teachers' consequent actions can all remain hidden. The 1989 configuration with 14 areas and 10 levels is already forgotten (DES, 1989).

Using culture and discourse —products of society that nonetheless shape that society— takes our attention to circulation of power through such practices and the ever present political dimension to their functioning[6].

SOCIAL JUSTICE: USEFUL NOTIONS TO DRAW FROM THIS 'TOOLBOX' —TONY WRITES

Rawls (1971) uses the metaphor of 'halving an apple' to explain the basis of his theory of social justice. If two people are sharing an apple, one person cuts the apple and the other has the choice of which half they want. The theory being that the first person will be as fair as possible in cutting the apple in order to ensure they receive a fair share. For Rawls, to build a just society, we should create a society as if our enemy would choose the position in which we are placed in the social order within that society. He also argues that inequalities in distribution within institutions or societies are only just if they benefit the least well off within that institution or society.

I would suggest that Rawls offers us ways to critically examine our institutions and our classrooms. We can try to apply his tests of justice and consider whether the decisions we take as to the arrangements within our classrooms and our institutions always benefit the 'worst off' amongst our learners. Would we feel comfortable if we thought that our enemies could decide where to place us, or our own children in order for us to learn mathematics within our schools? If the answer to either of these questions is no, in what ways would we alter what we teach, or the way that we teach it to accord to Rawlsian justice?

A Rawlsian view of social justice offers a useful model on which to build a theory of social justice and education, but sees the values of justice and autonomy as moral issues detached from everyday human behaviour. Gilligan (1982) challenged this narrow view of social justice asserting that for many women, the notion of care

[5] Levels 1/2/3 are 'expected' descriptions of attainment for 7 year olds; levels 4/5/6, perhaps for 11 year olds; and level 8 is a high standard for 16 year olds.

[6] Where Foucault uses the term 'discursive practice', Lacan and Marx use 'ideology'. Althusser provides a definition of the joy of ideology as the pleasure, the non-critical tautology, in saying 'yes, the way I see the world IS the world.' Althusser (1994) runs parallel with the compulsive, the necessary with which Foucault invests discursive practices.

is a key to the way that moral decisions are made. The push for autonomy within a society leads to a detached view of an individual, living within a hierarchically ordered society, whereas the values of care and attachment create a world of individuals within an attached network of relationships. Incorporating the idea of 'care' within a social justice framework offers extra possibilities for transformation rather than adaptation. This perspective also begins to suggest a shift from the view of 'equality' as 'equal turns' to social justice as a transforming power.

For me, a social justice model is based on the assumption that people are alike in some ways and different in other ways, 'people should be treated identically in ways they are alike and differently in ways they are not alike. Relevant differences are respected and treated fairly, and justice is achieved' (Bennison et al., 1984, p. 3.) Here, social justice is a way of working that accounts for, and works with, the links between oppressions, inequalities and exploitations that we see inside and outside our schools and classrooms.

This concept of social justice represents a shift in thinking away from equality in classrooms. Equality can suggest a norm towards which we should strive. It does not easily accept and value difference —although attempts have been made to address this issue through slogans such as 'equal but different'. The recognition of connections between the experiences of different groups in our classrooms should in fact lead to a clearer understanding of the oppressions and injustices within our institutions.

Models for research in social justice?

Young (1990) defines 'five faces of oppression' which she suggests will ensure there is no reduction of oppression to a single shared experience. These 'five faces' are exploitation, marginalisation, powerlessness, cultural imperialism, and violence. McCarthy (1990) sees four types of relations within schools that govern actions and interactions between pupils and pupils, and pupils and teachers. These relations govern the production of inequality in the school setting but are nonsynchronous in terms of race, sex and class. The four relations are competition, exploitation, domination and cultural selection. Here we see that educational research for social justice refuses to be tied down to a simple model, but offers us many domains for action.

For McCarthy injustices are played out in the mathematics classroom through the four relations above. Developing these ideas further offers me traits, which may be observable in the classrooms in which I work and which are linked to my model of social justice. A competitiveness that leads to individuals or groups becoming isolated from mathematics and from mathematics learning. This competition can be seen as a competition for access to education, a competition for credentials from education as well as the competition for the scarcity of resources and teacher time. The relationship of domination of one group over another in the classroom; of one teaching style over another; of teacher time; and of resources clearly takes us back to the exploration of power relations. Exploitation in schools is evidenced in schooling as preparation of individuals for 'appropriate' life plans rather than the rational life

plans of Rawls. The relationship of cultural selection has echoes of Bourdieu's ideas of cultural capital.

For this project I will use McCarthy's relationships as a lens through which I can view texts for evidence of injustice being perpetuated through mathematics education practices.

THE REGULATION OF POWER/KNOWLEDGE: USEFUL NOTIONS TO DRAW FROM FOUCAULT'S 'TOOLBOX' —TANSY WRITES

In this section I will discuss Foucault's conception of power and what I believe it can offer mathematics education research.

> Early attempts to do this (inject power, as the energy behind social, economic, and cultural movement, into theory) within educational theory often conceived of power as a scare commodity, like money or cultural capital, which people 'have' in relative amounts. (Appelbaum, 1995, p. 37)

Foucault attempted to rethink the nature of modern power, rejecting totalising schemes that anchored power in ruling or dominant classes and that saw power's effects as entirely repressive He developed perspectives that interpret power as dispersed, productive and dynamic. In this he abandons individualistic ways of viewing power and offers an alternative conception of power —understanding it as a property of relationships, that is, not invested in one individual to exert over another. He writes:

> What characterises the power we are analysing is that it brings into play relations between individuals or between groups [...] The exercise of power consists in guiding the possibility of conduct and putting in order the possible outcome […] (Foucault cited in Dreyfus & Rabinow, 1982, pp. 217-221)

The exercising of power produces what is held to be knowledge; what is the right interpretation; the valued act or utterance within that practice. From this Foucault collapses the distinction between power and knowledge; that is, to begin with a single category of power/knowledge. More complexly, he connects this single category inextricably to a notion of the production of a sense of self that he refers to as 'the production of subjectivity'. For Foucault's analysis these are inseparable facets of the modern human condition. This conceptionalisation of functioning of power provides me with tools with which I can develop a more recognisable account of the effects of the relationships in many mathematics classrooms. Others in mathematics education have found the theories of Foucault valuable in providing new understandings on 'the production of subjectivity' in education. These authors have written about this for mathematics teachers and learners. Walkerdine's (1988) major analysis of 'the developing child' shows the ways in which psychology has produced this 'subject' as its object for scientific investigation. She describes how the practices of mathematics education become sites for the production of 'the self-regulated child'. Dowling (1998) has also written about the production of mathematics and the learner of mathematics within mathematics teaching texts and, for example, Klein (1998) considers the production of the mathematics teacher's

sense of self through investigatory teaching approaches. When these authors write about subjectivity, they inevitably engage with the circulation of power and the production of knowledge.

It is the slippery and inconclusive nature of Foucault's conception of power and its inseparable meshing of the people, their actions, their relations, their subjectivity, their institutions that can cut through the habitual and 'obvious' interpretations made for a site. For these reasons it can be applied valuably to the sites of mathematics education and can offer a fresh view of my own professional contexts.

Foucault (1977a, p. 218) describes particular 'techniques of power' and invites an examination of how power relations function at the micro level within institutions. Gore (1998) did this for sites of pedagogic practice and claims that 'techniques of power' can be seen to appear in sites as varied as a physical education classroom and a feminist reading group. Using these 'techniques of power' as tools[7] gives me a way of looking at teaching and learning interactions, and particularly, teachers' talk about how they plan their work and how they view their practice. In the transcript reading that I offer later in this chapter, I aim to establish whether these techniques of power are readily recognisable in pedagogic interactions in mathematics classrooms.

My theoretical tools

Power is productive

For Foucault power is internally contradictory. Organised forms of knowledge, working together with their associated institutions, have significant effects on people and their possible actions, repressing and enabling.

> If power were never anything but repressive, if it never did anything but to say no, do you really think one would be brought to obey it? What makes power hold good, what makes it accepted, is simply that fact that it doesn't only weigh on us as a force that says no, but that it traverses and produces things, it induces pleasures, forms, knowledge; it produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repressive. (Foucault cited in Rabinow, 1986, p. 61)

When power circulates it determines, to some extent, possible ways of acting and limits what can be done; but it is also the mechanism that enables one to act. My analysis of the discursive nature of professional practices also attends to the simultaneity of how human beings —teachers and children— are defined by discourse's use —that is in fact defined by human beings— whilst at the same time the discourses describe them.

I have experienced this constitutive nature of discourse when, as a researcher observing in a classroom, a child has tried to elicit 'my help':

[7] Foucault in discussing the role of theory (Foucault & Deleuze, 1972b) made the invitation to use his theoretical notions 'as a toolbox'.

> I am watching the interactions of a group of 5 children near to me. One of the children, Sarah, tells another, Tom, that he can't keep having a go, that it's supposed to be turns. She turns to catch my eye. 'It can't always be his choose, can it, Miss? He's got to pass it on. Tell him' and then requests that I 'help' them. I have entered this room as a 'researcher'. I know, however, that I have not shed my 'teacher' self as I pass through the door. I become acutely aware that my response will define who I am in this classroom context. It will also determine my possible future actions. Sarah's query positions me as 'teacher'. Teachers care about fair play. Teachers tell children things. Teachers help. If I help them organise the cards I am indeed a teacher (and will not be able to ignore her subsequent questions). If I stay silence, avoiding her gaze, then I do not help. I am seen not to care about the fairness of their actions - so I am not a 'teacher' (and might not be invited to contribute or to talk with her in the future). I find it very difficult to resist responding (From my research journal).

For this child I will define myself as 'teacher', or not, by my response to her. At the same time I contribute, through that response, to the definition of what a teacher is and does.

Normalisation and surveillance

Any mathematics education discourse positions and categorises children —and teachers— in particular ways. For example, children are often portrayed as the ones who 'have difficulties' or 'misconceptions'. Much of Foucault's work is focused on two particular techniques of power that can bring about this pathologising: *normalisation* and *surveillance.*

The process of normalisation is the mechanism that categorises people into normal and abnormal. Linking the notions of normalisation and power as a productive network reveals the process that determines what is considered to be valid knowledge in the classroom, how that knowledge can be expressed and by whom. It is the process of normalisation that determines who is included and who is excluded in this discourse —who 'has the difficulties' and who does not.

Foucault (1977a, p. 184) also claims that examination '[...] is a normalising gaze, a surveillance that makes it possible to qualify, to classify, and to punish. It establishes over individuals a visibility through which one differentiates them and judges them.' The specific forms of normalisation, individualisation and totalisation, are interesting. For totalisation a group specification is given, asserting a collective character. This forms a readily recognisable element of pedagogic activity where 'we' or a class name is used in addressing whole groups of participants. For example, 'Well done, 3W. I' m pleased with the way that you moved back to your desks'. Individuals and their behaviour is ignored or erased by this statement. It permits regulation of the group behaviour and assertion of the group's characteristics and subjectivities. Children claiming an individual voice could find themselves excluded from the group and from the classroom culture. Individualisation, the technique of giving individual character to oneself, may be an attempt to resist unwelcome totalisation. However it can also be a way of drawing

attention to a child's deviance from the classroom norms and establishing abnormality —again a common classroom practice.

These techniques of power form my toolkit for considering the effects of mathematics classroom practices drawing from the transcript given next in this chapter.

TRYING OUT THE TOOLKITS

Tony uses McCarthy's relationships of competition, domination, exploitation, and cultural selection to look for traits that may be observable in the classrooms in which he works. In the same way Tansy works with a Foucauldian conception of power as productive and looks for patterns in classroom interactions and teachers' descriptions of their work where techniques of power, particularly forms of normalisation and surveillance, might be recognised.

This section is made up of extracts from Tony's research journal. This contains significant —to Tony— sections of group discussion, which he has transcribed. He has also included some of his reflective commentary on these discussions. This is followed by two readings of the text: a social justice reading from Tony, and a Foucauldian reading from Tansy.

Tony's journal

> 22/8/95 Yesterday we had a school-based training day. One of the aims was to try and move away from standardised tests or traditional beginning of term 'find out what they know' tests to assessment based more precisely on the needs of the kids or the information required by the teacher to help them plan.
>
> We offered many exemplars of these types of assessments, which seemed to be enthusiastically received and agreements were reached to trial certain materials. However today some teachers have immediately fallen back on their 'traditional' methods. Do they see training days as detached from their' real lives' of teaching and so forget very quickly?
>
> I notice also how embedded these testing practices are in the learners' image of schooling and learning. The children have to find ways of making sense of these practices within a wider context. —Here, that is a school which professes an ethos of learner-centred teacher behaviour.
>
> 22/8/95 The kids do not seem threatened by the tests however some move themselves to sit next to someone doing the same test so that they could work together, despite (the teacher) deliberately alternating mathematics and language reviews to stop 'copying'. This didn't disturb (the teacher) in the slightest —in fact (the teacher) was quite amused by it.

Although the school ethos of 'learner-centredness' remains, an assessment system which could not be described as learner-centred has embed itself in both learners'

and teachers' common sense. This is only exposed when alternatives are offered to both teachers and learners. This can be seen in the ways that these teachers do not develop learner-centred assessment as part of their everyday practices and in the ways pupils resist the implementation of these testing practices.

At a focus group meeting with 6 11-year old children we discussed the process of SATs (National Standard Testing at age 7, 11 and 14). This is an extract from the discussion:

Group voices:	Err - quite easy
Tony:	Somebody said easy - why were they easy?
Lucy:	I dunno - we've like done em before haven't we?
Mehnaz:	We haven't done 'em before, that was revision.
Rupa:	We did that Science test revision thing.
Tony:	Right.
Lucy:	Science Test B was easiest because we had just done that work really recently.

There followed an excited discussion about what had been easy and what had been hard. Kenny explained that he was trying to forget all about the tests as quickly as possible. There was general agreement about how useful it had been to have covered the topics on Science test B, electricity and evaporation, just before the test. So the immediate reaction appeared to be one of enthusiasm to discuss the process and the questions on the test rather than resistance. Even Kenny and Lucy, previously the most resistant to ideas of testing, joined in enthusiastically. I also noted how at this early stage the practice of doing past papers as revision had already been accepted by the group as 'good practice' and the thing which made the tests easy. The discussion moved on,

Tony:	Do you think the SATs are a good way of testing?
Group (in unison):	Yeah
Tony:	Why do you think they are a good way of testing? Kenny?
Kenny:	Because it will give you a fair idea of what your, like, gonna get, when you are in senior school. When you come to do your exams.
Tony:	Right. Other reasons why they are - go on.
Lucy:	Because, they like see what sort of level you are on, instead of just saying, oh you did a level 2 in your infants so you're all on level 2, you've got to get like a different level.
Tony:	Right so you know where you are. Sairah.
Sairah:	It helps you practice for bigger exams in the future.
Group (several):	Yeah
Tony:	Right, any other reasons. They are reasons why they are a good thing to do maybe. Because it helps you practice, it tells you where you are, I've forgotten what you said Kenny.
Kenny:	It will give you a fair idea of what exams will be like.
Tony:	Right, so now you will all be given a level. Do you think the level you will be given is true, is a fair reflection of what you actually are?
Imran:	No.
Tony:	Let's go round. Imran, you said no. Why is it not?

Imran:	Because on the day your nose might be blocked or you might not be feeling well.
Group (giggles)	
Tony:	Right. So you might not do as well as you could do.
Imran:	Yeah.
Sairah:	Because the test might not be on the things you know, it might be completely opposite to the things you know and there might be some of it on another sheet that you know all about and on another sheet that you don't know anything about so you might get lower marks on one and higher marks on another.
Mehnaz:	Yeah, its like er if you haven't learnt something and like it just like new and your like erm what am I going to do and everything if you haven't like learnt it before.
Tony:	What about you Rupa?
Rupa:	Don't know.
Tony:	If it's not a fair way, what would be a fairer way of finding out what level you are?

The discussion that followed showed the group seeing the school curriculum defined by the test. They suggested that you should be tested on what you had just learnt and that the teachers should make sure they covered everything that would be on the tests each year. The group was all clear what level they should be —'we should all be at level 4'— although Imran described this as boring. 'It's boring being level 4 because that's what everyone is apart from some clever people.' Kenny went on to say that he had been estimated at level 2 by his teacher, when asked how he felt about this he said, 'If I come out as level 2 I will kill myself.' Rupa had also been estimated at level 2 by the teacher, she chose not to describe her feelings on finding this out.

> 9/5/96 (Parmjit's journal) I'm just a little bit nerves about the S.A.T.S. because I think I might get a really bad score or level. I hope I get at least level 4 because I think it is my ability but I will be very happy if I get a level 5.

Parmjit is clear that the 'expected' level is level 4. When the children discussed other forms of assessment it becomes clear that one form of mathematics learning is measured in terms of process and outcome, and another in terms of scores. For example:

> 26/4/96 (Sangeeta's journal) So far I have done (an) activity […] called 'the Give away'. I liked this a lot because there was a lot of designing involved in it. You had to make a poster to show any product of your choice was a good buy. You could make it up or you could choose one that is out in the shop. I made one up which was called Filo.

Sangeeta goes on to describe her poster in some detail, then she writes:

> Last week we had a Maths practice sat. I got 33/40 which was alright, but I wasn't happy. Yesterday we did a practice spelling sat. I got in that 23/30

> which is level 5, but I still wasn't happy. Also last week we had a Science Sat which I got 35/36/44 which I wasn't very pleased with either.

Later in her journal Sangeeta contrasts coming 'highest in the class' in spelling with doing 'the comic survey'. She is 'pleased' to be highest in the class and 'excited' about her comic survey.

A social justice reading from Tony

The children in the first extract seek to mitigate against the competitive nature of the tests through subverting the teacher's strategy. They are supported in this act of resistance by the teacher. In this way the teacher and pupils act co-operatively to support the stated ethos of the school that acknowledges values of social justice. Not supporting your school friends would be seen as 'unfair'. However when it comes to high stakes National tests the focus switches. The competition is now legitimate. Copying would be unfair. Kenny describes SATs as a 'fair' way of testing even though he thinks he may be pathologised as 'below average' by the results. This suggests a dilemma for the pupils. Their learning is legitimised through competition in 'proper exams'; exams that will help them in the future because they fit their models of what it will be like in 'senior school'. They also recognise the arbitrary nature of the levels that will describe them in educational terms. Indeed, two of the group are defined as 'not normal' by Imran as they are not level 4. A definition which they unsurprisingly find uncomfortable but not necessarily unfair. The impact of this competition can be seen in individuals who come to blame themselves for lack of success, or who feel excluded, or are literally excluded from academic success.

What I describe as 'traditional' assessment practices dominate both teachers' time and pupils' curriculum experience here. This means that although the teachers leave space for discussion of alternative assessment practices and can articulate the usefulness of such alternatives, they are unable or unwilling to create the space necessary in the classroom. Because of this, the shift in emphasis towards learner centred assessment cannot be made and the status quo remains unchallenged. The second section from the journal shows how traditional assessment practices that prioritise performance over process come to dominate the ways in which the pupils can evaluate themselves as learners. The final extracts from the pupil journals show how they began to link ideas of ability with the measurements SATs would provide, as opposed to the type of self-assessment that the teacher was trying to encourage. Here the high stakes testing is dominant to the extent that the teacher's values are marginalised in the pupils' understandings of the purposes of education.

The assessment practices documented here can be seen as sites in which life choices are legitimated. These life choices may begin through school selection, through expectations which pupils will take with them into secondary school in the form of National Curriculum levels and in terms of their own expectations. The SATs defined as useful by Kenny as they will give you an 'idea of what your [sic] like, gonna get, when you are in senior school. When you come to your exams.' In

this way the results in these SATs will ensure that Kenny makes 'appropriate life choices'. This is not the process of rational life choices suggested by Rawls.

The pupil's acceptance of predetermined cultural norms can be heard in the discussion around the fairness of SATs. Those who are likely to achieve levels at or above the 'expected level' accept the competition as fair, they accept the result as 'I think it is about my ability.' There is no questioning that this particular method of testing may favour certain groups of learners or marginalise others. Sairah begins to show an understanding of the hidden pedagogy around national testing. She foresees potential problems around lack of content coverage, something that can be overcome through the use of study guides or out of school support. Something that ties very closely to ideas of cultural capital. For Imran the main problem he foresees is feeling ill on the day of the test, something over which he has no control and a suggestion that falls outside a cultural norm of the group to the extent that they giggle in response. Another form of cultural selection can be seen when Umna describes the different emotions she feels within school. She has worked hard on her project and is pleased with this. However this counts for nothing when it comes to the SATs. She is frightened by the testing process and doesn't want to do them even though she knows she has to.

A Foucauldian reading from Tansy

In my reading of Tony's journal extracts I look for techniques of power: surveillance, examining, classifying, a play of totalisation and individualisation, and normalisation.

At the start, Tony talks about his training day; *'Do they see training days as detached from their 'real lives' of teaching and so forget very quickly?'* I recognise here the workings of discursive practice(s) —putting in order the meanings that teachers make and how these are related to patterns of classroom practice. The training appeals to teachers' concern for children as individuals —a learner-centred discourse. This leads to enthusiasm about alternative methods —the rich information about children's individual needs will be valuable to teachers' future planning— but does not dislodge the need for 'find out what they know' tests. It has not disrupted their knowledge that there is a need to view the class as a whole and talk about 'what they know'. The contradiction that Tony sees does not disable these teachers —their everyday practices fudge over the disjuncture. Tony might be aware of the way that children become classified and then included or excluded from future opportunities by the regulating effect of this 'whole class' gaze. He searches for alternative, richer assessment strategies. The teachers he describes are caught up with the common sense of 'traditional methods' and can seesaw between individualisation —concerned with particular children— and totalisation —treating the class as a whole—, valuing one while doing the other.

> The kids do not seem threatened by the tests however […] This didn't disturb (the teacher) in the slightest —in fact (the teacher) was quite amused by it.

This is a story of regulation through written tests. Several colliding techniques are operating to bring about normalisation of the learners and the teacher. Through this, aspects of their identities are constituted and they come to know the truth of who they are in this educational site. It is the final written test script that determines who and what these learners are. This erasures any information about what each child understands and can do —there is only their script. Does the teacher recognise this 'surveillance' and attempt to disperse its point of action? The gaze can shift to the class as a whole and possibly the teacher her/himself and avoid exclusion of any one child. This illustrates well the foregrounding of texts and language in a postmodern discursive analysis and the effective erasure of the subject —as an individual person— that can result.

The most significant use of the Standard Assessment Test results is to examine the school and its teachers. Does this account for the 'not disturbed' response from the teacher: that what individual children do does not directly matter to the school? At the same time, written tests are a surveillance technique that **can** bring about the regulation of individual children. It is in the collision of this totalising and the individualising that the students can effect their resistance to the implementation of this examination —a transgression of the ordering of the classroom space. Interestingly, it does not reclaim their voice but it exploits their invisibility. Does the teacher's pleasure arise from this contradiction?

In the focus group discussion a group voice is identifiable. It asserts 'The tests are easy and fair. We find them easy; we are not crushed by them'. The pupils are also aware of the individualisation that can be effected through the scripts. This attributes a level to each child('s name). The technique of normalisation can be clearly traced out in the effects of the SATs. A 'normal' child will achieve level 4 at Key Stage 2. If a child is given a lower result then they are not up to the required standard. They become 'abnormal'. These primary school children are acutely aware of this defining effect, of how it determines how their efforts past and future are interpreted, and of ways in which this determines their future position. This is so, despite what teachers or parents might do to reassure them that these test results will not determine their future. Their results may put one of them outside —e.g., Level 2. Abnormal. But the tests are seen to make a false offer of redemption: 'if you work hard enough, practice, revise, then maybe you will be successful in the next test, and be back 'inside". They can see specific cases where the tests are not fair, but the only way back in —from the outside— that they talk of is a retiming of the tests. If you talk differently, i.e., not in terms of levels or SATs, then you are mad. To deny SATs is to be totally silenced.

It is interesting to note that attempts can still be made to reclaim part of their sense of self from this process. Success at the test is to do with revision and familiarity —not directly, for example, their ability or their understandings. At the very least they give the pupils a fair idea of what they are like in terms of work and practice. Did I work well? Did I practice enough? This is surveillance: the anonymous SATs writer might be both the examiner and arbiter. This attempt at distancing themselves from the process of normalisation is not (completely) successful: 'I will come to know my place/my position in the order, that is, whether I am normal'. There is an erasure of self through the levelling and normalisation: 'I

will become what I do on that day'. Is it melodramatic to echo Kenny —if that proves to be 'abnormal' you might as well be dead?

WHAT WE HAVE DONE, WHAT WE MUST DO, WHAT WE WILL LOSE

What we have done

Foucault has outlined how every education system is a political means of maintaining or modifying the appropriateness of discourses along with the knowledge and power that they bring with them (Foucault, 1971, p. 46).

We have said that there is a need for mathematics education research to think differently about its concepts and objects of study in order to become aware of why the process of schooling, and of learning school mathematics in particular, has the effects it does on groups of our children. It is also necessary to think about how this process and the possibility of alternatives can remain hidden.

For example, we argue that a model of 'the social' where the individual is separated from the 'outside', and where that outside is seen as forming the individual produces disjunctures in research analyses. This consequence is often ignored in the desire to make generalised claims about the causes of our failure to help children to learn mathematics. We have argued that we must struggle to find more valid ways of examining our practices and that, to do so, we must acknowledge the discursive nature of the field in which we operate.

As a start we have provided an illustration of this by tracing some of the facets of the process of schooling through discussion of a range of tools or strategies related to Tony's transcript. We have discussed how these strategies can reveal the discursive practice(s) of mathematics education and the ways this can position people within the classroom, effecting their actions and determining what each has to say to be heard. More importantly, we have begun to identify some tasks to engage in as part of that struggle. Our commentaries on the transcript point to important, though not easy, steps that we can take to re-problematise aspects such as assessment practices.

> The work of an intellectual is not to mold the political will of others; it is, to re-examine evidence and assumptions, to shake up habitual ways of working and thinking, to dissipate conventional familiarities, to re-evaluate rules and institutions and starting from this re-problematisation [...] to participate in the formation of a political will. (Foucault, 1989, pp. 305-306)

Through this project we have not aimed to order or interpret the 'reality' of mathematics education in schools. Indeed, we do not believe that there is such a thing as reality of experience within our schools —rather a multiplicity of 'realities'. Neither have we tried to understand the lived experiences of pupils and teachers; rather we have searched for ways of producing knowledge —about our classrooms— that reveal their constructed nature and help all of us to start to think about alternatives. This form of knowledge involves a very different attitude towards meanings, so that our aim is not to describe and understand but to criticise and

transform. Clearly results take on a different meaning here. Ernest (this volume) reiterates this when he points to the care that needs to be taken with knowledge claims in social research in mathematics education.

We have worked with Foucault's vision of normative nature of assumption, that for the vast majority of learners, school is an 'unreal' world —where they are not the 'normal'. Through our readings we have come to know better what we must work through if we are to come to terms with the 'conditions of education' as we have established them, and the representations and practices that have given rise to our current maths education practices. We have shown a possible way to rethink our curriculum and our practices, to undermine common sense ideas about children, teachers and mathematics. This gives some insight into why we are not helping many groups of learners learn mathematics. Moreover, we cannot claim to have 'solved' our concerns and questions. We acknowledge the provisionality of the claims that we make here but argue that we have offered support to our insistence to abandon normalising frames.

What we must do and what we lose

In taking up the challenge to examine how we live our lives, both professional and personal, we have to be prepared to engage with complexity. We have to find ways and terms that acknowledge the multiple positioning of the actors in the mathematics classroom and see and work with diversity. We think that the toolkits we have illustrated offer a way in to this.

We also need to abandon normalising assumptions and ask the hard questions of why we act the way we do, why we construct the identities/subjectivities we do. And as researchers we have to be wary of hidden assumptions about looking for generalisable knowledge and seek alternative ways of validating our work.

So what might such a hard, but interesting question, be for mathematics education? We have asked questions about the process of perpetuation of injustice and now need to search for domains within which we may have the power to intervene. Our work suggests that there is much we can do within schools to challenge injustice, by examining the curriculum —and assessment procedures— we offer learners, by being critical about the practices we adopt, and by being aware of the social and political structure and contexts within which we operate. This indicates the field from which we can specify further questions.

For ourselves as researchers this involves confronting the inevitable privileging of the readings we present here. This could take the form of such questions as: As interviewers are we colluding in the 'naturalising' of the assessment process? How would the learners have talked about assessment had we acted to disrupt this? How could we have revealed this possibility through our analyses?

In summary, by exploiting the lack of stability of many of our professional notions, we might open up spaces from which we can counter pathologising, ill-posed problems, and look for sites of resistance. So the challenge for educational researchers interested in redefining the experience and effects of learning mathematics is to locate ways in which the curriculum and its assessment, pedagogy

and social and cultural environment can be reconstructed to allow all teachers and learners to confront injustice as they experience it. And to participate in that reconstruction and confrontation through research practices. Not an easy task to describe or take on —but we suggest an exciting task.

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THIRD DIALOGIC UNIT

RESEARCHING IN CONFLICTIVE SITUATIONS

The recognition of the social and political complexity of the situation where research and intervention programs happen has led the authors of the chapters in this dialogic unit to be reflective about the process of knowledge production and its implications. Considerable research in mathematics education is conducted in classrooms where researchers seek to test hypotheses or interventions. More often than not, such research projects are conducted in safe, stable environments where the researcher seeks to reduce the effect of uncontrollable variables. This type of research is prolific in international literature in mathematics education and its outcomes have represented significant advances in the field. However, when further examination of the research environment is considered, the recognition that the research was conducted in a relatively 'safe' and purportedly 'neutral' environment is important. This raises serious questions within a socio-political approach. First, the very same adoption of a socio-political approach denies the possibility of conceiving a context as 'neutral'. All social situations, no matter how 'quiet and stable' they may appear, are plagued with conflict. The theoretical lenses used to 'see' a situation reveal this characteristic —as much as a theoretical frame rooted in a psychological-mathematical approach may underplay it or make it invisible. Second, a socio-political viewpoint raises doubts as to the validity of the outcomes of 'context-blind' research for learning and teaching in mathematics education. As the authors in this book keenly address, context is a constitutive element of the 'text' being researched, and, therefore, it is critical to the research outcomes. Moving out of the safe terrain of 'politically neutral' contexts generates issues that researchers need to recognise.

The authors in this section consider what it is to undertake research in highly conflictive contexts where it is impossible to be 'blind' to power relationships, since the latter are central to the lives of the people with whom they work. Both chapters discuss the micro and macro considerations of power and how these impact their work with the participants in their projects. Núria Gorgorió, Núria Planas and Alan Bishop base their reflections on their experience in an action-research project seeking to bring change and improvement to the mathematics education offered to immigrant youngsters in a Catalonian school district. They identified a series of tensions that they had to tackle when working with teachers and students involved in the project. One tension the research team encountered was the divergence between the actual situation in the schools and the situation that they wished would happen.

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*. 105—106.

The tackling of this dichotomy —analysed in detail by Skovsmose and Borba— implied not only a negotiation between the researchers and the participants, but also a struggle with the local educational authorities. Another dichotomy dealt with is the distinction between theory —or the interests of research— and practice —or the interests of collaboration with teachers and students in the project. Once again different elements of the 'context' such as the recognition of teachers as 'legitimate' research partners by academic authorities contributed to the complexity of this well-known tension in action-research projects —as mentioned by Atweh. A very important tension, though less discussed in mathematics education literature, concerns the consideration of students as more than 'research objects' but as central actors within learning-teaching and research processes. The tension emerging from the difference in cultural background between students, teachers and researchers challenged not only the researchers' preconceptions about the students, but also fundamentally the research strategies that they had planned to use as part of their study. This tension draws attention to the fact that the research process needs to be constantly re-constructed given the conflicts and disruptions emerging along the way. This point, also explored by Vithal in relation to a changing context such as the new South Africa, highlights a new aspect of context that other chapters have not considered: context is theoretically and methodologically significant.

Gelsa Knijnik invites the reader to a different location: the Brazilian countryside and the struggle of the Landless People Movement. Her work in mathematics education from an ethnomathematical perspective with the people of the movement has made her aware of the fact that any kind of 'mathematical formation' cannot be isolated from the struggles of these people within both macro and micro contexts. Many of the conflicts experienced by the landless people in their overall political fight for the right to land are deeply connected with clashes emerging in the mathematics classroom. For example, the relevance of a particular curricular activity centred on the exploration of similarities and differences between traditional school mathematical knowledge and in-practice mathematical knowledge becomes evident in relation to the advantages and disadvantages that each type of knowledge may offer to people when solving a real problem emerging from the Movements' political struggle. That school and in-practice mathematical knowledge is power-knowledge, which people use to position their actions and discourses in more or less influential sites, becomes palpable for both the people and the researcher.

Central to both chapters in this unit are considerations about the conflictive nature of the context in which the research takes place and how this makes it impossible for the researcher to leave in tact fundamental assumptions about the process of knowledge production through research. The chapters alert researchers to the importance of their own learning through participation in research and fieldwork: learning is not unidirectional and researchers working with a socio-political field cannot remain unscathed from their work when they engage in emancipatory research projects. Neither can their work, theoretically and methodologically, remain untouched by the dynamics of the context in which they act. This dialogic unit contributes both to the discussion about the centrality of context, and to the issue of the researchers' visibility which will be the focus of the next dialogic unit.

NÚRIA GORGORIÓ, NÚRIA PLANAS AND ALAN BISHOP

DICHOTOMIES, COMPLEMENTARITIES AND TENSIONS

Researching mathematics teaching in its social and political context

The growing immigrant population in Catalonia is experiencing the inadequacy of the educational system in general, and of the teaching of mathematics in particular. Most of the ethnic minority students are from Magrib (North Africa), but the presence of students from other countries in Africa, Latin America, Asia and Eastern Europe is also significant, not to mention the Catalan Gipsy students.

In 1997 a project was commissioned by the Ministry of Education in Catalonia, an autonomous region in North-eastern Spain, concerned with mathematics teaching in schools that have large numbers of immigrant students. The project is concerned with finding more appropriate ways to teach mathematics to immigrant students, both in primary and secondary schools. The project is funded by the Fundacio Propedagogic, a Catalan private foundation devoted to education. During the first two years of the project there was also some financial support from the Ministry of Education. It is based at the Universitat Autonoma de Barcelona, and the local research team, led by the first author, consists of academics at the university, among them the second author, and secondary teachers. Two of the secondary teachers were given time-release from their school commitments for one year. The third author assists with the development of the project as a research consultant.

The research team works in several schools in Catalonia, most of which have a high proportion of immigrant students. The research is collaborative not just in the sense of working in a team, but also in the sense that the project team works with the teachers and their students in the schools. More details about the project and its theorisation may be found in Gorgorió, Planas and Vilella (2002).

Explaining the difficulties immigrant children have in these schools in terms of cognitive deficit is, in our view, too simplistic and questionable (Ginsburg & Allardice, 1984; Nunes, Schliemman & Carraher, 1993; Rasekoala, 1997), particularly because this interpretation itself has social implications in the way it projects particular expectations onto diverse cultural groups. In our view, mathematics education should be interpreted as being embedded in an understanding of the social and cultural reality where it takes place (Oliveras, 1996). Thus the starting point of the project for the team was to consider the cultural contributions of ethnic minorities and of the different social groups as a source of richness to be maintained and shared. We did not see cultural differences and the cultural conflicts

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 107—123.

arising from them, as a 'problem to be solved' nor as a 'diversity to be treated' but as a potentiality.

The stated goals of the project are, among others, to analyse the difficulties that immigrant children experience when learning mathematics in Catalan schools, and to identify their potentialities as well as their knowledge; to come to know the values, expectations and beliefs that they associate with in-school and out-of-school mathematics; to analyse the dynamic of the mathematics classroom from the point of view of the different interpretations of the various norms that regulate it; and to identify successful teaching strategies, such as teaching methods, students grouping, content selection and class dynamics.

However, perhaps more important than the stated goals, for this chapter, is the fact that the whole development of the project is based on five crucial assumptions. First, we understand mathematics to be a cultural product (Bishop, 1988), that is, our understanding of mathematics acknowledges and shows the relationships between mathematics and culture, in other words, the cultural and social origins of mathematics. We also understand culture in its broadest sense (Geertz, 1973), in which culture is a pattern of meanings historically transmitted that are embodied in symbols, through which human beings communicate, perpetuate and develop their knowledge of, and their attitudes towards, life. We approach mathematics education from an integrated perspective that takes into account cognition, values and emotions (Abreu, 1995); this means that constructing and using mathematical knowledge goes beyond a purely cognitive aspect of human capacity; it involves affective and social factors linked to particular contexts. We understand socio-cultural diversity as implying different participants' interpretations of the norms that regulate the dynamics of the mathematics classroom (Yackel & Cobb, 1996). The social context of the mathematics classroom includes the social norms of functioning and the sociomathematical norms that regulate the mathematical practice itself. The understanding and the interpreting of those norms change according to the initial meanings of those who interpret them. Finally, we understand linguistic diversity as a socio-cultural factor that conditions mathematical learning (Ellerton & Clarkson, 1996). The different languages, as vehicles of the learning in the mathematics classroom, act as mediators between the goals and the visions that the teachers have and those that students have.

This then is a basic outline and ethos of the project. The remainder of the chapter will contain more information about the project but this will be presented in the context of the pertinent social and political issues arising in the course of the project. The chapter will concentrate on the discussion of the challenges that working in a clear situation of political and social conflict —such as the education of immigrant children— posed to us as researchers and to our conceptions and actual actions when interacting with the many actors involved, directly or indirectly, in the project. We engage in this discussion by identifying five sets of dichotomies, complementarities and tensions concerning the different fronts and actors on which our research activity had an impact.

THE SOCIAL MEANING OF THE RESEARCH PROJECT: WHAT IS? VS. WHAT MIGHT BE?

As a first dichotomy, or complementarity, research in mathematics education can be understood in two major ways (Bishop, 1992). On the one hand, there is research that is concerned with the study of the present mathematics educational situation and its complexities. On the other hand, there is research concerned with future possibilities and alternatives in mathematics education (e.g., Skovsmose & Borba, this volume). However, when researchers want to consider the social aspects involved, it is difficult to talk about 'what is' without at the same time suggesting or implying 'what they think, or wish, might be'. From that perspective, in this section we will consider the social dimension of mathematics education in the sense of Bishop (1988), as furthering the development of society and of the individual, and we will take into account that if research has to have a social meaning we will have to recognise the societal, institutional and cultural contexts in which the teaching and learning take place.

At the societal level we can see immediately the tensions between 'what is' and 'what might be'. Indeed it is also necessary to be aware of 'what is not', when interpreting 'what is'. For example, the Catalan educational administration has always had a clear policy regarding the right of all children to attend school to the age of 16, regardless of the legal situation of their parents. However, it is also the case that, for the most recent immigrant children, due to the family re-groupings in many cases, there are delays in their schooling arrangements that can hardly be justified. Despite having laws that regulate the right of those children to attend school, the reality is that society and the government do not make the effort, nor create the necessary means, to make this right effective.

Regarding the institutional level, there are again tensions between 'what might be' and 'what is' which have strong implications for both teaching and researching mathematics education. As a result of the rapidly increasing and uneven immigrant situation, schools' populations located in what have become 'ghetto areas' have changed rapidly. We can find public schools in Barcelona that contain nearly 90% immigrant students. Moreover, since immigration is often linked to economic deprivation and social risk in the areas where immigrant students live, what could be 'normal' schools turn out to be what we could name 'ghetto' schools, with huge implications for the kind of research that could take place there.

A further aspect of context concerns the negotiations with the educational bureaucrats in order to carry out the research. The official regulations of the educational system do take into account the diversity of the students, but old beliefs have not yet been abandoned by the administration. For instance, the implicit theories underlying the administration's proposals for dealing with cultural differences are based on 'deficit theories'. In many working documents of the educational administration, immigrant students are referred as 'students with special educational needs', a description that is also used for students with specified cognitive deficits or physical handicaps. At the pedagogical level, the difficulties that many teachers encounter today, especially when facing mixed abilities groups and multilingual or multiethnic groups, are due in part to the fact that the official

documents describing the reforms or the in-service teaching programs do not help teachers to overcome the 'invisibility' of the social and cultural factors that crucially affect the teaching and learning process. The data coming from our project show that, too often, the teachers associate the idea of 'immigrant student' with the idea of 'disruptive student', and that, most of the time, teachers project negative expectations onto their immigrant students.

There is no doubt that research in mathematics education cannot remain indifferent to this whole situation. All the decisions, both methodological and practical, that the team has made throughout the development of the project, have had to take into account the 'what is' and 'what might be' political and social aspects that go far beyond the domain of the mathematics classroom. These aspects constrain and structure not only the teachers' practices but also the researchers' goals and methodological approach.

In our research study, focussed at the pedagogical level, we have gathered information about both 'what is', and 'what is not' the present situation regarding the teaching and learning of mathematics in classrooms with large numbers of immigrant students. We have also collected data that show that the situation could be improved in different ways through implementing different teaching strategies and content selection (e.g., Gorgorió et al., 2002; and Gorgorió & Planas, 2001). The results of our research, and those of similar projects developed in other contexts (e.g., Abreu, Bishop & Presmeg, 2002), give an idea of 'what might be' possible in such classrooms.

However, even if from the project we can suggest and propose what could be done to improve the situation, it is the Catalan society which has to decide, if it wants as a reality, what we consider 'might be'. Furthermore, it is the Catalan educational administration that has to make available not only the means to research further the ideas from our project, but also the means to disseminate and to implement the ideas that we are generating.

RESEARCH AND THE 'REAL' SOCIAL WORLD: RESEARCH VS. PRACTICE

A second dichotomy which relates to the first and which became very obvious from the start of the project concerns research and practice. This is a dichotomy which has been fully documented, both in mathematics education and in general education. It is clear however from various research summaries (Bishop, Clements, Keitel, Kilpatrick & Laborde, 1996; Grouws, 1992) that this dichotomy is no longer acceptable, and that there is an urgent need to strengthen the complementary relationship between educational research and practice (Hoyles, Morgan & Woodhouse, 1999).

In any educational research that wants to call itself socially committed, the need exists to know more about teachers' perspectives on practical issues that researchers could seriously address, and for counting on their expertise and knowledge to find ways to research those issues and to interpret the results (Bishop, 1996). As Ruthven (1999, p. 212) states we 'have the challenge of finding an appropriate transposition

of the methods, evidence, and findings of research into the currently very different context of school practice'.

Given that the main aim of the project is to 'promote changes' in the educational context, since the beginning we considered, as Zuber-Skerrit (1996) does, that the best overall methodological approach would be that of action research; that is, research done by people on their own work, following an essentially critical approach to schooling, and with the explicit aim of improvement (see both Atweh and Knijnik in this volume). We consider, as Cohen and Manion (1990) did, that action research is a powerful alternative both to external research that has little connection with classroom realities, and also to subjective practice, which omits external observers who can ensure triangulation of the data and its interpretation.

However, rather than discussing here the recurring issues associated with doing action research, which, on the one hand, are well documented (see, for instance, Cohen, Manion & Morrison, 2000) and that, on the other hand, have been presented elsewhere concerning our project (Gorgorió & Planas, 2000), we would like to highlight the political context and its influences on the research process. For example, it rapidly became clear to us that the development, if not the survival, of action research in this situation would not be possible without the contribution of the different educational administrations. They need to facilitate it by such means as giving economic support, reducing the teachers' school schedules, facilitating their attendance at conferences, promoting in-service working groups, reforming rigid working structures, and restructuring the power hierarchies as also pointed out by Baumann (1996). So, having received the request to address an issue that is mainly connected with schools, we argued with the Catalan educational administration the necessity of working in a team that included different members of the educational community. The negotiation with the educational administration resulted in a collaborative team, and with such a team we consider that we not only have the support and expertise from the university level, but also the knowledge and expertise of the school practitioners, and, what is more important, we have been considering issues directly related to practice.

However, doing research within a collaborative model with in-service teachers, and having teachers participate in action research projects implied changes that, at least in the Catalan context, still needed to be justified within both the university community and the school system. Elliott (1989) concluded from different examples that action research is only reluctantly accepted or considered because of the complexities of the many value decisions that this kind of research involves. This conclusion —still valid nowadays— is reiterated by Atweh (this volume). For instance, our university system, both academically and administratively, is still reluctant to accept in-service teachers as full members of research teams. Academics at the university do not consider that school teachers have enough knowledge and expertise in the field of research. Neither does its administrative system consider school teachers as members of research groups for the provision of grants. Moreover, within the school system, teachers in mathematics departments, principals and inspectors and, more globally, the educational administration, find it difficult to accept and justify teachers devoting part of their time to research.

From another perspective, the complexity of the social context makes it difficult to do the research; as it creates many tensions between the research activity and the people affected by it. Because of the roles and responsibilities of the educational administration, the characteristics of immigrant students and their families, and the ethical ideas of what we understand as research, we faced several challenges. In particular, how to achieve a full acceptance of the project by the educational politicians by convincing them that our project is not only 'politically correct' but also fulfils a real social need. We had to encourage the school principals and inspectors to accept the project, and to convince them that the mathematical education of children within a multicultural framework was, and still is, a priority at the present time.

We also had to gain the trust of the students, and establish the credibility of the research team with their immigrant parents and with significant members of the different communities. The methods used within a qualitative research approach, whose 'objects of study' are individuals, obviously require their acceptance to be 'studied' and their willingness to cooperate. To study real mathematics classrooms we needed not only the engagement of teachers but also of students, and therefore we had to convince them of 'our good intentions', to overcome some practical matters related to different cultural traditions or personal situations —e.g., accepting to be videorecorded—, and to overcome the linguistic barrier —e.g., finding ways to analyse the interactions in small group work when they took place in a language unknown both to the teacher and the university researcher. Moreover, due to the unstable social and economical position of the students' families it became difficult to focus on some case studies —e.g. some students 'disappeared' during our study.

The team negotiated strongly, and continues to do so, to change what initially was a policy driven 'research' project into a research project with no inverted commas. The initial request from the administration was to create 'ready-to-use' materials to be given to the teachers having immigrant students with an incomplete knowledge of Catalan language. Behind the request for 'packaged materials produced in different languages' to be spread throughout the schools are many of the hidden assumptions of the educational administration. The Catalan educational administration obviously wanted a quick answer to society's demand for caring better for the immigrant children.

Since the goals of the research project were not fully appropriate in the eyes of the educational administration, we had to argue with their representatives on many points, in particular on the focus and the methods of our research. The educational administration was reluctant, for instance, to accept that part of our study was focusing on 'ghetto schools'. It was reluctant to provide help for overcoming the linguistic barriers within the schools, or to make available any general data about the students. We also had to convince them of the need to use some particular research procedures —e.g., videotaping mathematics classes— and to facilitate the collaborative research process —e.g., providing a partial time release of the teachers involved.

After five years of working on the project, we are aware that there are still plenty of unresolved issues, which are of concern not only to us, but also, we suspect, to anyone involved in such kind of research. For example, up to what point can the

research on mathematics education reflect the real needs of the society where it takes place? Who determines these needs? Who is in charge of deciding which changes should be implemented in the mathematics curriculum in order to guarantee mathematics learning for **all**? Why, so often, does the educational administration pay so little attention to what is being done in the research field? Is it the fault of the university researchers themselves because of failing to communicate their results in such a way that they can be useful for the classroom? Or is it because the research questions emerged far away from the reality of the classroom?

RESEARCHING TEACHING CO-OPERATIVELY: STUDYING TEACHERS VS. STUDYING WITH TEACHERS

Our starting point when structuring the research team was to consider that since it is the teachers who should benefit from the outcomes of the research, they were important in the research, not just as 'subjects'. At present, research agendas are still dominated by the researchers' questions and aims, and not by those of the practitioners, if we can make that distinction. However, we are addressing a crucial issue related strongly with social demands within a particular context. Change being the final goal of the research, the teachers' contributions to clarifying the points that should be addressed and how to address them were crucial.

The issues addressed in the research are actual teachers' problems that have to do with actual teaching constraints and limitations. It is essentially the teachers who are more aware of them and who can conform more to practitioners' criteria and methods for addressing this issues. We agree with Ainley (1996, p. 18) that 'far from leading me to feel that I must deny my identity as a teacher in order to be an effective researcher, I see the skills that I have as a teacher as crucial in enabling me to frame such interventions effectively [...] To be an effective researcher (and perhaps also an effective teacher) I believe that I need to be aware of the attractions and constraints of both roles'.

Moreover, school teachers' questions and explanations derive from a knowledge domain that is distinct from, and complementary to, that of university workers isolated from educational system realities. Within the research team collaboration means that everyone's view is taken as a contribution to understand the situation; and theory and practice are seen as two interdependent, yet complementary, aspects of the change process. Moreover, school teachers, as full members of the research group, legitimate and facilitate the contact and the communication process with other teachers; and they also help with disseminating the ideas from the research and the innovation proposals.

Besides that, the presence of teachers on the research group strongly enriches the process of interpreting the situation and the triangulation of the data, illustrating the complementary points of view of university researchers and school teachers. For example, the following 'vignette', which forms part of one of the interviews with the teachers, illustrates a pattern that we have found repeatedly in the interviews:

Interviewer: How are you getting on with your classes? Do your immigrant students follow the mathematics you teach them?

Teacher: It is very difficult for them. They find it difficult to concentrate, to behave, to respect me and to respect their mates, they find it difficult to realise that they are in another country, and that here things are different...

I: What do you mean?

T: For instance, if you give them homework, they do it at the Mosque with the Imam. And then, if you find something which is not exactly correct, and you mention it to them, they feel it is an offence to the Imam. They are too arrogant.

I: Arrogant?

T: Yes, yesterday, for instance, Kamrum, a Pakistani student that arrived here two years ago, left the class.

I: For any particular reason?

T: Well, I gave them a problem to solve and suggested that the students work in small groups to think about it. Kamrum refused to work in the group, where there were three girls. I explained him, once again, that things here are different from the way they are in Pakistan, that we have to leam to work with boys and girls... But, while I was saying that, he took his notes and his book and left the class...

I: And he was all the time in the corridor...

T: Yes... and when the class was over, he asked for permission to come in and showed me the problem solved. It was correctly solved. In fact, he solved it in a very clever way. It is a pity that he is so arrogant!

From the perspective of the university researcher one could easily generate other explanations for Kamrum's behaviour, inferring for example how often cultural conflicts within the mathematics classroom do not appear significant to the teachers. The research has clearly shown how difficult they find teaching multicultural classes; yet in many cases the teachers revealed to us that they were not aware of the fact that the mathematics classroom is a cultural and social scenario. But these are all researchers' interpretations and constructs. The team's real research task has been to try to understand how the participants in the classroom encounter interpret the 'conflicts' that arise in their classrooms, how they often do not see them, or perhaps how they often choose to ignore them (see Gorgorió et al., 1999 for details), and why they see classroom events in the terms that they do.

Thus the collaborative work allows us to take into consideration not only the factors that condition teaching practice, but also the connections with published theory. Both of these play an important role in shaping the research, by establishing the possibilities, limitations and constraints of the context, and also by offering the dimensions of generality that give sense to the research. The study thereby has become both an analysis of practice and a search for explanations towards the development of theory.

One of the needs we were aware of was how to ensure a distance between the simultaneous roles of being both a teacher and a researcher for both the school and university members of the research team. In particular, we had to deal with the tensions between the teachers' responsibility to the students and to the research. For

example, there were tensions regarding issues of students feeling reluctant to participate in the research, by, for instance, not wanting to attend a class if it was going to be videotaped. Our response to this kind of tension was this: The research team explicitly agreed that we all had a responsibility as teachers that was over and above that which we had as researchers, even if that could mean a 'loss' for the study.

We also had to face the obvious risk of bias when interpreting the data obtained from a classroom where the roles of teacher and researcher were played by the same person. Analysing the data obtained in a study developed on one's own class requires important control actions (Robinson, 1998). Discussing and contrasting the different points of view within the research team, having an observer in the classroom who is different from the teacher, documenting and analysing the development of the lessons through the video recording and the teacher's diary have all helped to control the biases.

After five years working, we are convinced from the perspective of our project, that the understanding of the situation and the outcomes we could achieve through a collaborative and action research approach, under a qualitative and interpretative paradigm, were worth all the conflicts we were facing in getting involved in action research (see Gorgorió, 1998 for details about the research procedures used). We are convinced that, despite the difficulties and tensions, it has been more important for our goals to study with teachers rather than to consider teachers only as objects of study, or having their roles consisting only in developing the researchers' proposals, without knowing the grounds for their actions.

RESEARCHING WITH STUDENTS: NORMS VS. DIFFERENCES

At the individual student level of the social dimension the research project relates to the diversity of the students, and to how to interpret this diversity. Aspects such as language, gender, age or place where they come from, shape and affect not only the research but also the teaching and learning. We have discussed elsewhere (Gorgorió & Planas, 2001) how the diversity of languages in the mathematics classroom conditions and affects the teaching and the learning processes. Gender differences are a reality in all mixed classrooms and many researchers have studied gender differences in the learning processes of mathematics. There is also a wide range of studies that document the mathematics learning processes throughout different ages. However, in the context of this research project we have to consider such explicit differences as gender and age from another point of view.

Gender and age differences are extremely important to us when related to emotions, beliefs and expectations, and therefore to cultural and social conflicts. In Western societies mixed gender classes are commonplace, but it is not the same in other societies, such as Muslim societies. When students come from a Muslim cultural background to attend Catalan schools they find it difficult to adapt to a situation where it is 'normal' to have working groups with girls and boys together. In Western societies, it is 'normal' that parents have positive expectations regarding the possibilities for both boys and girls to have an academic career, while in other

societies girls, from a certain age, are expected to leave school and to devote themselves to other activities. When girls come from this kind of social and cultural background into Catalan schools, and discover their potentialities, they often face a cultural conflict between their two coexisting worlds: the family's world where they are supposed to leave school early, and the school world where they have teachers who foster in them the interest in continuing their studies. The personal values linked to cultural backgrounds also condition the research and, in particular, the research methods. For example, while 'normally' students have nothing against being videotaped during a mathematics lesson, it was hard, if not impossible, to convince some girls from other cultural backgrounds to allow us to videotape them.

The social dynamic of the mathematics classroom also reflects the complementarity of 'differences' vs. 'norms'. When talking about differences in a social situation, we mean differences from the socio-cultural 'normality', where this is defined according to the assumptions and expectations of the individuals concerned. Thus, our teachers find immigrant students to be 'different' from what they expect students to be. Immigrant students find their teachers 'different' from those they were used to, as they do the dynamics of the classroom and the school they are in. The interactions among students and between students and teachers are also culturally 'different' as are the relationships between parents and the school system.

As a result of the empirical work done so far, we can claim that the different interpretations of what is 'normal' concerning the social dynamics of the mathematics classroom among its members can interfere significantly with the actual teaching and learning process. Moreover, the different interpretations are clearly present, whether 'visible' or not, in all mathematics classrooms, both in 'ghetto' schools and in 'normal' schools. In the following paragraphs we show how the social norms that regulate the dynamics of the mathematics classroom can be differently understood by its participants, both in a 'ghetto' school and in a 'normal' school (see Lerman & Zevenbergen, this volume, for a similar discussion).

As part of the work of the team, we hold regular in-service sessions with groups of teachers. In one of these sessions one of the members of our research team doing action-research in a 'ghetto' school, explained a situation of cultural conflict in her class in the following way. She asked her students to work in small groups to solve some mathematical problems, and she explained to the students that in such a way they could help one another. However, two of her students, Imram and Nadia, refused to work in small groups. When the teacher insisted, Imram said *'I do not want, if you insist I do not come back to school',* and Nadia's answer was *'If I have any doubts I ask you, you are the teacher, that is why you are here'.*

Rosa, one of the teachers participating in the in-service sessions, said at one of the subsequent meetings: *'I had never thought that some of my students would feel so uncomfortable when working in small groups, but they just accept it to please me!'* In fact, one of her students told her during an interview: *'Working in small groups is very nice, but in one's life one has to do real things alone [...] Moreover, in the examination we do it alone, don't we?'* In her class, the students accepted the norms that regulate its dynamics without apparent conflicts, but this acceptance did not mean that the students did not experience conflicts arising from their different

interpretations and valorisations of the norms. The conflicts are less 'visible' in this class than in a class with a high percentage of immigrant students where, from the start, the teacher expects the students 'to behave differently'.

On another occasion, a teacher member of the research team presented an example of a conflict within her immigrant students' mathematics classroom: Nadia refused openly to share her mathematical knowledge with her peers by saying *'To learn with them? They do not know a single thing about mathematics, they always fail their examinations!'* Whenever some of her peers contributed to the whole group discussion, Nadia got very anxious, because she really wanted to have an answer from the teacher on the blackboard, *'Come on teacher, we will not have time to finish it, do it yourself on the blackboard'*. While her peers discussed the problem, she did the homework from other subjects. She explicitly and noisily put her books on the table to show her disagreement, in fact, to show that she did not accept the other students as valid discussants of mathematical knowledge.

As a reaction to that explanation, one of the teachers in the working group, Rafa, explained that he suspected that some of his students also experienced similar conflicts. He is teaching in a secondary school located in a wealthy neighbourhood, which is well known for having high scores in the external examinations. At the next meeting Rafa came with the example of a brilliant student, who engaged in the following conversation:

Rafa:	Do you take notes when your friends explain how do they solve the problem?
Student:	No, I wait until I know it is correct...
Rafa:	And how do you know it is correct?
Student:	Because you say it!!
Rafa:	But, sometimes we have two or three groups presenting different ways to solve the problem all of which are correct and I say that you may choose the one you like the best. Which one do you choose then?
Student:	[smiling] Montse's one [Montse is the girl with better grades in the class]
Rafa:	And when neither I nor Montse present a solution?
Student:	Then, I copy nothing and when I come back home I ask my father!
Rafa:	And what happens all the time while your friends are presenting their solving processes? Is it useless?
Student:	[smiling] It is your problem, you will not finish the program!!

These are a few of the examples that we have collected throughout our research that give us evidence of the existence of cultural conflicts within the mathematics classroom, in terms of the norms of both the social dynamics and the mathematical practices. Our research has also shown that cultural conflicts exist both in schools with immigrant students, whether they are a majority or not, and in schools with no immigrant students at all. This fact reaffirms our belief that the outcomes of our research might well be beneficial for all teachers and in all classes regardless of the number of immigrant students present.

CHANGING THE SYSTEM: PACKAGED MATERIALS VS. TEACHING STRATEGIES

As has been pointed out, the initial request from the administration was for 'packaged materials produced in different languages' to be spread out in the schools. We understand that behind this request was, on the one hand, the idea that once having overcome the linguistic barrier all the problems would be solved and, on the other hand, the idea that there was a need for materials to help immigrant students to reach 'the same level' as local students.

On the basis of the assumptions of our research project, stated earlier, and in the light of our findings, we have argued with the administration that 'ready made' materials would be of little help for teachers and students. Instead we argued that any change towards an improvement of the learning experience of immigrant students should come through developing and disseminating teaching strategies that facilitate the discovery of cultural and social conflicts and their positive handling, and through giving clues to teachers that allow them to create their own materials adapted to their own classroom's realities.

In the process of searching for teaching strategies suitable for multicultural classrooms, we organised a working group of in-service teachers, with the aim of, on the one hand, discussing and analysing their actual needs regarding teaching in multicultural situations and, on the other hand, to experiment with teaching strategies and learning activities that were suitable in their particular contexts (see Meaney, this volume, for further discussion on working with teachers in similar contexts). The examples we have presented concerning norms and differences are from this working group and illustrate how it developed. From this work we have been able to offer some recommendations for teachers on how to deal with the diversity they have in their classes. These recommendations include adopting an approach to the curriculum that is non-reductionist in its contents, articulating different meanings for every mathematical idea, and using a participative methodology to promote the contribution of all the students.

To summarise, the following are what we consider essential conditions to help to overcome the 'invisibility' of the cultural difference and to 'positivise' it within the mathematics classroom, thereby facilitating the immigrant students' learning process:

- to clearly make explicit, as far as possible, the implicit rules that regulate the classroom social dynamics and the classroom mathematical practices,
- to allow the acceptance of out-of-school mathematical knowledge and skills in order to reach those whose family's culture is far from the school culture,
- to acknowledge and rehabilitate the mathematical knowledge linked to every culture in order to ensure the survival of the different mathematical models that relativise the unity of mathematics,
- to accept the challenges of cultural diversity's fact in order to promote a process of establishing a real cultural interaction.

CONCLUSIONS: 'TOP-DOWN' VS. 'BOTTOM-UP'

Social change via educational research is a difficult and time-consuming process. Moreover, we agree with Ahmed (1987, p. 81) that 'improvements and change can only be sustained if teachers in the classroom believe in and support the developments taking place. Impositions from above are therefore unlikely to work'. In particular if research has to have a direct application in school practice it requires, as Ruthven (1999, p. 12) states, 'supporting work within schools directly aimed at the improvement of mathematics teaching through scrutinising current practice and its outcomes, and the identification and the appraisal of viable adaptations and alternatives'.

Moreover, development through research of a social nature is a process which requires overcoming some obstacles, as Ruthven (1999, p. 12) points out in the phrase following the one previously quoted: 'An initiative of this type would, however, be likely to encounter two particular obstacles, one is the currently restricted range of research tools tuned to collecting and analysing evidence about teaching and learning under typical conditions, the other is the lack of a powerful means through which teachers can actively explore and evaluate alternative perspectives and practices in mathematics learning'.

Developing socially-oriented research that analyses mathematics classroom practice under its normal conditions, and through the methods that are presently available, is a long process. It takes time to get data from classroom observations, and to analyse them in a collaborative framework in order to have feedback for the possible modifications to explore. Having teachers participating in research teams takes time, time that has to be allowed to them by reducing their teaching hours. In short, time is probably the most crucial resource for developing any research agenda which aims to reform educational practice.

Rephrasing Lappan (1997, p. 233), time is required:

- for politicians to understand the ideas of the reform and to figure out and accept what they might mean for the existing practice, and to create opportunities for administrators and teachers to learn about these ideas,
- for all those involved in the reform process to learn the knowledge required,
- for teachers to grasp the reform ideas and to come to understand how they might reshape their existing practice around these ideas, and
- for educators to reflect on their attempts in carrying out these supposed improvements.

We would add to those points that, it is not only time that is required, but also big efforts need to be made by any individual researcher in order:

- to encourage the educational system to accept the crucial role that research may play towards a positive development of reforms, by making researchers' voices heard and understood;
- to encourage the whole research community to accept that doing research related to education is not only a socially committed activity, particularly for research

that refers both to teachers and students, and to their immediate environment, but also that it is a political process because it involves promoting changes that will affect others;
- to accept, deal with and make explicit the limitations and constraints of doing research in socially and culturally complex contexts; and
- to find alternative methods and approaches that, while still being acceptable within the research community, are fully respectful of the social and cultural idiosyncrasies of the persons and their situations.

After just a few months working on the project we were already conscious of how difficult, if not impossible, it would be to change the beliefs of the educational administration about the mathematics learning of immigrant minorities. In order to seek acceptance of the ideas emanating from the project by the educational politicians we had to convince them that our project would be of benefit to all the students and not only to those belonging to minority groups. Based on the evidence from the project, the research team strongly believes that an inclusive approach to both content and methodology in mathematics classroom, will be beneficial not only to children who are 'culturally different' but also to the children of the Catalan communities because, in particular, it will make them aware of learning in a non-ethnocentric context, which has respect for other cultures and which also enlarges their understanding of mathematics as a cultural product. However, what was so clear to us from our research, seemed to be hard for the educational politicians to understand.

At a certain point we realised that, if we wanted to convince them of the importance of the project, and of its development, we would have to 'sell' it to them by, on the one hand, basing our discourse on what they considered to be 'important issues', and on the other hand, by 'proving' that what we were saying was applicable to all classrooms. Therefore, our first argument with them was based on their priority of developing in youngsters the value of respecting diversity, including cultural diversity. Since attaining 'interculturality' is a goal for the Catalan educational administration —even if the meaning they gave to that word could be debated— we argued with them that mathematics teaching could help in the process towards 'interculturality' only if it was first acknowledged that mathematics 'also' has cultural roots. Therefore, we 'showed' them, through many examples coming from our data, that mathematics is a cultural product.

As a second step, we had to 'prove' to them that any classroom could be considered as being 'culturally diverse', even if all children belonged to Catalan families. To do that, we showed them the many examples that we had as a result of experimenting with the same teaching strategies and learning activities both in schools with a large number of immigrant students and in 'normal' schools. That way we 'proved' to them that socio-cultural diversity was a reality in all mathematics classrooms and that it was the source of conflicts that interfere with the learning processes. The only difference was that in 'normal' classes the diversity and the conflicts were less visible, as has already been mentioned.

In order to spread the outcomes of our efforts, we are disseminating our results and analyses through journals and in summer schools for teachers. Both in the

publications, and in the in-service courses, we present our work with two goals. Our first goal is to make the teachers aware of the fact that the basic assumptions of our project are real issues in every classroom, and that taking them into account can contribute to making mathematics learning become a possibility for **all** students. Our second goal is to spread the idea that cultural and social diversity, far from being a problem, can be a source of richness if the teachers can take the advantage of it. However, since most of the teachers feel they have little knowledge about how to develop teaching strategies, create materials or adapt curricular content on the basis of socio-cultural diversity, we have also published and presented the processes of working within the group.

Our experience of working groups with in-service teachers and of summer schools is based on the idea we offered, as a research team, to the educational administration to develop in-service programs that would work from the actual needs of teachers facing multicultural groups. These programs would help the teachers find more appropriate teaching strategies, based on our strong belief that if we want any educational act to be positive both for the individuals and their communities, it would be helpful to begin to consider the perceived differences associated with different socio-cultural contexts as a source of richness, rather than problems, in the educational context. The first stage of doing this is likely to include making the differences explicit to everybody in the teaching/learning process. Therefore, the basic goal of our programs is to give tools to the teachers to help the 'differences' to be made explicit, and to use them as starting points for building rich teaching and learning resources.

However, being conscious of how difficult it is to change the administration, and that often trying to change the system only from 'top-down' fails, our efforts are also directed to encourage the involvement of school principals and inspectors that have among their responsibilities the schools with immigrant students. Very often, they see the ideas underlying our project as a threat to the well-established tradition. Socio-cultural diversity and the learning conflicts arising from it are not a 'priority' for them, but rather conflict is understood as a 'disruption of' or a 'transgression from' the norms. It is often forgotten that the meanings of the norms are usually implicit, and that the teacher' interpretation of the norm and that of the student can be different. Even if we are conscious that not all the 'disruptions' and 'transgressions' can be justified by, or explained through, different interpretations of the norms, we believe that some of them could be minimised through making explicit, and negotiating, the norms with the students. It is our plan, to base our discourse with school principals and inspectors on the importance of this perspective to convince them of the benefits of supporting our project and facilitating its development.

Finally despite all the obstacles, the tensions and the challenges we have had to face in developing our project, after five years' work we are convinced of the importance of continuing and spreading it, and involving in it as many practitioners as possible, as well as seeking a full appropriation of it by the representatives of the different levels of the educational system. It will certainly be a long and difficult process, but we are convinced it is worth the effort.

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GELSA KNIJNIK

LESSONS FROM RESEARCH WITH A SOCIAL MOVEMENT

A Voice from the South

Drawing on empirical data from a 10-year research program that I have been involved in with the 'Movimento Sem Terra' (MST), the Brazilian Landless People Movement, I discuss the pedagogical lessons I learned from this research experience. These lessons are connected to two main themes: 'To learn in the process' and 'To understand knowledge in a political perspective'. These topics form the basis of the arguments that I develop throughout the chapter. As a very personal story, the genre of my writing is, to a certain extent, different from that of the usual mainstream research texts. This change in style is critical for a book such as this one where there is a strong recognition of the socio-political dimensions of mathematics education. For me, the personalising of the text represents a very political act because it highlights the fact that knowledge production is not a neutral activity. This personalisation reveals the subjectivity of the researcher, her/his political stance, and the ways of interpreting the world; all of which imprint the topics and the methodologies that the researcher chooses within the research process[1]. Hence, what follows is both a very personal and political account of the mathematics education in a specific social movement in Brazil.

TO LEARN IN THE PROCESS

My intellectual trajectory as a researcher is strongly marked by my first experience with youth and adult education of the Brazilian MST in the beginning of 1991. At that time, I was in the small town of Braga, working at a MST teacher-training course. Late one afternoon, Father Sergio Görgen informed me of the ideas involved in the first National Literacy Project for adults in the Movement that was to begin soon in the southernmost state of Brazil. I was being invited to participate in it, as an adviser in mathematics education. In the ensuing discussion in which we negotiated my potential participation in the project, I raised my concerns about feeling inadequate and unprepared for the demands of such a project. However, he was resolute in his stance and answered emphatically: 'We intend to carry out this first national project. If you want to participate, and learn in the process, fine. If you

[1] A similar issue about the visibility of the researcher in her writing is discussed and exemplified by Chronaki (this volume).

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 125—141.

don't, we'll do it without you.' The choice of 'learning in the process' was perhaps one of the most important life lessons I have learned and it stays with me today — carrying into my work as researcher in the field of mathematics education. The process is particularly powerful when working within a social movement in the southern half of the world.

The movement to which I had been invited, at the time unbeknown, is now internationally acknowledged for its struggle for land reform. In a country with the highest concentration of land ownership in the world, this land reform has as its fundamental basis the goal to enable a more equitable distribution of wealth and promote social justice. The MST has also been acknowledged internationally for prioritising education as one of the central dimensions of its struggle, in particular, the need of children, youths and adults for mathematical education (Caldart, 2000). This fact is even more relevant when the official data on schooling in Brazil are considered. Out of a population of 160 million, 20 million report that they do not even know how to write a note (IBGE, 1990). These figures represent a significant portion of the Brazilian population, particularly when the demographics of the cohort are considered —most of them are young people (adolescents) and adults. These people have been excluded from participating in the education system and are even further excluded from participating in many other facets of life beyond the education system. As such, the MST serves an important and critical role in the education of a significant sector of the Brazilian population.

The principle of 'learning in the process' was one of the foundations that supported that first project. The project has three distinct roles embedded within it. There are the students for whom the project is focused. The teaching roles are supported by two other roles —the monitor and the adviser. The monitor assumes the role of teacher, preparing and giving classes to the students, whereas the adviser assumes roles of discussing the guidelines of the project and helping in the pedagogical issues connected to its implementation. My role within the project was that of an adviser. Over a period of two years approximately 100 monitors and 2.000 students were involved in the initial start up of the project. Now, 10 years later, the principle still lives on in the work of the approximately 1.200 monitors and 20 thousand learners who participate in the youth and adult education projects of the Landless Movement. Students, monitors and advisers must 'learn in the process'. This principle is seen as relevant because the working conditions within which we are located are quite vulnerable. This vulnerability comes from a number of areas and hence, key considerations must be accounted for when projects are developed. First, there is not a great tradition in Brazil regarding youth and adult education projects in rural areas[2].. The projects see this sector of the Brazilian community as a most important consideration since they are not catered for otherwise. Unlike other countries, as a system, we had little experience in working with the special needs of

[2] It is important to stress that Paulo Freire's work had —and still has— great influence in the Brazilian educational context as well as in the Latin American one. His ideas concerning popular education inspired most of the adult educational projects which have been developed in the south of America. Nevertheless, at least in Brazil, before the emergence of the Landless Movement there were only few isolated experiences of educational projects for adults in rural areas.

this sector of our communities. Second, even less experience has been accumulated in work organised *by* a peasant social movement —and not *for* it. This emphasis of moving away from 'for peasants' to 'by peasants' represents a significant ideological shift and has important repercussions for how learning is conceptualised and organised. At a more pragmatic level, we had to consider the level of education of the people who were to act as mentors/teachers for the peasants. For many of the monitors, in many parts of the country, their level of schooling did not go beyond the initial grades of elementary school. As such, we needed ways to enhance their learning in order to support their colleagues. Accordingly, the 'learn in the process' model was ideal.

In the first years of the project, we were committed to the 'learn in the process' principle. We were aware that Youth and Adult Education should be attuned to the Landless Movement pedagogy, in particular with its educational principles[3]. These principles have guided the Landless Movement work in education. They constitute points of reference for the implementation of Youth and Adult Education projects in the Brazilian camps and settlements[4]. This implementation has occurred rather unequally, in part due to the heterogeneous levels of organisation of the educational staff of the Movement in each state of the country. It was further compounded by the improbability and difficulty of obtaining advice from universities and non-governmental organisations connected to popular education, as conceived in Latin America.

In relation to the more explicit focus on mathematics education, from the beginning of the project we identified the theoretical perspective of ethnomathematics as the one most directly connected with the educational principles of the Landless Movement pedagogy (Caldart, 2000). As noted by many of the authors in this volume, ethnomathematics recognises the histories and contexts of the people with whom it is located. We wanted a mathematics that was relevant, meaningful and purposeful for our people, so ethnomathematics was the most appropriate form of mathematics to consider. However, at that time and still today,

[3] These principles are now expressed in the following points: relationship between practice and theory; methodological combination between teaching processes and training; reality as a base of the production of knowledge; socially useful formative contents; education for work and by work; organic link between educational and political processes; organic link between educational and economic processes; organic link between education and culture; democratic administration; self-organisation of the students; implementation of pedagogical collective and continued teacher education; research attitude and skill; and combination of collective and individual pedagogical processes (MST, 1996).

[4] The settlements are a result of continuous land occupation advanced by the families belonging to the MST. These occupations are used as a means of pressuring the reform of the existing (ill) land distribution. Historically, occupation took place only on large, unused extensions of land. However, recent occupations have even taken place on the sides of roads and highways. These attempts of occupation are violently repressed by police forces through confrontations that may result in the death of many occupants. One group of families —sometimes exceeding a thousand people— organises several occupations that may lead to obtaining ownership over an extension of land and, then, founding a settlement. The process of occupation constitutes a relevant organisational experience for the community, on which the pillars of a settlement organisation is built. It is in the settlements, over which people have gained ownership, that the families, members of the MST, start producing means for subsistence.

the 'application' of this perspective to youth and adult education is not a simple operation. In part, this is due to the very specific characteristics of a peasant social movement such as MST. Such challenges are typically posed by the contexts within which the peasants are located so that a common ethnomathematics for all people was not appropriate. Rather we needed to identify the appropriate ethnomathematics for the communities within which we worked. This demanded that we would need to work closely with, and develop an intimate knowledge of, our communities in order to be able to identify what might be the best forms of ethnomathematics for that community. We have dealt with three different 'new' challenges.

The first challenge has to do with the 'new' social realities produced by the movement, which require new pedagogical answers. This challenge can be understood by means of the two complementary axes that the MST has defined for education: the *struggle for the right to education* and the *construction of a new pedagogy*. The first axis deals with the different ways in which the movement has pressured the government for giving their children access to formal schooling. This struggle has been especially tough in settlement areas where not only material conditions of living are precarious, but also the lack of qualified teachers is acute. From the very beginning of the land occupation educational work, even in an informal fashion, was carried out in the settlements. However, it was only in 1996, after an intense process of pressure to the government, that the first 'moving school' for children was established in the country, in the state of Rio Grande do Sul. The 'moving school' is characterised by being in constant movement —as families occupy new territories, the school needs to move with the families, thereby ensuring students have constant access to education. The school is currently organised —in December 2001— in 15 MST settlements in the Brazilian state mentioned above. It involves approximately 600 children and 60 educators. Most of these teachers have no formal pre-service teacher education. Concerning youth and adult education, there has also been a growing pressure for the government to support literacy projects within which numeracy has occupied a central place.

The second axis expressing this 'new' social product within the MST deals with the construction of a new pedagogy. Unlike traditional forms of pedagogy which are often seen to be fairly politically neutral, the MST has been developed from very different social roots and hence the new forms of pedagogy are, and must be, radically different. Being located in and born from contexts with so much adversity and in struggles of such as radical nature as those within which the movement operates, education gains a yet clearer and stronger political dimension. This politicisation of education requires, from all areas of knowledge and in particular from mathematics education, the adoption of approaches that allow articulating the local world of each settlement and the immediate demands of the struggle with educational processes. This, in turn, opens possibilities for students to understand the larger world —outside the settlement— and engage in further studies. This process of articulation has given birth to a new pedagogy that, as I have previously mentioned, follows an ethnomathematical approach.

The second challenge tackles the 'new' brought by ethnomathematics, a relatively recent area, whose formulations are still, in a sense, incipient, particularly in relation to its curricular implications. Authors such as Vithal and Skovsmose

(1997) have pointed in this direction. The studies that I have developed together with the MST (e.g., Knijnik, 1996b, 1997, 1999, 2000) have analysed, problematised and provided further understanding of the connections between ethnomathematics and pedagogical processes in mathematics.

The third challenge that we have been forced to face relates to the recognition that the first youth and adult education project refers to the implementation of a 'new' attitude and research skill. This is one of the founding principles of the Landless Movement pedagogy. Within this process, it is expected to develop with the youth and adults particular attitudes related to how to use the research methods in collecting data for planning youth and adults' educational projects. Moveover, it is expected that they can follow the implementation of these projects through as consistent methodology that gives them conditions for evaluating their social, educational and cultural repercussions. These attributes of 'attitude and research skill' have challenged us to understand knowledge within a political perspective. In dealing with these challenges, I have learned a few lessons. They are at the core of the next part of this essay.

TO UNDERSTAND KNOWLEDGE IN A POLITICAL PERSPECTIVE

In recent times, mathematics education has had to deal with issues that were previously absent from the debates in this field of knowledge. Among these issues are the processes of social exclusion. Studies on students' mathematical performance in many countries have shown that this school subject is one of the elements that has strongly contributed to school failure and mortality. In the case of Brazil, the results of the examinations organised by the SAEB —Sistema de Avaliação da Escola Básica (Primary School Evaluation System)— have contributed to increase these problems. This type of assessment system should not only be criticised for the political purposes that inspire and motivate them —which are aligned with the guidelines of the World Bank— but also for the methodologies that they utilise —which are in agreement with national curricular standards that have been centrally conceived and that homogenise the contents of schooling[5]. These assessment systems also should be critically examined in relation to the way in which they make explicit the failure of many children and young people in learning mathematics. The learners who are doomed to failure in these systems come mainly from socially-disadvantaged groups. The result of such assessment practices is the creation of yet more exclusion for these children and young people who come to internalise their failure as a natural or normal process rather than something inherent in the assessment practices.

In part this is due to the hegemonic role and status of mathematics in our present societies, in a time marked by rapid and profound technological innovations. These innovations are producing new challenges for humankind. For example, Moses and Cobb (2001) use data from the Department of Labor of the USA in order to show that, currently, in that country, 70% of all the jobs require technological literacy. Furthermore, they argue that around the year 2010, all jobs will require technical

[5] An analysis of this issue is presented in Knijnik (1997).

skills; and at that time, we do not even know what the 80% of future jobs will be. Faced with these data, the authors ask:

> Who's going to gain access to the new technology? Who's going to control it? What do we have to demand of the educational system to prepare for the new technological era? What opportunities will be available for our children? (Moses & Cobb, 2001, p. 22)

I would add to the thinking of these authors another dimension on this theme of new technologies and changes in the work world, which I consider equally important for the field of mathematics education. This is a matter of problematising and critically examining these technologies, asking ourselves how they are being used, what interests have guided the research that supports them, and what portions of the population have benefited from such technologies, in terms of quality of life. I argue about the importance of looking critically at what has been named 'technological advances', not looking back with regret and intending to return to a past marked by manual labour. My attitude is that of avoiding the glorification of such advances, by rejecting a naive position in relation to the vast web of interests that guides their production and dissemination. For educators, it is relevant to ask ourselves about the contributions we can give to the new generations in order to avoid their school failure in mathematics. One of these contributions has been indicated by authors such as Moses, when he speaks of access to mathematical literacy, in opposition to the failure that has characterised 'peripheral' countries such as Brazil —and that is also present in multi-ethnic contexts within 'central' countries such as Spain (see Gorgorió & Bishop, this volume). Another contribution, which I mentioned earlier, is the need to be suspicious of this discourse regarding the new technologies and the advances fostered by them. This discourse is usually not problematised, and we need to start questioning it. A third contribution would be to think more carefully about the evaluation processes that we have used in our school activities, and also in the evaluation processes present in the national elementary, secondary and university-level exams currently held, for instance, in countries like Brazil, as I referred before.

It is through the processes mentioned above that mathematics, as a field of knowledge, has contributed to social exclusion processes. Among such processes there is one which is certainly not new but at which we, educators, have recently looked at in different ways. I refer to the exclusion produced by knowledge. This might better be said if we rendered it in the plural: the exclusion produced by knowledges. It is a matter of examining the social effects produced by the presence, in the school curriculum, of a particular set of knowledge —which includes those that are authorised to circulate— and also those that have been silenced. The question I am interested in problematising is how exclusions are produced also in what Frigoto (1995) very appropriately called the 'school floor', especially paying attention to the role that has been played by the mathematics that we teach in these 'school floor' exclusion processes.

It is not only new techniques, or techniques implemented in the best way, that will be able to prevent failure at school. The time has gone when everything was simply a matter of how to best teach the usual contents, and to decide what games and practical materials would make our students understand more mathematics. It is

obvious that we want them to know more mathematics. But, at the birth of a new millennium, there are different demands, different issues that affect even so called 'hard' fields of knowledge such as mathematics.

We seek to escape technicism, to look at people's lives, the ways they are in the world, their ways of signifying life, in other words, their culture. Our challenge is to root mathematics education in culture; culture being understood here as something produced by people and social groups that is not determined, with a closed meaning for once and for all. Culture here is *not* considered something consolidated, a finished product transmitted from one mind to another. This form of conceptualisation of culture implies comprehending it as a conflictive, tense terrain, in permanent dispute for the imposition of meanings.

Moreover, from the ethnomathematical perspective, the concept of culture also moves away from the conservative view of 'the heritage of humankind'. Considering that this cultural heritage is a social production resulting from all our efforts, the expression thus supports the argument that humanity as a whole has the right to access and use knowledge created by humans. Nevertheless, the expression *cultural heritage of humankind* is very often identified only with academic mathematics. It is precisely this identification that masks power relations that, in turn, legitimise one very specific way of producing meaning —the Western, white, male, urban and heterosexual one— as the cultural heritage of humankind. By providing visibility to other mathematics besides the academic one, ethnomathematics discusses precisely this apparent 'consensus' on what counts as *cultural heritage of humankind.* It is, in fact, a part of a broad, heterogeneous production, precisely the part produced by hegemonic groups. What groups must remain silenced and hidden, not represented in the school curriculum also in the field of mathematics? The expression: 'Our job as teachers is to bring to the curriculum, to teach at school, the mathematics that has been accumulated by humankind', must now be questioned. Precisely this is what is now at stake: What mathematics has been called 'mathematics accumulated by humankind'? The important question now becomes 'which sectors of humankind are represented in the mathematical knowledges represented in school curriculum and how relevant are they to our students? What groups have remained silenced, concealed, under-represented in the school curriculum, also in the field of mathematics? As we ask ourselves these questions, we are establishing close ties between mathematics and culture, that is, between mathematics and the ways in which people give meaning to the world. In so doing, we attempt to problematise what authors such as Walkerdine (1988), Frankenstein and Powell (1997) and D'Ambrosio (1999) indicated as the characteristics of school mathematics: Its marks of eurocentrism, whiteness, middle class-ness, maleness, and, I have added, urbanity. Who has remained concealed? The non-European, non-white peoples, women, gays, lesbians, and bisexuals, and the rural groups. It is the cultural productions of these groups that have been systematically excluded, and have remained outside the school curriculum. It is precisely these under-represented (or non-represented) groups whose mathematics must become central to the MST project —particularly when considering that the students come from non-European, non-white and rural backgrounds. Questions as

to the representations of their knowledges, and in particular, their ways of mathematicising their worlds becomes central to the projects.

The branch of mathematics education that has been especially attentive to these questions is ethnomathematics. Since it is a very heterogeneous field constituted by many different approaches, I do not intend to present here a concept that will sum it up in a unified way. What appears to be present in most of the ethnomathematics studies is the idea that this approach attempts to identify and celebrate the ties between education and the culture of the social groups. It seeks to do this by establishing connections with the history of these peoples, their present and past histories, and their traditions, including their ways of dealing mathematically with the world. These processes have throughout history, been absent from the traditional school curriculum. The history of mathematics that we tell our students is the history of official knowledges, of dominant knowledges. In formal school we teach the dominant ways of reasoning, as though they were the only way of thinking about the world, in particular dealing mathematically with the world. It is in this sense that we say that ethnomathematics seeks to deal with the non-official history of the present and the past. This is done by providing visibility to this present and this past and, in so doing, ethnomathematics advocates the position that mathematics is a cultural production. This position is in stark contrast to what is commonly viewed — mathematics as a consensus (or as a Platonic creation) thereby reifying it as the supremacy of what has become legitimate because it is superior from an epistemological standpoint.

The arguments I presented here are the basis of the work I have been developing with the MST. Among the many episodes that I have collected over these ten years of study and research, I selected one to present in the final part of this section because it seems worthwhile to analyse it from this theoretical standpoint. It constitutes a piece of a project developed during one whole school year, involving all the students of a school in a MST settlement.[6]

The Melon Project: A case study

From an ethnomathematical standpoint, it is essential to identify the practices of the people and the ways in which they mathematise their practices. One such practice was identified —the production and marketing of melons. The pedagogical work focused on the follow up of melon production in the settlement, an activity that directly involved the teachers, the pupils, and the families that lived in the settlement, along with other melon farmers in the region whose children attended the local school. At the time of the year when the project commenced, melons and their production and marketing were key activities within the community so it seemed like a relevant and meaningful project for ethnomathematics. In each grade, the pedagogical work was performed with different characteristics in terms of content and teaching approach. This was due less to levels of schooling than to the characteristics of each teacher —who all happened to be women. Typically, the

[6] This episode is also analysed in Knijnik (1999), in which I emphasised another theoretical aspect of the research.

practices the teacher adopted were influenced by factors such as where they came from —whether they belonged to a settlement or not—, their academic formation, and their political involvement with MST.

Intentionally, I tried to work carefully in order not to become the person who guided the pedagogical process that developed in each grade. Working *with* the teachers, I discussed with each teacher separately, and also with all of them collectively, their proposals for the activity. The pedagogical work was expected to develop over the growing period and would end in the beginning of December, when the settled families would sell their produce. However, an event, which occurred in the late afternoon on Saturday, November 23, disturbed the settlement's life, and consequently impacted significantly on the work we were carrying out. A heavy hailstorm lasting 15 minutes destroyed most of the melon production in the settlement, estimated at approximately 300 tons. The very means of financial stability and their purpose for living and working was effectively destroyed. The desolation, which dominated the whole community, was the hallmark of that difficult time for all of us –students, families, communities and the MST team. After the storm and when the students returned to class, the impact of the hailstorm on the pedagogical project was profound. The young people of the more advanced classes refused to discuss the subject. As one of them said: 'What has happened has happened, one cannot think about it!' They would not even analyse the size of the loss. As one of the girls said: 'It was a total loss, this was greater than knowing how much the total was!' The effect of the loss on the community and the students profoundly impacted on their desire to have this focus in their mathematics.

'Learning in the process' never became more apparent —not only could we not continue with our pedagogic project due to the emotional turmoil caused by the hailstorm, we had little idea of how to move forward. The significant presence at the school on the following Monday was a woman from the settlement, a mother and grandmother of pupils. She brought to the scene a discussion, which I considered the 'key' to carrying on pedagogical work: the question of agricultural insurance[7]. This theme became a linchpin for our pedagogical projects as it allowed us to take up the discussion on melon production again in the various classes, even though the assessment of the losses, which more directly involved mathematics, was impossible to perform having met with resistance on the part of pupils.

The pedagogical work for that school term had been significantly influenced by the hailstorm but its effect on the community was profound. There was great despondence among those women and men who had worked for one whole year and now had to reorganise to survive with the losses, and to plan new planting under even shakier financial conditions. Such strong emotions and social impact cannot be divorced from a pedagogical project such as ours as its impact is too profound.

In the follow up process that occurred from the time of tilling the soil until the (failed) sale of the fruit, pedagogical work was constructed from the perspective of

[7] The Agricultural Insurance Bill, proposed to the National Congress of Brazil by a MST Congressman, establishes financial guarantees for small farmers who have suffered losses in crop yields due to climate problems. The bill has been in Congress for several years and has met with difficulties in obtaining approval.

ethnomathematics. In this work, melon crops were examined not as material from which the school subject could be exemplified. Rather, the school knowledge was constructed at the heart of the follow up, examination and problematisation of the community's productive activity, taking this activity not as a source of inspiration or exemplification, but, on the contrary, considering it the central objective of study. This means that the perspective used goes against the idea that the 'world outside' the school, in particular popular practices and knowledges of the socially subordinate groups, could serve as what Mellin-Olsen (1987) described as 'intellectual material', that is, as a point of departure for teaching school mathematics. The melon planting process was not a tokenistic focus but an integral and important aspect of the survival and success of the community and hence served a real need for using and understanding mathematics in a very real context. As I have suggested elsewhere (Knijnik, 1997), the key expression to be problematised here is 'point of departure'. If we say that we simply begin with practices and knowledges of the group with which we are working, their culture, their ways of living and signifying the world, we are considering that these are only the initial point of a rising trajectory which would lead from the lower point to another that would represent surmounting it, to wit, learning other ways of signifying the world. These ways are produced by a rationality that originates and is impregnated by the Western academic mathematical knowledge, strongly marked by its Eurocentrism, whiteness, heterosexuality and maleness (Walkerdine, 1988). In fact, what I sought to build with the settlement school community —here seen as the full group of female and male teachers, female and male students and their families— was pedagogical work that would centre on the main productive activity of the community.

The effective participation of the community in the pedagogical process —especially the melon producers— was constructed in order to qualify the ways of production, and not simply for the purpose of collecting elements that would enable the study of mathematics. This qualification occurred, however, in a dynamics that paid attention to the traditional practice of the group that, when problematised, could be better understood by the practitioners themselves. This occurred, for instance, in measuring the lands to be farmed. There were settlers who used what I have been calling 'popular methods of land 'cubação''[8] (Knijnik, 1996b, 1997). The first of these methods can be described as follows, using the words of the peasants themselves:

> Here is a land with four walls. First we add all the walls. Second, we divide the sum by 4. Third, we multiply the obtained number by itself. This is the 'cubação' of the land.

[8] Land 'cubação' is a term used by the Brazilian peasants to express the act of finding the area of a piece of land.

A second method[9] was presented by one of the settlers as follows:

> This is the land with four walls. First we add two of the opposite walls and divide them by two. Second, we add the other two walls and also divide them by 2. Third we multiply the first obtained number by the second one. This is the 'cubação' of this land.

The modelling involved in each of the methods was discussed with the students of the higher grades. The 'changes' produced by each method were examined. Further, it became necessary to compare the two methods and discuss the advantages and limitations of each of them. At the same time, what some groups of the MST call 'book mathematics', i.e., official school mathematics, was taught and more precise methods for land measurement were presented. However, in the settlement there were peasants who used other processes to measure the land. Some delimited a hectare of land to be farmed using the '100 by 100' square —i.e., 100 metres by 100 metres. Sergio, a young settled peasant, used as a parameter to determine the size of the surface the 'tractor time used to hoe'. According to him, 'one puts the tractor on the land. Working with it for 3 hours one has exactly one hectare'. The issue of measuring land by time was analysed together with the students and peasants. What, at the beginning of the work appeared to be an 'impropriety', began to be better understood by the group as examples of linear distances expressed by measures of time were brought up for examination. With increasing intensity, currently, in the city and in rural areas, what really matters is time used to perform a given displacement. For purposes of cultivation, the hour of tractor use is more relevant data than the precision related to square meters of land. As Sergio said, 'a few metres more, a few metres less, it really does not make much difference'. Possibly this was what in fact occurred in the reality of the melon crop. One of the lessons that I learned from the settlement community is that, depending on the purposes for which land is measured, the need arises for greater or lesser precision, and therefore, this is the need that will determine which is 'the' best method to use.

This episode that I have just described is an example of the ethnomathematical approach that I have been developing with the Landless People Movement. It can be summarised as:

> The investigation of the traditions, practices and mathematical concepts of a subordinated social group and the pedagogical work which was developed in

[9] This method has not only been used in the southern region of Brazil. In the coastal area of the northern region of the state of Pernambuco, the sugar-cane growers of the municipality of Igarassu use it in their farming activities, as shown in Abreu and Carraher's (1989) research. Sotto (1994) also refers to the use of this method in peasant communities in different regions of Chile. A historical reference must also be made. In ancient Egypt, the calculation of quadrilateral areas was performed by identical procedures to those involved in the method which I am showing here (Peet, 1970, p. 93; Coolidge, 1963, p. 13; Vasconcelos, 1925, p. 72). Peet presents the issue when he analyses problem 51 of the Rhind Papyrus, which deals with the area of a 'triangle of land' 10 khet high and 4 khet used as a base. In the context of this problem quadrilateral areas are discussed. He writes that historical evidence has led researchers to claim that the method was already used in the Ptolemaic, Roman and Coptic periods of Egypt, for taxation purposes. Peet adds that landowners in ancient Egypt probably knew that their calculations produced areas larger than they actually were. The landless people, however, did not realise this.

> order for the group to be able to interpret and decode its knowledge; to acquire the knowledge produced by academic Mathematics; and to establish comparisons between its knowledge and academic knowledge, thus being able to analyse the power relations involved in the use of both these kinds of knowledge. (Knijnik, 1999)

Performing pedagogical work with an ethnomathematical approach, as defined above, enabled the recovery and interpretation of the popular methods for land 'cubação'. Thus, it contributed for these knowledges to be learned by students who did not yet know them, or, when they did, were not aware of their approximate character. Like other elements of the culture of subordinate groups, popular land 'cubação' methods tend to disappear, due to the characteristics of oral tradition. This is, in part, due to the fact that they are not legitimated by the dominant culture and, consequently, are not included among the contents usually transmitted by school. The students' testimonies have shown that practices of land 'cubação' —important for their farming activities— are being 'wiped out'. In this sense I interpret the value attributed by the Landless Movement —and also by myself— to the recovery of some of their traditions. The question that must be asked here is why carry out this recovery. In other words, to what interests does it respond?

Of course this is not a question of glorifying popular knowledge, in order to use this glorification to close the subordinate groups into ghettos, in an ethnocentric operation that reinforces the social inequalities. At the same time, I have also taken care not to glorify academic knowledge, as 'the' great metanarrative capable of explaining and presenting solutions —preferably a single one— for all problem-situations of the concrete world. When analysing the knowledges produced in Academe, in specific contexts, their disadvantages are pointed out. They valorise particular world views while excluding others —such as the cubação— as if they have no real value. Such a process certainly disadvantages particular groups of people and their knowledges. In terms of the knowledges I gained from my involvement in this community, I now see that when working with notions of area, for example, it is no longer a matter of 'enlargements' in the areas of land surface, but of expending efforts to perform more complex calculations, that, depending on the shape and ends for which the land is being measured, are unnecessary. Thus, in given contexts, popular mathematics is that which presents the best credentials.

The pedagogical work from an ethnomathematical perspective enabled a double movement: from community to school —insofar as school knowledge was produced based on the reality of the settlement— and an inverse movement —insofar as the work performed at school had repercussions on the life in the settlement. What was at stake in carrying out this double movement was the construction of ethnomathematical work that would not be limited strictly to the school context and also would ultimately constitute, above all, a perspective that would reinforce only the hegemonic ways of learning and teaching mathematics, marked by the Western, white, urban, male culture (Knijnik, 2000).

It is in this sense that I consider the importance of ethnomathematical thinking. Its value comes from its potential to problematise the scientificity, neutrality and asepsis of academic mathematics, and brings to the scene 'other' mathematics,

usually silenced in school, as the cultural production of non-hegemonic groups. This is not, however, a mere attitude of 'benevolence' toward the excluded. We educators, who from an ethical standpoint are co-responsible for the great massacres that were thus far and are still being, committed by humankind, are also participants in small daily massacres, such as those practised in our classrooms, on the 'school floor'. We do so through the pretence that those knowledges have not even existed, nor exist, and, with our authorised voice as teachers, we value only the erudite knowledge of the Western culture. This differential valuing is done not because they are superior from an epistemological standpoint, but because they are those practised by the groups that are legitimised in our society as those that can/should/are capable of producing science. We are directly implicated in the processes that oppose or favour what sociologist Souza dos Santos (1996), so properly called 'epistemicide' —the destruction of the knowledge of a given social group— whose most radical form is genocide, where not only minds and hearts, but also people's bodies are eliminated.

Our role in these processes of inclusion or exclusion of knowledges in the school curriculum is, above all, political. Such processes, defining which groups will be represented and which will be absent in school are, at the same time, a product of power relations and producers of these relations. A product of power relations, since it is the dominant groups that have the cultural capital to define which knowledges should legitimately be part of the school curriculum. They are also producers of power relations because, for instance, they have influence on success or failure at school, they produce very specific subjectivities, positioning people in given places of the social space and not in others[10]. These issues lead me to understand knowledge in a political perspective.

ENDING REFLECTIONS, OPENING QUESTIONS

In this chapter I intended to present some ideas constructed from the lessons I have learned in my research with a social movement. Such lessons —to learn in the process and understand knowledge in a political perspective— have allowed me to continue my activity as a researcher together with the MST. They have also challenged me to search for an understanding of the mathematics education that takes place within the Movement. In this final section I present some of the elements that constitute mathematics education in this context and raise some of the questions that they pose.

The first element tackles the dynamic embedded in a social movement with the characteristics of the MST and in the ethnomathematical approach that I have developed together with the different groups with whom I have acted. The issue at stake here is the 'historical urgency' —to use the formulation of a peasant in the MST— of the movement for providing to its members immediate possibilities of taking professional posts that demand, for example, their participation in

[10] These issues have also been discussed and exemplified by Cotton and Hardy and by Wiliam, Bartholomew and Reay (this volume) in the case of dominant school mathematics practices in England.

examinations to enter the public service. In these examinations there are mathematics tests that usually demand the mastery of a list of contents that are important from a very conservative view of mathematics —i.e., the algorithm for finding the square root of a number. This tension is present in the teacher training courses that the MST offers to the teachers who will take a post in the settlements. How is it possible to pay attention to, on the one hand, the 'urgency' of preparing people to cope with the demands of this kind of examinations and, on the other hand, develop meaningful pedagogical work from an ethnomathematical perspective? How to articulate two approaches that differ fundamentally in their conceptions, given the restrictions of time and resources? Is it possible to think that the 'historical urgency' that the peasant referred to is precisely one of the political dimensions of knowledge? These are questions that have not been answered yet. But once intellectuals and members of the MST are aware of the necessity of 'learning in the process' and of understanding 'knowledge in a political perspective', it is possible to start searching for solutions —always partial and provisory— that could allow building an education that makes sense to these people and their struggle.

As a researcher, a second element of the mathematics education in the MST that has invited me to reflection is the understanding of the limitations and possibilities of participation in the educational process carried out in the movement[11]. I have constantly questioned myself about the role that I can/should adopt as an intellectual who researches *with* —and not *about*— a social movement. Since the beginning of my work with MST, I was aware that in the context of the struggle for land, there was no place for a researcher who would merely be concerned with describing and analysing reality in a supposedly neutral manner, and who would not be involved in the specific educational problems of the movement. My academic trajectory indicated that my research activity would not be centrally descriptive. Rather, the application of 'scientific' data obtained in research into a kind of social engineering which would lead to the improvement of the actual mechanisms of society would be relegated to a secondary instance. Unlike research in the United States, positivist thinking did not heavily influence the Brazilian tradition of social research[12]. The belief in neutrality as a guarantee of scientific rigour —a heritage of this current of thought— in our university milieu, did not have the force which it still carries today, for instance, in more conservative North American academic institutions. As clearly indicated by authors such as Popkewitz (1993), this belief cannot be sustained. From the analytical standpoint, it does not take into account a vast literature on language and theory of representations. Furthermore, it neglects the fact that research and the researcher are producers and product of historically situated social relations.

Moreover, the privileged position from which I speak to MST students in the different projects in which I am involved cannot be avoided. Acknowledging me as a voice with a different 'weight' in the educational debate about their pedagogy, I

[11] This topic has been extensively discussed in Knijnik (1996a). I summarise here some of the central arguments in the discussion.

[12] Freire (1970) and Fals-Borda (2002) are representative Latin American thinkers that advocated for a non-positivistic interpretation of the social sciences. Their work is a hallmark in the research that has been developed in the field of education.

have been trying, with humility, to problematise my own voice. I have rendered my sensitivity more acute so that the 'authorised' discourse of a white, urban, middle-class, university-researcher woman will also be permeable to 'other' voices. In considering myself a 'specific intellectual', in the Foucaultian sense (Foucault, 1989) of the term, I am taking myself as an educator who performs

> a much more modest role, much less universal and much more local [...] a much more symmetrical role in relation to the other participants in the social struggles in which she is involved, in the sense that her knowing, her vision and her discourse owe as much to the interests of power as to those of any other participant (Silva, 1994, p. 251, my translation).

It is not a question of creating illusions regarding the process of the insertion of an intellectual in a social movement. My experience has indicated the complexity of this process and has challenged me to learn to deal with a permanent tension between the dimensions of commitment and autonomy to the specific demands of the social movements with which I work.

In referring to the dimension of commitment I try, following Popkewitz, to emphasise my connection with concrete aims and with contingent characteristics of the struggles of the social movement. I keep in mind that the commitments of intellectuals are 'historically situated, provisional and connected to regional practices, through which social life is structured' (Popkewitz, 1991, p. 241). This perspective opposes the notion of a 'universal' intellectual and problematises the privileged role which such a person would play in indicating the paths to go, guided by universal ideals and utopias.

As far as autonomy is concerned, I consider, as Popkewitz writes, that the autonomy of intellectuals should not function to destroy their commitments or exacerbate disenchantment with the world. Autonomy should lead them to acknowledge that the authority on social symbols is part of the battle in world production. Education research and its researchers are part of the political practices which are being disputed. Those who have the authority to speak, and who are authorised speakers, are important elements in the construction and reconstruction of society.

It is from this perspective that I take on the relevance of the political role I perform as an educator. Aware of my responsibility as a 'specific intellectual', I have tried to guide myself by the principles of *effectiveness* and *honesty,* as enunciated by Blacker (1994, p. 168): Effectiveness in 'specific competency', in 'specialisation which does not imply compartmentalisation'. Honesty covering an 'attitude of alertness' —which means paying attention to the consequences of my own theoretical practice, of how the results of what I am producing are used— and the 'effort' —which means to be persistently performing vigilance in good faith required to support this consciousness. This honesty does 'not signify an effort toward self-knowledge; a Hermeneutics of the self, nor any other search of truth' about myself. As Blacker (1994, p. 169) says:

> The ethical work of the specific intellectual is the strict practice of honesty and of the several forms depending of the field involved, which this may take on(...) The university researcher, for instance, should possess profound, varied

> knowledge of the object of study (how can one be effective without competence?), to ensure that the work has some strategic consequence (otherwise, why make the effort?), and pay attention to discussion (if any) about the work. There is no place for dilettantism, for 'knowledge for knowledge's sakes', or indifference toward the audience or the public.

The work I have been doing with MST and the lessons I learned from it provided me with the challenge of exercising this ethical work.

AKNOWLEDGEMENT

I would like to thank the editors of this book to their contributions for the first versions of this chapter. My special gratefulness to Paola Valero, who not only gave her personal and intellectual energy to discussing the ideas 1 am presenting here, but also translated into English parts of this text.

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FOURTH DIALOGIC UNIT

ADDRESSING THE RESEARCHER'S POSITIONING

As highlighted in previous chapters, the researchers' positioning in relation to research participants is an important issue that needs to be questioned in mathematics education research. Within the wider field of educational research, there have been debates about power relationships between the 'researchers' and the 'researched'. One arena of debate has been that of 'research ethics' in which there has been a scrutiny of the types of social studies that can be undertaken, how the research should be conducted and what the responsibilities of the researcher(s) will be in relation to their participants. The aim of this discussion has been to protect academic institutions, their staff and research participants from potential abuses emerging from the minefield of power, which the very same act of inquiring opens up for all people involved. A shortcoming of this kind of ethical discussions is the assumption that it is possible to 'objectivise' and 'structure' power relations through the implementation of formal procedures and legal agreements. The ideal that participants should be able to enter a research project fully cognisant of the process, the assumptions, the outcomes, and so on, so that they can make an informed judgement as to whether they are willing to commit to the project both physically as well as ideologically falls short in grasping the complexity of the evolution of power relationships in the research process. Furthermore, in many research projects, the focus may be evolving so that what the participants originally committed to may be at variance with the eventual direction and actual outcomes in spite of the best intentions of the research team to ensure that the participants are informed. Another arena of debate about these power relationships has been research practices themselves. Within the field of education, a significant body of research exists that documents the relationships of power within the research process. Poststructuralist, postmodernist, feminist and multicultural writings have been particularly illuminating in how power is realised and exercised in research.

Much of the research in mathematics education that adopts a socio-political approach has also demonstrated how power is realised in and through the practices of mathematics education so as to position learners in relation to the discipline and outcomes —consider for example the chapters in the first dialogic unit. These studies have been instrumental in gaining an appreciation and critique of the socio-political dimensions of practices within the field. From this perspective, questions are posed that challenge why such practices can continue in spite of sustained

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research showing the problems of the practice in terms of learning —cognitive and affective— for students. As such, consideration is made of how students come to be positioned as outsiders, aliens, and marginal in mathematics education practices.

When reflecting on the research process, there has not been an extensive writing addressing the evolution of power relationships between researchers and participants and revealing the actual way in which such relationships are shaped in concrete research projects. The two chapters in this unit carefully address this issue, which has previously been suggested as being of paramount importance in socio-political approaches. While adopting different foci to their considerations of 'otherness' in the research process, Tamsin Meaney and Anna Chronaki discuss how the researcher, her decisions and actions can construct different spaces of participation for those involved in the research.

Anna Chronaki takes issue with the 'self' – 'other' relationship and its evolution during her Ph.D. research project. She starts from a consideration of her own self and how her observations of schools and classrooms got an initial meaning through the contrast of her own experience in Greece with the new context and culture in which she was conducting the study (England). This first level of sense making of the 'other' was completely mediated by her 'self'. This level is of significance since it exposes the fact that all impressions, interpretations and 'data' are constructed on the grounds of the subjective frames available to the researcher given her 'being'. Furthermore, the dialogue between a 'self' positioned as an 'outsider' and the 'others' who are positioned as 'insiders' of a community of practice allows to reveal aspects of practice, which cannot be easy to identify for both researchers and practitioners who consider themselves as being familiar with a situation under research. Chronaki shows how the first contact with the 'other' got transformed during the research process until positioning her in an influential and dominant role, given her role as a critical partner in search of explanations for the reasons and motivations behind teachers' action in the mathematics classroom.

Taking her work in cross-cultural settings with indigenous people in Australia and New Zealand as a point of departure, Tamsin Meaney discusses her experience in bringing a curriculum change in mathematics in collaboration with a Maori community. In contrast to Chronaki's initial disadvantaged positioning, Meaney's point of departure was that of 'advantage'. Her being White, of Anglo-Celtic background and in the pursuit of a Ph.D. degree, positioned her in a dominant role in relation to the expectations of the community regarding the new curriculum. Her struggle was precisely that of constructing a balanced power relationship that allowed the community to lay the principles for a mathematics teaching in agreement with their being Maori. During this process, Meaney encountered a series of ethical dilemmas concerning her decisions and methods, and she found the necessity of continuously negotiating her role in face of the community's expectations —and sometimes against her own interests as a researcher.

This dialogic unit illustrates the contention brought forward by previous chapters about socio-political approaches in mathematics education research: The examination of power in the practices of researchers is as necessary and essential as the study of power in students' and teachers' practices of mathematical learning and teaching.

ANNA CHRONAKI

RESEARCHING THE SCHOOL MATHEMATICS CULTURE OF 'OTHERS'

Creating a Self – Other Dialogue

Characterising teaching is a complex task. Equally complex is the task of researching as an ethnographer, since the present day societies, and consequently, the embedded 'school cultures', are not neatly bound. The context in which my research study took place was of no exception. This chapter discusses the experience of taking the role of an ethnographer and researching in mathematics classrooms, not of the same country and cultural background from which the researcher arrives. This very issue of societal and cultural diversity between the researcher and the researched is not alien in ethnographic accounts of anthropological studies where the main focus is to characterise the culture of 'native' communities or social groups of a special interest. But, the focus of anthropological studies, during the last three decades, has moved away from studying solely oriental cultures, indigenous communities and under-represented groups and now embraces the study of communities familiar to the researcher or the community in which the researcher belongs and lives. In this sense, the disciplinary boundaries between anthropology and sociology have become diffuse.

In many educational ethnographies the 'natives' and the researcher share the same experience of the educational system and teaching practice (Woods, 1990; Heath, 1983). In those studies, the role of the ethnographer is often played by a researcher who is —or has been— a teacher or an educationist in the same educational system[1]. However, given the importance of inter-cultural interactions in the research domain —i.e., inter-cultural research collaborations, postgraduate programmes, mobility of educationists— as well as the multicultural synthesis of classroom practices, there is a growing need for researchers to explore and understand educational systems and ideologies which differ from their own. This, in consequence, creates an additional demand for unravelling and understanding how this process takes place. Furthermore, in mathematics education research the adoption of ethnographic methods to inquire mathematics education practices as well as the growing internationalisation of the field (Atweh & Clarkson, 2001) demand our attention on the politics of the role and positioning of the researcher.

[1] As an exemplification of this claim, one can mention the movement of 'action-research', where the researcher is the teacher. Of course, there are debates concerning the 'authenticity' of claims made by an 'insider' in educational settings (see Hammersley, 1992).

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This issue although highly debated in ethnographic and general educational research —as I will discuss in the following section— has not been systematically addressed in published mathematics education literature.

Some of the chapters in this book represent an advance in such direction. Meaney's study (this volume) with a school in a Mäori community is an ethnographic study in a non-Western setting exploring how teachers, parents and the researcher are all involved in developing the mathematics curriculum. Although alien to their culture, she was able to enter the school community and participate in their practices as an outsider and expert in mathematics education. Knijnik (this volume) also discusses her awareness on her position as a 'white, urban, middle-class, university-researcher woman' when working with landless peasants in Brazil. I also had a similar role as an ethnographer exploring the mathematics culture of another group. But, unlike Knijnik, Meaney, and many other indigenous ethnographies, my entry position was not of a Western origin whilst my research setting was Western. As a Greek woman entering a British context, I was positioned as marginal. Thus my role could be perceived as having weaker status and raises questions concerning the relative power of the researcher in the ethnography.

However, the developing relations between researchers and researchees are far more complex than a static weak-strong power role influenced solely by the status of the culture of origin. All participants hold different expertise of knowledge —e.g., knowledge of the field of mathematics education, knowledge of the structure and culture of the setting— and therefore power shifts can be witnessed in the process of researching. A narrative of the research evolution is presented in the form of a dialogue amongst a number of issues concerning the position of the researcher. This dialogue takes place as a conversation between 'self' and 'other', between the researcher's personal context and the context of her research setting, between inside and outside positions, and between interpretations based on first images and insights and interpretations based on later critical reflections on key incidents.

This explicit discussion of the researcher's positioning and the narrative of how the self-other dialogue is being constructed emphasise the socio-political character of the research process through which knowledge about mathematics education develops as part of specific practices (see also Gorgorió & Bishop, this volume). It allows a holistic view on the complex features of coming to research in praxis. Discussing the interplay of relations and ethics amongst participants is a political action that involves a serious consideration of the contextual —social, cultural, historical, political— features of the settings —i.e., classroom, school, community. To explore the researcher's positioning itself is to agree that any research praxis is value-laden from its very birth. The researcher is not a neutral instrument, but an interactive and historical human being who influences and transforms the development but is also being influenced and transformed by the study.

WHAT IS THE 'OTHER'?

The search for 'personal identity' has been attributed as one of the major reasons for creating the dichotomy between 'self' and 'other'. By means of understanding what

I *am not,* I can understand what I *am.* Such psychological split up is experienced and expressed by infants as a way of making sense of boundaries within their physical and social surroundings. On a societal level, the separation between 'we' and 'others' has been used to establish notions of ethnicity and national identity (Said, 1978; Lloyd & Duveen, 1992, cited by Paechter, 1997, p. 5).

Anthropology, in particular, has been constituted as a field on the notion of studying the culture and civilisation of 'others'. The methodological paradigm of distinguishing 'self' and 'other' has been for years a predominant one through the exploration and discovery of 'other' cultures and civilisations (Wolf, 1982). In particular, the 'other' has been searched away from the self, as if the *distance* that produces and constitutes this 'otherness' can provide clearer understandings about the culture observed. The assumption that 'self' and 'other' are in fact disconnected entails epistemological and ethical pitfalls and has been problematised in writings of alternative anthropological paradigms where the ethnographer is an 'insider' such as, the 'anthropology at home' (see Willis, 1974; Clifford, 1988).

As far as it concerns the epistemological grounds of this assumption, there is currently expressed doubt[2] as to whether the ethnographer as an 'outsider' can indeed learn something substantial about the culture of the group that she or he studies (Gefou-Madianou, 1998). There are concerns that the nature of knowledge produced by an 'outsider' cannot be valid and its outcomes cannot be used reflectively by members of the cultural group studied. As a result, it has been claimed that the ethnographer whose origins are rooted within the cultural group and has the role of the 'insider' is in a better position for providing trustworthy information (Kuper, 1998). As examples of ethnographies where the researcher takes an 'insider' role in mathematics education, can be the study of investigative work in mathematics teaching by Jaworski (1995) and Boaler's (1997) study of gender and setting in mathematics classrooms. However, greater appreciation about the ideological diversities within cultural groups is evidenced in a counter response to the above view by arguing that in fact there can be no distinction between 'insider' and 'outsider' (Narayan, 1989). In other words, the ethnographer as 'insider' can equally arrive at invalid, biased and misleading interpretations when he or she fails to encounter the multiple perspectives of varied subgroups and fails to recognise the dynamic interrelation amongst them. This line helps us to appreciate that the ethnographer's 'view of mind' —perspective and political positioning— is far more important than his or her cultural origins and location in —or out of— the field.

Further, the ethical and political dimensions of distinguishing between 'other' and 'self' is that these categories can easily become structural and linguistic forms that serve the politics of discrimination and exclusion (Said, 1978). The 'other', especially in the course of descriptions of some places of a special anthropological interest, has become a symbol of exoticism and orientalism. This has been done extensively by positioning the 'self' in a central and superior socio-cultural group through which the non-Western countries were treated as inferior, native, primitive

[2] For a detailed discussion concerning the different positions and the flow of current debates in anthropological ethnographies, see Gefou-Madianou (1998).

and thus 'other'. This then could result in viewing the indigenous group as 'under-developed' or one which through the 'imperialist eye' deserved to be brought onto a civilised level (Derrida, 1974). Meaney (this volume) is conscious that her Anglo-Celtic origin may come in conflict —in terms of participants' expectations and understandings— with Mäori culture when entering research in their community and explains that the process of accommodating the researchees' voice and coming closer to them is neither easy or straightforward.

Through this methodological gaze, the country of Greece has often been portrayed in Europe as the 'exotic' and 'primitive' place suitable for summer vacations and tourism. Such is the image that dominates in public rhetoric. Even more dangerously, such views can be espoused by policy makers and people in key positions within the country. As a result, the identity of the 'other' is being formatted and labelled by the civilised and superior 'self' and its voice becomes weak, almost unheard, and subordinated to the views and decisions of those who act as 'self'.

Beyond indigenous communities, categories of 'otherness' have also been constituted amongst groups of people within the same culture of which gender, race and class are only a few distinctive examples. Paechter (1998, pp. 5-6) based on Foucault (1979, 1980) discusses how Woman has been —and is still being— constituted as 'other' and explores the power asymmetry entailed in such a discourse. It becomes evident that there is a need to move beyond a discourse that emphasises the 'self-other' separation. This need becomes progressively more urgent since geographic distances amongst countries have the potential of becoming more and more easily grasped —e.g., transportation and the internet. Also, a noticeable diffusion of cultural backgrounds —due to mobility, refugees, and immigrants— constitutes the basis for modern societies nowadays. There is, also, a greater awareness about the simultaneous existence of multiple ideologies and groups of interest within the same cultural group, and so talking through a polarised discourse may be not only superficial but also unjust.

Marcus and Fisher (1986) have argued the need to search for the 'other' in the self and for the 'self' in the other, recognising that phases of 'separation' or 'detachment' can arise. Indeed, in my ethnography, the danger of construing the category of 'other' could be due to at least three positionings that I as 'self' can espouse: a) cultural origin —being Greek and researching within British classrooms—; b) professional status —being a postgraduate researcher and observing teachers' practice—; and c) gender —being a female working with male teachers. However, in all three cases, the 'self' is not of a Western or superior origin, as noted by the indigenous anthropology discourse. On the contrary, all these possible positions can be —at different times— conceived as inferior ones and therefore as construing a weaker status for the 'self'. In my particular case, the 'other' becomes for a change not the 'primitive' but the superior 'Western' setting.

The observant, I as 'self', could come to represent a less privileged background[3] not only in terms of educational provisions, but also in broader societal terms. All these possible positions can result into possible actions of power —or weakness— and control —or lack of control— over relations with research participants and disciplinary knowledge. Meaney (this volume) describes her experience of her role with teachers and parents from the Mäori community in very similar terms and explains that ethical dilemmas can occur concerning whose story would count more and why.

This ethnography involved me being in classrooms not of my own origin. Classrooms to which I was neither familiar —to start with— nor accustomed to their rituals, their 'ways of doing things'. Classrooms that were not part of my own school experience and therefore could become 'not-me' classrooms[4]. The 'afresh' reaction of an 'outsider' ethnographer is that of creating a *distance*[5] and thus creating the category of 'other'. I certainly experienced this stage as the description of my very first impressions show below. But, recognising the 'other' can also be the first step for creating awareness about the *distancing* that this conception may create. Could this then be used constructively for discussing its substance and exploring the multifarious nature of the 'other'?

In this text, I use the category of 'other' as a tool for deconstructing my field experiences. The notion of 'other' here is used as irony[6], in the sense that the intention is, despite my weak status positions, not to separate 'self' and 'other', but rather to reconcile them. So, the category of 'other' is simultaneously recognised and deconstructed. Given the 'less-privileged' status of 'self' the nature of interpretations produced needs to be reconsidered and one may ask questions such as: What would be the nature of the process of interpreting and deconstructing the 'other'? How, in fact, the 'other' of a superior status can be deconstructed? In what ways could the 'self' encounter the 'other' in the course of these interpretations? And what does this process tell for the 'self'? Furthermore, one may also ask questions about the implications of this deconstruction for building research relationships with mathematics teachers in the school, and, more precisely, for inquiring the mathematics classroom.

[3] Although the Eurocentrism of mathematics education is often associated to a strong resonance with a Greek tradition, the contemporary context of mathematical education in Greece does not resemble the European scene (see Chronaki, 1999). Therefore, in terms of educational provisions, structuring and organisation, Greece does not belong in the Western paradigm.

[4] Paechter (1998, p. 5) argues that '[a] very important early experience is the difference between me and not-me'.

[5] Distancing from the field of research can have both negative and positive consequences. On the one hand, it is seen as a necessary process for developing authenticity (Hammersley, 1992; Ely, Vinz, Downing & Anzul, 1997). But, this same process can involve detachment, mis-representation of the group studied and the constitution of ethnography as 'authority' (Clifford, 1988).

[6] Atkinson (1990) refers to the use of 'irony' as a textual strategy in ethnographic writing.

THE CONTEXT(S) OF THE STUDY

> Today we visited the Green Valley school. A local bus brought us to the small rural town and then we took a short walk to the school, only ten minutes. The school was located at the bottom of a small hill. A beautiful secondary school that everybody in my country would envy. Even the most well off private schools that effortlessly try to meet North European and American standards. Trees and freshly cut grass everywhere and lots of playgrounds for pupils. And a real two-room library. Going into the classrooms, the scenery looks even better!! Resources everywhere! Even the mathematics room has its own resources! Saying this, I just realise that there exists a mathematics room. A room totally devoted to mathematics! Full of books about mathematics, different textbooks, posters and materials to work and play with. Hey, this can be a paradise!! (Impressions from my first visit in a British school; Diary, Summer 1988)

I remember vividly how stunned I was during that my first visit to UK schools. The bulk of my observations were focused on the material appliances I saw in the school and I was left with images of the wealth of books and the availability of materials such as paper in all different sizes and colours, tangrams, wooden geometrical shapes and varied other instruments. Being deprived of such 'luxuries' in my own school as a pupil and later as a teacher[7], my attention was immediately drawn to those things. Feelings from that visit contained a mixture of joy and disappointment. 'Joy' to experience perceptually all these wonderful conveniences, and 'disappointment' when I realised that most of the state schools in my country, which are under-resourced, could not even dream of this paradise. Schools are struggling to cover basic maintenance problems such as heating and broken windows[8]. A centralised curriculum provides a single textbook to all pupils free of charge in the name of social equality, but makes no further resources available. This textbook, as a sole resource, deprives imagination and creativity. Although there is a formally established office in the Ministry of Education for the allocation of resources in schools, this office is under-functioning and mainly caters for the distribution of textbooks. Still, the school conditions in Greek schools can be described as far better when compared to some East European countries —e.g., Albania— or South Africa (Naidoo, 1999). The majority of schools have the basics. Teachers have a chalkboard and pupils have their desks and textbooks. But can these 'basics' ever be enough?

My research, influenced unavoidably through lived experiences in the context of my own country, was coloured with a desire to explore the potential of resourcing

[7] We need to mention here that even though the Greek state schools are by and large under resourced, there is availability of resources —e.g., extra curriculum books and materials— in the private sector and in bookstores.

[8] Of course there are under-resourced schools in Britain, too. And the present conditions, resulting from a policy which favours educational privatisation, have caused many schools to close down. But, by and large, the mathematics curriculum still retains some practical orientation and leaves greater autonomy for the teacher to use multiple resources in teaching when compared to South and East European countries.

and its specific focus became the construction of innovative curriculum activities in the teaching of mathematics. One may claim that for such a research interest the British educational setting could be favourable due to its flexible curriculum and its long tradition in teachers' planning their own curriculum and using multiple resources for their teaching —e.g., the Nuffield Mathematics Teaching Project in the mid 60s (for details see Moon, 1986). A scholarship came as a ticket for me to undertake this journey.

But, by the time I started my research, in the early 1990s, new educational policies —such as the Education Reform Act 1988, see Flude & Hammer, 1990— had brought a series of changes in schools, initiating a long period of uncertainties concerning the roles of schools and teachers. Britain obtained, for the first time in its history, a National Curriculum addressing all main subjects that prescribed content areas and guidelines for teachers to follow. For mathematics this meant the re-organisation of the curriculum around content areas —including number, algebra, shape and space, handling data and measures— and specific levels of attainment (see Dowling & Noss, 1992). At the same time, teachers were called to make extensive change to their practices and were recommended to endorse a mixture of styles of teaching such as exposition, discussion, consolidation of fundamental skills and routines, practical work, problem solving and investigations (Cockroft, 1982). Ernest (1989, p. 5) acknowledges not only the great influence of the Cockroft report in the British mathematics education scene, but the widespread acceptance of the teaching methods introduced.

This was the particular context in which I embarked upon my research and which had meant to provide an additional influence in my thinking and practice. This new climate, although still favourable for practical work in mathematics teaching, meant that schools and teachers had to invest a great amount of their time towards a major re-organisation of their school curriculum and practice in order to meet accordingly the new recommendations and statutory orders. This situation could not easily permit teachers to get involved voluntarily in the devising of innovative materials. Teachers seemed to be more concerned at that time on how they could conform their teaching to what their school perceived to be required. This often meant an emphasis on improving their teaching by adopting new teaching styles —e.g., investigative and collaborative work— and re-organising their subject curriculum to fit the National Curriculum structure. Getting involved with inquiring mathematical content and new forms of its representation was less of a priority. This meant a shift on the emphasis of my initial interest from *resources*[9] towards *teaching with*

[9] Adler (1998b) discusses viewing resources in a more elaborative view including not only material appliances but social and cultural resources such as language and languaging. I could note that in a similar sense, the shift in my research incorporated a shift from a focus on resources production towards the pedagogical use of resources in classrooms (see Chronaki, 1997).

resources[10]. The teachers were not my partners in creating the resources, but were certainly partners in my inquiry about how they used a particular set of resources in their teaching. As a result, I could claim that both contexts, the personal culture where I had experienced mathematics education, and the cultural setting of the present ethnographic study, where I have entered a more systematic inquiry, influenced simultaneously the nature of the research and my role in varied dimensions —e.g., positioning, interpretations, relations, decisions about what counts as key incident.

DECONSTRUCTING THE 'OTHER': TOWARDS A SELF-OTHER DIALOGUE

Below, I am describing this process of interpretation as a deconstruction of the 'other'. This is based on a narrative concerning the 'self' experiences about the significance of certain structures in school life, and, the process of being and communicating with teachers. This narrative reveals and makes explicit distinctive features of the culture(s) in which mathematics classrooms are embedded including the relations between participants. It describes relations, structures, functions, values and views through a comparative analysis of experiences in two contexts —the personal and the researched. This may seem irrelevant to an audience in mathematics education that focuses mainly on the type of activities produced by teachers and learners over specified content. For such an account the reader can find details in published work elsewhere (see Chronaki, 1997, 2000). But, for a book that explores the socio-political dimensions of mathematics education, and especially for a section that is particularly focused on reflecting over methodological aspects of the research process, the present chapter is written so that to present a reflexive account of the research story.

As such, it is necessary to move beyond exploring mathematical content *per se* and embracing a host of factors through which mathematical content is being shaped and organised as part of the school practice. The main reason is that the constructed mathematics classroom culture cannot be seen in isolation from the broader pedagogic context. Although, there are particular aspects on which one needs to focus, for example the curriculum, the kind of activities, the teaching methods —or texts, meanings and interactional practices in Bernstein's words (Bernstein, 2000, p. 105)— all these live and evolve within the boundaries and constraints that the wider pedagogic ethos of the school prescribes. In fact, the ethos of this context regulates how specific mathematics practices are being recontextualised in lessons and communicated amongst participants and also determines the nature of texts and meanings produced (see Lerman and Zevenbergen, this volume for a further

[10] In England, the establishment of a partnership between a researcher and a teacher involves more than just an expressed interest and availability of time. It also involves institutional support for the researcher and teacher —e.g., authority or status that would legitimise the process of involvement or the necessary funding support that would cover teachers' time in the particular research. Having the status of a foreign research student could not help to establish such a partnership as the production of curriculum materials involves and requires. Thus, I had to rely on teachers' co-operation and involvement on a different level.

discussion of Bernsteinian constructs in mathematics education). When the emphasis is placed on methodological aspects of researching mathematics education, wider pedagogical aspects cannot be subordinate to subject specific ones. This is particularly relevant in research studies, such as ethnographies, where the researcher is required to spend a considerable amount of time with teachers and pupils in school communities. A number of methodological issues emerge that determine the nature of outcomes and the ethics of the study. For example, how entry and access to the practice is being negotiated, how relationships are being established, what the roots for the interpretative inquiry are, and on what basis final arguments are being constructed.

The focus here, is thus, to narrate how the inner 'self-other' dialogue is being experienced. And in that research journey, aspects of the broader pedagogic context played a role not only for identifying the underlying backgrounds on which interpretations of subject specific aspects are made, but mainly for establishing what is the researcher's role when exploring the mathematics education culture in a country not of her origin. This particular part does not provide information about how one can teach better or worse certain mathematical content, but instead provides reflections over the process of researching mathematics education practices.

I hope that through this explicit account of the process of research exploration, I can offer ways in which 'self' and 'other' can meet and the unavoidable —or sometimes desirable— 'distancing' between the two can be bridged. Further, it can assist to raise questions and challenge oversimplified methodologies of certain studies that claim to research ethnographically and/or comparatively mathematics education settings but avoid to explain and/or problematise the position of the researcher in that process. The process of researching within a specific culture — either familiar or foreign— is not simple and raises complexities and dilemmas that are often silenced over the expense of neatly presented outcomes. In addition, it is aimed that besides unravelling these complexities, other researchers —and readers— who undertake similar paths will be benefited.

The unbearable silence of images: Uniforms and alien movements

Pupils' movements in particular triggered me. I had noticed, in my first visit, that pupils were moving into a different room at the end of each lesson. I wondered what that wild and speedy pupils' running through corridors could mean. Was it part of the school's 'sporty' atmosphere? Why do pupils have to change classrooms so often? It was difficult for me to realise why they had to pack their bags and move onto another room. In my country, pupils are allocated one room and, in secondary school, it is the teacher who moves around classrooms. But here, I was observing the opposite. The organisation of the school curriculum around subject departments meant that the mathematics department had its own room for meetings and for teachers' work and also particular rooms were allocated for teaching the subject — e.g., the mathematics room. This meant a better organisation around the available resources and pupils' work that, very often, was displayed on the walls.

But there was another reason for pupils' moving around. And this was the school's approach to set pupils in groups of their perceived ability. British schools have a long history on 'streaming' and 'setting' pupils in ability groups[11] which was abandoned in the 1970s due to recommendations of the Plowden Report (see Flude & Hammer, 1990): 'the 1970s and early 1980s witnessed a growing support for mixed-ability teaching in the UK' (Boaler, 1997, p. 125).

'Student movements' had particular significance for my study when the teachers preferred to work with higher sets and insisted that these sets will also be best used for the study. The main reason put forward was that pupils were more motivated and interested in their mathematical work and could cooperate easier. When I asked to visit lower sets they did not deny, but their preference was not to include them in the study. It was not difficult to appreciate teachers' hesitations. Since the study was not geared to explore pupils' learning per se, but teaching in a wider context, they did not feel comfortable to expose themselves within a class where students can create risky situations for their professional status —e.g., discipline problems, lack of motivation and interest. For them working with a higher set meant working within a safer environment that could be easily controlled and influenced. Although important, at that stage, it was not easy for me to insist for including a variety of sets.

Uniform was another striking difference. All pupils, boys and girls had to wear a uniform. Some looked very smart in it, but others had to use a suit or pair of trousers one or two sizes smaller or bigger —possibly due to family budget— that made them look very uncomfortable. Inquiring about the use and role of uniforms one of the typical answers I used to get from teachers was that *'they are part of the tradition'*. My first reaction was that this might be a very simplistic approach. How can a 'school uniform' be such a strong part of the tradition and then if it is part of the tradition why not using a 'period costume' instead of a formal suit? What type of tradition is it and moreover, whose tradition? Immediately, I was bombarded with images of my own schooling. Uniforms were also used in Greek schools, but, in the secondary level, they were addressing only the female population. Girls, had to wear a specific uniform that consisted of a dark blue dress and a white —or sometimes blue— ribbon for the hair. This one uniform became obligatory during the years of dictatorship and a similar style was used in all schools around the country. The boys did not have a particular dressing code, apart from being required to have a rather short haircut. The rationale was that pupils in school should look tidy and especially girls —by wearing this dark blue dress— should cover their femininity. This was regarded as the means for promoting —and sustaining— a certain ethos in the society through the school.

The significance of these images is that they originate from 'self'. Whilst observing British pupils running around in their uniforms, my personal experiences did not allow me to accept un-problematically that 'uniforms' and 'setting' were just parts of the tradition. I could not neutralise them as a ritual part of the school culture.

[11] Boaler (1997, p. 142) explains the difference between streaming and setting: Setting is 'when students are put into groups according to their perceived ability for individual subject areas'. And streaming is 'when students are placed into one 'ability' group that they stay in for all of their subject'.

Digging deeper, alternative explanations were offered by the teachers in the school in the form of personalised beliefs or 'narratives'. For example, a uniform can be taken as a symbol of 'readiness to work'. Such a view is mirrored in many professions in British society and beyond —e.g., not only the medical profession, but cleaners, porters, post office employees, etc. It signals discipline, readiness, cleanliness and tidiness. Another importance of the uniform is that it indicates a sense of belonging to the same group. Sharing the same *'school tie',* as some teachers expressed, could mean the support that old school mates can offer to each-other in life and in careers. This very fact can sometimes be a motivation for parents to enroll their children in specific schools, not so much for academic reasons but for socialisation.

Overall, the central systemic educational policies seem to entail a much greater power over individuals and as such particular groups can make use of them to promote their interests. For example, the dictatorship regime in Greece used such a policy on dress code for imposing control and discipline to the society through using the school community. The Education Reform Act of 1988 has been constructed and used by liberal governments in the UK to further progress in society through the increase of individual competitiveness which, can be clearly realised in the level of school communities —e.g., pupils' setting. As a result, 'uniforms' and 'setting' for pupils are not innocent parts of the educational structure, but they can serve to shape pupils' identities and academic experiences —of which mathematics is no exception (Wiliam, Bartholomew & Reay, and Cotton & Hardy, this volume, discuss and evidence the connection between structures, cultures, and students' identities and learning).

What is of noticeable importance here, is that the school community —teachers, pupils and leaders— has the potential for problematising these issues and for raising a 'voice' about their pedagogical and sociological consequences. Thus, 'resistance' can be expressed. As an example, I can cite John, a boy in Grade 2, who was known as rebel because he used to wear his trainers under the uniform trousers. For him to wear the trainers instead of his standard black shoes was a form of 'resistance'. In this way, John could show disapproval of the dress code uniformity within that particular school. Other forms of resistance, often met in mathematics classrooms, are non participation, not asking questions, not sharing interest, not expressing one's own views, expression of boredom and indifference (see e.g., Mellin-Olsen, 1987). Although, the system exercises 'power' the individual or the community have the potential to 'resist'. This points out the urgent necessity of having and making available productive channels —e.g., dialogue, collaboration, self-image— where the local significance of such 'power' and 'resistance' can be reviewed and reconceptualised.

Being with teachers: Staffroom, meetings and the curriculum

The staffroom, teachers' ways of behaving and talking, as compared to home experiences, seemed, for me, to entail a certain degree of formality and portrayed a well-organised life around the school itself. Although, teachers had their individual

style in mathematics teaching (see Chronaki, 2000), their ways of being and communicating with colleagues in the staffroom or the mathematics department revealed aspects of the subject's social organisation. For example, how the national curriculum becomes the school mathematics program, how knowledge and expertise over parts of the curriculum are being distributed and how the daily timetable is being structured. In the staffroom, there was a relaxing atmosphere in the seating room where teachers could have their teas and socialise during breaks. In the same space, there was a small kitchenette for them to make their drinks and store their mugs, as well as some more private areas where they could sit quietly to read and write.

Apart from friendliness and politeness, I could also discern *'formality'* in teachers' school lives of which, the serious look of their dressing and certain means of communicating —pigeon holes and formal meetings— were striking differences to my 'self' experiences. The dress-code addressed mainly male staff, apart from physical education and art teachers, who all wore suits. The tie was an important part of looking smart in school. I remember, Peter, one of the mathematics teachers, was particularly unhappy with this stress on 'tie' and was even considering looking for a different job. Pigeonholes and formal meetings are very practical means for certain types of communication. Messages can be exchanged through pigeonholes without requiring physical contact and this can save time when busy as well as to retain privacy. Arranging meetings is unquestionably a channel for public debate where problems and solutions can be discussed and decisions can be made. As such, they are both means of convenience. At the same time they constitute parts of the systems' formality. Sending and receiving post in writing looks serious and can also serve as a proof text. Reporting ideas in a formal meeting colours the process of information give-and-take as official.

All mathematics teachers used these channels for their work when preparing the school curriculum, the timetable or varied other activities such as lesson tasks, mock-exam preparation and extra curricular activities —e.g. mathematics clubs. The degree of teachers' involvement and responsibilities was based on a strict hierarchy based on experience and seniority and was geared to promote the standards, values and aims of the mathematics department. For example, in one school there was more emphasis on pupils' success in exams, whilst the other was more concerned with improving teaching strategies and focused on using open-ended and investigative work.

In contrast, my home experience of mathematics teachers' work follows a much more informal pattern. The mathematics teachers did not have their own room, no hierarchy exists within the school —e.g., head of mathematics department— and their responsibilities were not addressed in a formal way. Teachers' work is discussed and debated more through national or regional subject associations rather than through school meetings. The school-based organisation of the subject program and timetable was based on an informal and emergent allocation of tasks and roles for which ultimate responsibility has the school director.

These thoughts can be viewed as 'mis-interpretations' of the school culture I was observing if seen separately from 'self'. Had I been an insider, could I have been able to perceive images of suits, pigeonholes and meetings as 'formality'? Maybe

yes, may be not. I arrived at this interpretation, not as a matter of rationalising, but because I could *feel* the formality of the culture as a result of its contrasting effect to my 'self' experience. The state schools in Greece are mainly resourced with the 'basics' which often means desks, chairs and the textbook. Teachers in the state schools have no 'space' for relaxation, socialising or privacy. Their working status is that of a civil servant which, despite the low salary, allows them a sense of job security and autonomy. Administrative work and pastoral care are part of their duties, but the school does not require them to portray membership and representation of a community. British teachers' conformity to the school rituals and the resulted 'formality' in their style, could also be conceived as a result of their relation with the school as the employee —and thus as authority. Schools in Britain consist almost of self-sustained communities, in the sense that pupils can study, play, lunch —or have dinners— and do their homework or library search within the school. Teachers' recruitment —and redundancy— is made locally by the school board with no ministerial involvement —as happens in the case of Greece. As a result, teachers need to conform to and obey a number of rules. The immediate sense of 'formality' in British schools and the contrasting sense of 'informality' in Greek schools can be the outcome not just of personalities —north and south Europeans— as mostly advocated, but also on the ways in which the teachers relate to the school system and on how their roles and responsibilities are conceived. In fact, Valero (2002) in her study of mathematics education change in schools in Colombia, Denmark and South Africa has shown how these organisational differences impact teachers' cooperation and their possibilities to make sense of the challenges of teaching mathematics in their classrooms.

Getting closer with teachers was part of overcoming the initial stage of negotiating the research and being accepted not so much as part of their culture, but as a respected member of their company in the staff-room. Meaney (this volume) has described her role as 'outsider' and explains the constraints raised in terms of getting entry to the culture. Despite the difficulties that such a role entails, I, also, did not try to play the role of a colleague. Being aware of my position —a Greek female postgraduate student— and of the possible fragmented understandings that this could generate, I did not try to pretend being 'knowledgeable' of their practices or being a full participant in their culture as an 'insider'. Instead, I chose to be open about my own subjective interpretations and tried to seek —whenever possible— their alternative views. During that process, access and acceptance were not automatic gains. In fact, there were distinct 'cultural barriers' that needed to be faced, understood and crossed throughout the research. One such barrier was language use. Language is a resource, but communicating in a language not of your own can also entail difficulties in various ways[12]. For example, it is one thing to speak the language fluently and, read and write sophisticated texts, and another to understand the colloquial and other expressions that are pertinent to a specific local culture. Pupils and teachers would sometimes assume that I knew certain movie

[12] Adler (1998a) has explored the issue of language use in multicultural classrooms of mathematics and has pointed out a number of dilemmas faced by teachers. Here, language use is being discussed as a resource for enabling the teacher-researcher communicative process.

stars or the context of some popular TV series, for which I happened to have no idea. As a result, they would use certain expressions expecting me to know their meanings.

Teaching strategies was a main part of my conversations with the teachers as well as discussions over the types of activities used to address certain areas in the mathematics curriculum. There was an emphasis on offering opportunities for pupils' experiential and practical work and especially hands on experience, and connections with real life. In contrast, the Greek mathematics curriculum places more emphasis on pupils' training with abstract work, formal argumentation and proof, and work within the mathematical domain. Concerning the aims for content coverage, it was noticeable that the demands of the Greek mathematics curriculum, at least in the lower secondary level, were much higher, placing more demands on pupils success with formal and abstract mathematical thinking. But, the British curriculum offered more opportunities to pupils in making connections with extra curriculum work as well as mathematical applications in other contexts such as the information communication technologies.

Whilst discussing with teachers about their work, it was obvious that we did not use some words with the same meaning —e.g., pedagogy or didactics. The case of specific terms was another dimension in which language and cultural differences could be realised. One such example was the different use and meaning of the word 'didactic'. This term is of particular interest since etymologically it has a Greek root. Didactic in Greek means the art of teaching and its root comes from the verb *διδασκω* meaning teach. In the continental literature of mathematics education, 'Didactics of Mathematics' is the expression of addressing Mathematics Education itself as a field of scientific research (Biehler, Scholz, Strasser & Winkelman, 1994). In English, the term 'didactic' has a very different meaning. It is a value-laden term that signifies formal, traditional teaching —in an expository form— geared towards rote learning, repetition and training of skills. Simon, one of my teachers, had described himself as being 'didactic' in his teaching with a sense of 'guilt'. He actually said:

> So, I do that. I get the children to try to come up with the information. I get in a process if you like of feeding a question or suggestion and try to get people to respond. I don't just say this [...] how you do variants, you always draw from the north line, you always measure anticlockwise [...] Not quite like that. But, once I've elicited the information from the kids and then gone over to make sure that they know it, we will do some, perhaps we will do together some work on the board. Actually, about the bearing, I probably ask them to copy some work on their books, and have an example. And I don't see any harm in that. I think that's quite good. To discuss it first, to get information from the kids, to actually do an example on the board for them, to get them to copy that in their books. Then to get them to do their own examples [...] I mean, that's what I call 'didactic' teaching (Simon, 7/12/93).

It is striking, when one searches the literature in mathematics education, to notice that 'didactic of mathematics' is the way that most other European countries, apart UK, refer to mathematics education —e.g., *didaktik* in Germany and Scandinavia, *didactique* in Francophone countries, *didáctica* in Spain and Portugal, and

διδακτικη in Greece. I was aware of this difference, but Simon's comments made me realise its 'British' connotation. The word is assigned a negative value, and as seen above, Simon felt guilty of being perceived as 'didactic' and all the way through tried to defend his own 'didactic' teaching as being not as negative as the term implies. It became clear to me, that a possible usage of such a 'negatively' oriented notion by the British mathematics education community could cause misunderstandings between teachers and researchers in the country.

Teachers also wanted to explore me. There was a genuine interest from their side to find more about myself, my schooling experiences, and my future plans. In my case, teachers —and pupils— were interested in our cultural differences; we talked through breaks about different traditions, and ways of celebrating feasts, as well as differences in climate. We even exchanged recipes about Christmas cookies with one of the teachers! I was also asked by a pupil to be interviewed about tourism and travelling in Greece, as they had a project on Europe at the time. Some pupils were surprised that it snows in Greece, and that there are skiing centres too! I believe that this interaction helped to maintain communication with them. Measor (1985, p. 62) also argues that: 'Sharing interests and talking about them with interviewees is an important element in building a rapport, so that people will talk to you'.

One issue that mainly appeared in our conversations was the mathematics curriculum. Teachers —and sometimes pupils— would refer to mathematics as being the 'Greek Subject'. Such comments were intended as 'praise' for me concerning the connotations of such an hegemonic curriculum subject to the historical past of my country. They seemed to believe that mathematics must be easier for Greek pupils since the Greek alphabet letters are being used in so many algebraic expressions and symbolism and much of mathematical content is related to those ancient Greeks guys such as Pythagoras. Accordingly, I should feel proud! But, I did not. I was interpreting their comments as an internalised perception of an oversimplified version of a 'History of Mathematics'. At times, I could feel them picturing me as the descendant of those mathematicians and philosophers and staring me with excitement —or fear— and even considering me guilty for all the suffering that mathematical illiteracy has caused to children in the world! However, the development of mathematical science in the histories of ancient civilisations (Restivo, 1992) as well as contemporary choices world-wide about what content mathematics education curricula should contain (Hoyles, Morgan & Woodhouse, 1998) are quite complex social processes (Restivo, 1990). Moreover, the content of the mathematics curriculum in modern Greece over the years has changed towards achieving a greater uniformity with that of other countries. It can be claimed that nowadays, it resembles more other European countries than its ancient past. And although its enactment in teaching practice is rooted in a traditional pattern the reasons are mainly socio-political and not cultural (Chronaki, 1999).

In this realm, we —the teachers and myself— shared a lot of information and a 'dialogical space' was created amongst us. By 'dialogical space' I mean that we had accepted that differences exist and we had implicitly agreed —without an explicit negotiation— the need for unpacking notions and exploring issues. These implicit agreements —or disagreements—, expressed in the form of nods, smiles and other

gestures, played an important part for creating consensus amongst us throughout the study. Meaney (this volume) has also valued them as essential data in the research process. However, talking about agreements and disagreements helped enormously not only in establishing trust, but in understanding each other and moving beyond the differences.

Asking 'whys' as openings for mutual learning and change

Asking 'whys' was part of the 'dialogical space' described above and provided an opening for mutual learning and change. The learning experience for me was part of my development as researcher. Getting into the classrooms and observing mathematics teaching was not enough. Finding out about what these observations might mean for myself and for the teachers who participated in my study seemed equally important. For example, two —out of three— teachers described themselves as 'traditional' and 'progressive' —teachers' own characterisation and comments on their style of teaching was based on simulated recall of their videotaped lessons. By asking 'whys' concerning specific local episodes in their use of theme-based activities[13] I could understand the significance of these two different pedagogical orientations in their teaching practices (Chronaki, 1997). These two orientations — embodied by the two teachers— were resulting in different experiences for pupils. The 'progressive' teacher was making room in lessons for exploring the theme and the mathematics related to the theme. In contrast, the 'traditional' one tended to overemphasise mathematical content in a manner that dislocated mathematics from the theme context. But, in the course of their interactions with pupils, both teachers followed a typical communicative pattern in which the underpinning mathematical formulas and terminology were mostly given from teacher to pupils —and not entailing part of a social construction with pupils. These observations raised questions not only about the potential for teachers and pupils, who live and work in current educational structures —e.g., framed by curricula and lessons—, to create 'dialogical' experiences in which mathematical knowledge can be co-constructed, but also about the relevance of the polar 'traditional' and 'progressive' orientations as a discourse for teaching styles.

Second, it was a learning experience for the teachers themselves, because asking them 'whys' encouraged them to reflect on their practice. For example, Peter said:

Peter:	I found myself saying things that I didn't realise I thought. And maybe I'll think I'm not that sure about it afterwards, which is quite good talking about it.
AC:	Why?
Peter:	Because you don't normally do it, because you're too busy.
AC:	So by talking about it does it help you with your teaching?
Peter:	Yes, it gives you the chance to reflect on what you're doing (Peter, 22/11/93).

[13] The curriculum activities that the three teachers used in this study were based on the theme of art, and in particular the connection between Roman Mosaics and Geometric Transformations (see Chronaki, 1997).

One may argue that the act of observing teachers and asking them questions entails a 'power exercise' on them. Hammersley (1992) claims that its ethics need to be considered because apart from being emancipatory —as Peter's case above show— it can also be exploitative. In fact, there were times, when I could realise teachers becoming tense if my question was addressing a sensitive part of their practice. For example, when discussing Simon's teaching style, he felt guilty of being 'didactic' and he made enormous efforts not to hide it, but to justify his choice in the context of the school curriculum.

Asking teachers 'whys' in the field made me realise that my 'self' position was even more complex than I had initially considered. Whilst, at first, I had thought of myself as being —or sustaining— in a less superior status —foreigner, student and female—, I could now feel that a more superior 'self' could emerge out of my 'why' questioning. Through my 'whys', I was exercising power on teachers[14] because I was going deeper and deeper, undressing their practices, thoughts and feelings. I was then transforming and changing the 'self' from a less privileged to a more superior status. Teachers felt challenged, and, it was this very challenge that also entailed learning potential for them. But, asking teachers 'whys' was not the outcome of a rational decision. I realised that being able to challenge them was the result of feeling comfortable with them and confident that the teachers would not feel as though they were being criticised by me. This was the result of an established relationship achieved through previous stages of the research. Asking 'whys' was also part of my own learning which I did not try to conceal. As Punch (1994, p. 367) says: 'Do we represent the colonial culture [...] or do we humbly present ourselves as learners?' I certainly went for the latter and I felt it was appreciated. Ultimately what seemed to sustain teachers' interest in the research was that each, for different reasons, viewed his participation in the research as an avenue for gaining something about professional development.

LOOKING *AT* AND LOOKING *THROUGH* THE 'OTHER'

In the above sections, I described what it means to research classrooms of a diverse cultural background for the researcher. Research involves exploration for understanding. Diverse cultural backgrounds may create barriers for that understanding. They enforce interpretations from different perspectives which, when left unexamined, can lead to misunderstandings. And although *distancing* 'self' from 'other' and encountering multiple perspectives are seen as important for enhancing the authenticity of ethnographic accounts (Hammersley, 1992; Ely et al., 1997) *detachment* between 'self' and 'other' can be particularly ineffective and unjust. Analysis here, was based on a narrative of my personal experiences in schools with teachers and aimed to deconstruct the 'other' by means of encountering 'self'. I hope that through this deconstruction, I have managed to highlight some very basic

[14] This 'power' was used differently by teachers in their class and I left that control entirely in their hands. In Peter's class I was introduced to the class as Anna, a research student and in Nick's and Simon's as Mrs Chronaki a researcher from the University. This was not only the way they saw me, but mainly the way that they wanted the pupils to see me.

points, feelings and experiences of that process. Overall, two distinct phases in that process can be highlighted; looking *at* and looking *through.* Looking *at* is the initial phase where the first positive impressions are created and is characterised by an intention towards mythologising the features of the 'other'. Looking *through* is a later phase and consists of a continuous demystifying of the 'other' and in which greater and deeper understanding and reconciling of 'other and 'self' takes place.

Reflecting on the process of looking *at,* it is noticeable that initial impressions tended to create a 'myth' about the new context. For example, in my case the new school context was viewed as a paradise. This was the outcome of attending in the new setting mainly those features that were missed from the 'home' setting —e.g., the lack of resources attracted attention. This served to create a 'mythologised' category of the 'other', which can be paralleled with that of 'exoticising' other cultures as primitives —e.g., indigenous anthropology. However, given that the 'self' was positioned as being less privileged and non-Western, the 'other' instead of being under-valued was over-valued. As such, it can be seen as falling in exactly the same pitfall of creating 'otherness' as much ethnography in the indigenous anthropology paradigm had done. This phase was also characterised by polarised feelings such as joy concerning discoveries in the new setting and disappointment when the luxuries of the new were compared to the familiar.

Looking *through* was a more constructive phase. It entailed deconstruction of the 'other' and demystification of its mythologised features. Looking *through* enabled me to see beyond material appliances, visual images of everyday life in schools such as pupils' uniforms, teachers' suits and educational structures —e.g., ways of working and roles undertaken. In both contexts, of 'other' and 'self', moving beyond these appearances made it possible to re-discover human beings and their ongoing struggles with systemic features. As said before, the deconstruction of the other involved encountering the self. This self-other dialectic took place as twofold; contrasting differences and searching for reconciling.

Contrasting differences were observed in varied incidents. Moving from first impressions towards authentic interpretations involved constant comparisons of these contrasting differences through an interplay of simultaneously looking at 'self' and looking at 'other'. It was more than just borrowing a new pair of lenses and seeing 'mathematics education' from two different perspectives. It involved taking the 'self' in the place of 'other' and being engaged in a double bounded process of mythologising and demystifying. My personal history and images of my own schooling acted as a 'mirror'. In other words, the images of the 'other' brought images of the 'self'. This forced me to engage in a constant inquiry of contrasting incidents. Pupils' uniforms, male teachers' suits, the layout of the staffroom, the pigeonholes, the movement of pupils in corridors carrying their bags and changing rooms between lessons were all novelties for me. Reflecting on these, made me question how this very different social organisation of contexts can really affect mathematics teaching and learning in profound ways. Further, the 'self'-'other' dialectic also involved a reconciling between the two. All these images could have been interpreted as part of the school tradition and pass unquestioned. But, seeing them through 'self' became objects for questioning. Deconstructing initial images involved deeper understandings not only about the represented structures but mainly

about their significance in the school context. In the anthropology of indigenous cultures, the methodological gaze used, follows a movement from 'self' towards the 'other' and back to the 'self', as a way for looking at the primitive as an object that informs 'self'. Recent forms of anthropology —e.g., anthropology at home— urge us to reconcile the relative relation between the two, to re-conceptualise these two categories and in fact to see the 'self' as other and the 'other' as self (Marcus & Fisher, 1986).

Moving beyond comparing images towards exploring the significance of the underlying structures within the school context was a way for moving beyond the 'self'-'other' separation. During this process, teachers and I engaged not only in exploring our diversities but also in appreciating the similarities within these identified diversities. It could be described as a deeply transformative process involving learning and change from and with each-other. At varied stages, teachers and I, had to consider shifting our retained —and sometimes prejudiced— views and see each-other from a different perspective. We had to move from the predominance of contrasting images towards the embracing of multiple perspectives as a means for interpreting these images.

AS A WAY OF CONCLUSION

Participating, observing and exploring the classroom of 'others' is becoming a growing need in our societies. We are currently faced with rapid changes, and the demands for further change in educational policies, national curricula, teaching practices, learning habits and our ways of communicating due to a plethora of technological and cultural artefacts. To these, one needs to add a growing interest —and requirement— for the mobility of educationists amongst varied countries both for teaching and research purposes. As far as research practice is concerned, there is a higher degree of interaction amongst people from different countries in formal forums of educational research dissemination, such as conferences and journals.

Mathematics education is not unaffected by all these. When pupils —and teachers— move from one country to another —or even from one school to another—, they have to face not only a new mathematics curriculum, but a new culture of mathematics teaching which is nested in the school culture. How do they experience the differences and how do they manage to bridge those with their personal experiences? Moreover, as researchers in these type of settings we cannot pretend that our role is neutral. Instead, it is a deeply social and political role in the sense that relationships are established and 'self' and 'other' images are constructed that influence and control the development of the whole research process. The researcher's positioning allows —and restricts— the selection, documentation, articulation, interpretation and reporting of key incidents in certain forms and not in others.

Nowadays, there is a wider appreciation of the vast differences in mathematics curricula, teaching practices and aspirations for learning amongst countries. For example, in the practices of mathematics education across European countries, very strong and diverse pedagogical ideologies and curricula differences can be charted

(Buchberger, 1992; Eyridice, 1995). However, these differences are mainly taken as distinct categories when research outcomes are discussed and their construction is not being unravelled. Generally, a potential for exchanging ideas and views is assumed but not problematised. Taking into consideration existent diversities, one needs to query how these inter-cultural exchanges and interactions are evolving. Based on the analysis within this chapter, I would argue that a greater understanding of this process involves a 'self-other' dialectic that emphasises not the separation between the two but their reconciling. The discussion here has suggested the need to move from looking *at* towards looking *through.* Looking *at* verifies and confirms that differences exist. Looking *through* helps to appreciate the existent similarities embedded within the structure of such differences.

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TAMSIN MEANEY

THE FLY ON THE EDGE OF THE PORRIDGE BOWL

Outsider Research in Mathematics Education

> An individual's primary motive for research may be founded in self-interest and professional advancement, for example, in satisfying the academic requirements for a degree. The researcher's social concern is expressed in his choice of topic. Balancing self-interest and social concern in a society that values individualistic striving is not a simple matter; the way this is done affects the social utility of research. But there are other considerations which affect the nature and quality of research. These include the individual's cultural baggage which determines his perceptions of people from different classes and cultures, his theoretical training and the constraints of time and money available for research. (Walker, 1979, p. 91)

Many of the students that I have worked with have been from a culture other than my own. More often than I would like to admit, these students have not done well in learning mathematics. Continuing to change only my own teaching within the confines of a curriculum designed around the background and experiences of urban students of Anglo-Celtic descent in Australia did not seem to be sufficient. Changes were needed but they needed to be made with information provided by the communities of the students. However, as these changes would affect my teaching I wanted to be involved. As well, as a member of the culture which could be considered to have done well out of the present determination of what was valued in society, I wanted to be part of a solution not just part of the problem (see Bishop, 1996, p. 17-18 for a description of why non-Mäori researchers should be involved in Mäori research). Consequently I decided not only to initiate these changes through the production of a resource to help a community to rewrite a mathematics curriculum but to evaluate the usefulness of this resource.

This was my plan but in order to fulfil it, I moved country from Australia to New Zealand and situation. I found myself as a full-time Ph.D. student not only having to make a whole range of other decisions about the project and the research but also to justify those decisions to myself, to the community with whom I worked, and to those people who agreed to supervised or advise me about the research. As I tried to balance all of these kinds of justifications, the project itself evolved and developed. It became obvious that my background and experiences were driving the decisions that I made. Chronaki (this volume) also mentions how her lived experiences influenced her research. Yet this recognition of the influence of the researcher's personal background is usually ignored in studies with justifications for the research

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 167—183.

coming only from an academic analysis of the literature. As I considered through journal writings what was happening, my interest shifted to how someone like myself operated who had roles that were at different times that of an outsider and of an insider. The curriculum development project was no longer the object being studied but became the context. It is over four years since I began the first draft of what has become this chapter and the thesis is now completed (see Meaney, 2001). As this is a book about researching the social and political dimensions of mathematics education, instead of describing the findings of the project, this chapter describes its evolution. It is the evolution that shows how I grappled with these dimensions in the research that I was doing.

Mathematics education cannot occur without people. In the cross-cultural situations in which I have worked, there have always been very obvious differences between myself and my students. These differences included our first languages, understandings of the roles of individuals and communities and the knowledge that we valued. In some ways this has meant that how we interacted could never be ignored. As Chronaki (this volume) also argues, this *social* aspect is perhaps not always so obvious in other mathematics education situations even if it is just as important. In describing the evolution of this research, it is hoped that others will gain ideas about some of the decisions involved in researching their own social interactions.

The chapter first provides a background to the curriculum development project and the decisions that I made about the research. Then it shows how the information gained meant that changes were necessary to both what I did in regards to the curriculum development task but also what I researched. Finally it looks at the importance of relationships between myself and community members and between community members themselves.

THE CURRICULUM DEVELOPMENT PROJECT

Australian Aboriginal parents had consistently stated that they wanted their children to learn Western knowledge but also retain their Aboriginality (Graham, 1988 p. 131). In other indigenous communities, it has been suggested that 'increased local autonomy and actively valuing elders' knowledge will strengthen indigenous schools' (Lipka, 1994, p. 14). From my own experiences, I believed that in order for indigenous students' results in mathematics to improve, it was essential that parents with teachers design a mathematics curriculum for their school. Parents, using their knowledge of their community and their children, could have a say not only in what knowledge was to be learnt but how it was to be learnt. However, many parents have little knowledge about curriculum development in mathematics. Although teachers could have some, support for this process was still needed. Therefore, I designed a stand-alone mathematics curriculum development support document, which I have called the *Framework*[1]. It raised issues that I believed needed to be considered within the design of a mathematics curriculum. Its role, however, was to allow the

[1] A web version of the Framework can be found at http://aitken.scitec.auckland.ac.nz/~tamsin/FrameworkWebsite/framework.html.

community to make decisions which suited their children and their needs (see Meaney, 1999).

Ph.D. requirements mean that there is a need to expand academic knowledge. It was made clear to me that evaluating the Framework would not be sufficient. In considering the evaluation, I had been concerned that, as an outsider to the community that was doing the curriculum development, the support I provided through it may not have been appropriate. As a teacher of Anglo-Celtic ancestry, I am an insider to the mathematics culture with expertise which indigenous people value but am an outsider to their culture which makes me unsure how best to make use of this expertise (Osborne, 1995, p. 16 has stated similar feelings about doing research in indigenous education). There also appeared to be a dearth of research on outsiders working with indigenous communities on mathematics education issues. It therefore seemed possible to expand academic knowledge by investigating the effect of these insider/outsider roles within cross-cultural educational work. Figure 1 sets out how the project was to operate and be researched.

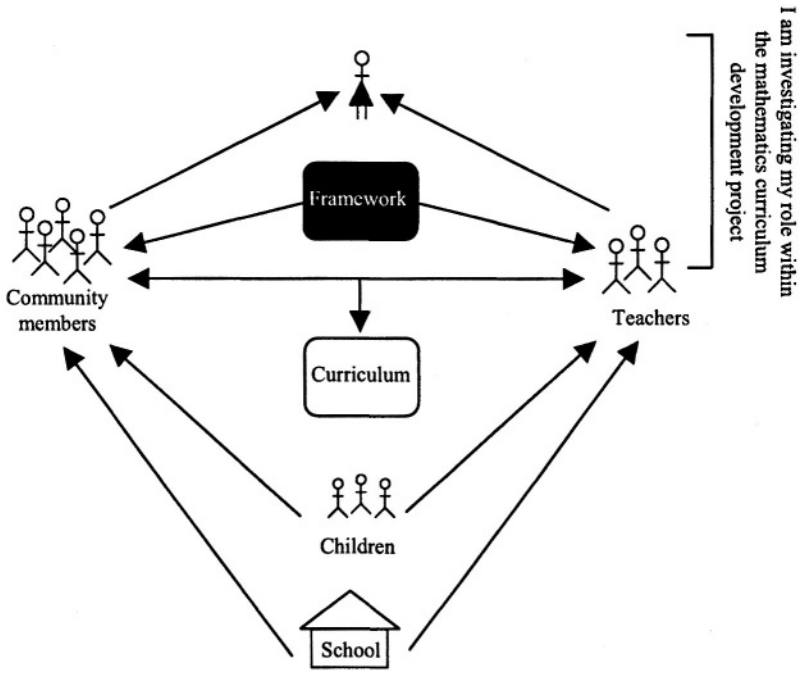

Figure 1: The development of the curriculum will provide feedback for the Framework and my interactions, but will not be part of the research.

I had wanted to work with a school community who had already decided to develop a mathematics curriculum. They would then have considered why they wanted to

participate and my role would be one of aiding rather than directing what they did. I anticipated that they would adapt the Framework to suit their own needs but that it would be possible to generalise its usefulness to a range of different communities.

Early in 1998, I discussed the curriculum development project with the principal of a Mäori school in a provincial town in New Zealand. The parents had placed their children in a kura kaupapa Mäori so that they could learn in Mäori, although not all the parents were fluent Mäori speakers. In kura kaupapa Mäori, 'the pedagogy of these schools is based on, but not exclusively, Mäori preferred teaching and learning methods' (Smith, 1990, p. 147-148)[2]. There is also an expectation that parents would have an active role in the decision making process within the school. In an interview, one parent stated that when her daughter was in a mainstream school:

> I wasn't as active a mother in school, I didn't ever question what maths was or what science was, I just went to the sports days. Kura kaupapa Mäori is something that we've only just begun to realise what a big commitment it is and how much harder it is but that's a good thing. I get a say, I get to contribute to what's happening with my kids. (P7: 2/12/98, p. 1)[3]

The principal had been involved in writing the Mäori version of the mathematics curriculum, *Pangarua* (Ministry of Education, 1992) and was disenchanted with some aspects (see Trinick, 1997 for a fuller description of this writing process). He believed that it would be beneficial for the school community to become involved in mathematics curriculum development and stated that:

> The New Zealand mathematics curriculum in Mäori, at the end of the day it was a translation exercise [...] it was quite unsatisfying. However, on the other hand as a principal of a kura kaupapa Mäori which is suppose to start from kaupapa Mäori, from a Mäori basis for learning, then the current one isn't really legitimate. (T3: 16/8/98, p. 2)

He expanded this by saying that

> I must put a footnote here, you know, sometimes we've looked so hard for this Mäori curriculum that when we find it we might not actually recognise it and go straight past it.

However, many of the parents had little understanding of what was involved in mathematics curriculum development and agreed to participate because it had been recommended by the principal. In an interview before meetings began, one parent in response to a question on why the community wanted to rewrite the curriculum stated:

> I don't know the difference [between the mainstream curriculum and a rewritten curriculum] so I can't comment on that. All I know is that our

[2] As a non-Mäori person, I have not expanded the description of kura kaupapa Mäori but instead recommend that people who are interested in these schools and their philosophies read Smith (1990).

[3] In excerpts from the transcripts of the meetings and interviews with parents and teachers the date is given and individual parents and teachers identified with numbers. The page number that the excerpt comes from is also given.

> headmaster, I'm quite confident that he's doing the best for our children. So I'm going to support this new framework idea. (P6: 23/8/98, p. 1)

When the curriculum development project began it soon became apparent that the decision to work with a community who had not had discussions about why a new curriculum was needed meant that my role became quite different to the one that I had envisaged. There are similarities between how these differences caused changes in my role and how the impact of the National Curriculum in the British mathematics education situation caused Chronaki (this volume) to adjust her research focus. Research participants will not only have beliefs about the research but will have a range of concerns which impact on how they participate in the activity being documented. This impact cannot always be predicted before research begins.

SETTING UP RESEARCH: AN ETHICAL DILEMMA

At the same time, as negotiations began with the school community, I was also negotiating with my supervisors on what my research would entail and making decisions about what information was needed, from whom and how it was to be analysed. These decisions would affect others as well as myself. In regard to research,

> there are always alternatives to be considered, choices to be made, and many times these choices involve other people. When these choices are value-related, we have an ethical problem. (Sowder, 1998, p. 428)

Research in cross-cultural situations, where there has consistently been differences in power, involves considerable thought about the questions of '*for what and for whom*' (Penn, 1999, p. 28). Equally important is the need to question the research methodologies used. Although my responses to these questions changed during the research, consideration of their ethical implications needed to be made continually. It was essential to discover the interests and needs of the other people in the research. Relationships with community members, both before the research began and those that formed during it, would be critical in developing an understanding of the situation. Each choice made had implications for all stakeholders —myself, community members and the academic audience for the research— and would be dependent upon whose interests were paramount. Nor were any of these decisions mutually exclusive but each one had implications for the others.

Deciding whose story to tell

Much of academic research involves standing back from a situation and telling someone else's story from a particular perspective (Smith, 1999, p. 42). In discussing the purpose of research in cross-cultural educational settings, Osborne (1995, p. 17) used the distinctions made by Singh (1995, cited in Osborne, 1995) between 'speaking not <u>about</u>, not <u>for</u>, but <u>with</u> oppressed minorities'. He felt that it was very easy to feel that you are speaking *with* when in fact you are speaking *for* or

even *about.* These are fine distinctions and they will be viewed differently by the participants in the research, the audience of the research and the non-indigenous researcher (Melrose, 1996, pp. 50-51). This is a different distinction than the anthropological outsider/insider dilemma described by Chronaki (this volume) as it is to do with who has the right to tell a story rather than whether the story told is an accurate representation of a situation because of the background of the teller. By focussing upon my role within the social contexts in which I worked, it would not be necessary to have to make such distinctions. However, by choosing to explore my role using an academic tradition within the requirements for a Ph.D., my ways of listening and learning were altered.

In Aboriginal society, different people own different knowledge and it is not appropriate for some people to talk about another's knowledge (Christie, 1985, p. 41). Knowledge is both restricted and personalised. Although my research involved reporting what others said, I did this only in regard to what I have done or contributed to the project. Wherever possible, community members' perceptions are presented in their own words. This decision had repercussions because it limited what was recorded and some of the information provided by participants, which could also be very informative about mathematics education was never made available. Writing and publishing these stories is often not a possibility for those outside academia because of a lack of knowledge of how to write a paper and a lack of valuing of how their stories could be informative. Jointly publishing papers is also problematic as it disrupts the confidentiality that university research ethics committees require from researchers (Sowder, 1998, p. 438). In joint publications, there are also issues about who benefits. Often researchers working in cross-cultural situations can be accused of exploiting the knowledge of others (Bishop & Glynn, 1992, p. 128; Jansen, 1983, p. 533). Certainly a university-based researcher can benefit from having a paper published, in terms of their career but the community itself may gain very little in the way of improving the mathematics education for their students (Walker, 1979, p. 92).

My decision to concentrate on my role caused endless arguments between myself and my supervisors over who was entitled to make the decision about what should be discussed. It also meant that anything to do with this research that has been published, including this chapter and the thesis, has had to be approved by the school community. Sometimes the principal read the article but at the last meeting of the curriculum development process I described to all participants the results contained in the thesis. Although any changes that they have requested have always been minor ones, it was important that they were kept informed. The school community, however, rarely had the same need for the separation between our stories. The following exchange happened in the second to last meeting while we were discussing how useful it was for my research to carry out a suggestion made by the principal.

> Tamsin: What you do is yours, [...] I have this horror of stretching what's mine and what's yours, and what I'm going to report on and what I leave as your business and what I leave alone. […] I'm quite happy to contribute anything which would be of

value but whether I would then use it for my own work, I don't know. But I've got a commitment to the whole process, and it's a bit like you saying whether the Framework's been useful for you, until you've got to the end of it, you don't know, well it's a bit difficult for me to say whether it would be useful for me until we did it because of trying to stay out of your business and just trying to concentrate on my business and I don't know when one stops and the other starts at times. It gets quite messy for me.

Parent 10: It's that English view of possession again.

Tamsin Well, it is but it's because of whom I am. There's been too many whitefellas come in and exploit indigenous knowledge and use it for Ph.D.s and academic things and the last thing that I want to do is to feel that I've done that, do you know what I mean? So, it is the possessive but I'm trying to do it for an ethical point of view and that gets me into worse troubles.

Principal: I don't have any problems with that, I think it's been mutually beneficial. (5/9/99, p. 51)

Decisions on how to carry out the research

The answers to my questions about research for what and for whom, were that the research was to be primarily for me so that I could improve my understanding of my mathematics education work. I wanted to investigate whether my insider knowledge could be of use to a community in which I was an outsider. However in regard to how this research would be conducted, I wanted to do it in a way that would give something of immediate benefit to the community. There is an expectation in many Aboriginal societies that if something is done for you then you will do something in return. This has implications for the type of research that is undertaken. Research which records a snapshot of what goes on in a community but which does not address ways of improving that situation, is one reason why some Aboriginal people are reluctant to have researchers in their communities. Walker (1979, p. 92) has stated similar feelings by Mäori people. Long term, this type of research could form the background to improvements but many communities feel that they have waited long enough to see returns from the information that they have already provided.

It was, therefore, important that the curriculum development project be as useful as possible. I needed to learn from the community as much as possible about how I performed my facilitator's role through the support provided in the Framework. This would be documented but would also allow me to improve it. This necessitated identifying the criteria by which they were judging my role. Although I had worked with several different indigenous communities, I had not worked before with Mäori people. There would be protocols and ways of behaving that I needed to learn in order to gain and understand this information. Bishop and Glynn (1992, p. 129) suggested that *cross-culturally competent* researchers needed 'an understanding of the importance of many Mäori concepts and their contextual relationships'. However, many projects undertaken with indigenous people are done within short time frames by people with no previous contact with the community involved, so I

wanted to see whether it was possible to learn these protocols while trying to participate in the project.

INTERACTIVE PARTICIPATORY ETHNOGRAPHY

The research methodology had to allow for all the different aspects of the research to be monitored but in a way that patterns and problems could be identified and changed if necessary. Often as an educator, I changed what I did as a result of an unexamined feeling that something was not working and could be improved. For the Ph.D., it was important to document the criteria used by myself and community members for doing my job well and any changes to the criteria as the process progressed. The research methodology needed to allow me to examine how my outsider status affected these criteria for both myself and for community members.

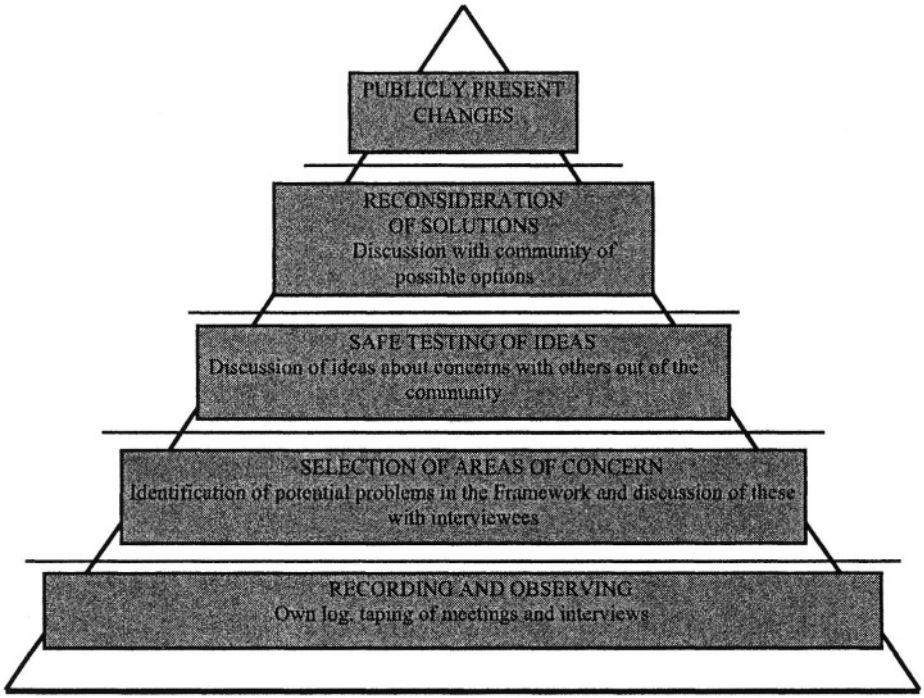

Figure 2: A pyramid of action inquiry processes used in Interactive Participatory Ethnography.

It was difficult to find a research methodology that would allow me to gather useful data and provide me with a way to analysis it. The study was with one community over a two-year period, so an ethnographic case study approach seemed the most appropriate. However, I was focussing on perceptions of my role rather than

insiders' perceptions of themselves (see Stewart, 1998 for a description of ethnographic research). As a result, I developed *Interactive Participatory Ethnography* which is based on ethnographic education evaluation (Fetterman, 1993), and Action Science and Action Inquiry research methodologies (see Reason, 1994, and Atweh, this volume, for a discussion of these). Although Action Science and Action Inquiry have been mostly used by managers working within large organisations, there was an emphasis on investigating the researcher's own biases in learning about themselves and how they were performing in their jobs which seemed relevant to this research. Reason (1994, p. 332) stated that 'one of the key skills in this process is to find ways of sidestepping one's own and others' defensive responses to the painful process of self-reflection'.

Interactive Participatory Ethnography uses a series of processes that are outlined in Figure 2. They are based on those of Krim (1988, p. 147) who described them as each becoming 'more reflective and proactive' as one moved up the pyramid. Thus the community members' comments during meetings and interviews and my own musings suggested areas of concern in the project. This is similar to Chronaki's initial analysis stage of *looking at* a situation. Things of interest are noted but it is not until the later stages that the reasons why they were noticed are understood. This understanding was developed from discussion in general terms with indigenous and non-indigenous colleagues outside the community. After further consideration, I described possible resolutions with colleagues and/or community members. From their reactions, a resolution was adopted. The Framework and/or my interactions were changed and these were publicly presented at the next curriculum development meeting.

Krim's research was based only on his own understandings of his work situation. I did not believe that my understandings alone would provide sufficient information for the Framework to be improved appropriately. As Hammersley (1992 p. 140) stated 'practitioners did not have "privileged insight" into the contribution that research makes to practice'. As participants and the clients of the support, it was important that the community also participate in the evaluation.

When working on other projects with indigenous people, I had often talked about problems when appropriate moments arose. However, as hard data was needed for my Ph.D., I choose to find out information through interviews at set times. Community members were aware that what they told me at these times could be quoted in my thesis. However, the repercussion was that I did my job differently as the research was very intrusive, and the relationships that developed between myself and community members were affected. Many of the people that Krim (1988, p. 154) was working with were unaware of his research until some of his descriptions of critical incidents were anonymously sent to his colleagues causing a major incident. Smith (1999, p. 138-139) related the story of how although an insider to a particular group as soon as she began to research the group, relationships with other members changed.

Following is a description of reactions to my role in the project. Community members' reactions to the Framework are not included here but are reported in Meaney (1999). It soon became apparent that the role that the community wanted

me to take was different to the one that I had envisaged for myself. Nor was it a role with which I was always comfortable.

COMMUNITY EXPECTATIONS OF MY RESEARCH

To become an insider in the project so that my expertise in mathematics curriculum development could be utilised, I needed to know what community members' concerns were, what role they expected me to have and how they expected me to behave. There had to be negotiation of these in order for them to be effective. Through respecting their concerns, I hoped that they would respect my expertise. One parent stated that

> it's like on the *marae* [ground before the meeting house], once you come into our thing for the duration of that stay, you must be an insider, a participant and if you start to dominate that consensus, which is how it operates, will soon tell you, it mightn't tell you verbally, but it will be told within things, so this is why I can't see it working if somebody like yourself says I've got to be an outsider. You've got to be part of it but the part that you do play which is your expertise will be regulated by the whole. (P1: 8/11/98, p. 9)

Community members had some specific ideas on what I could contribute to the process. One teacher stated at the beginning of the process that 'your ideas would probably be more than welcome to everybody' (T2: 23/8/98, p. 5). Later, the same teacher said:

> We need to look at other avenues especially, you know, with the type of kura that we are. No, I just think it's been a total advantage you coming in and opening our eyes to another world of maths, another outlook on it. (T2: 20/6/99, p. 5)

However the school community felt that they were in control of the decision making process within the meetings. After the fifth meeting, one community member said

> there is knowledge that you bring which is important to know about and if I don't agree with all that you say then I'll just leave it aside [...] But I need to be here to know what it is I'm going to accept from you and what it is I'm just going to put to the side for now. (P5: 20/6/99, p. 4)

Although I was seen as an insider to the project, my role was different. I was an outsider within the decision making process and it was important that I knew when to contribute and when to stay silent. Community members saw advantages in me being an objective outsider as I could give back to them their ideas without the passion that comes from being an insider. A parent stated

> because I think it happens everywhere, you know, you can get caught up in something and then you forget, after a while you lose the plot... So definitely there is a role for your perspective in helping to set up the curriculum. (P5: 23/8/98, p. 4)

The principal described it by saying:

> the fly on the edge of the porridge bowl is certainly doing a lot better than the fly in the porridge. The fly in the porridge is so concerned about difficulties, they can't actually see how close they are to the edge of the bowl or not. The one on the side can. (T3: 16/8/98, p. 9)

After the fifth meeting, one teacher in talking about the organisation of the meetings said

> you still need someone up the top there to actually move them along […]not you. I think you still need to keep out of the war zone, do your work on the outside. (T2: 20/6/99, p. 4)

The research methodology helped to identify community concerns, the role they expected me to play, and how they expected me to act. It was not always easy to decide whether I was willing to fulfil these expectations. Often I had to consider the ramifications for the curriculum development project, the research and my status within both. This information also rarely provided me an understanding of the criteria they used to judge the appropriateness of my role. The next section describes what I did during the project.

THE REALITY OF MY ROLE

I had expected to be an observer during meetings, but in fact performed several different tasks. Sometimes tasks would be suggested to me that I did not accept. As a result of interactions with the Ministry of Education, the principal suggested that I contact them about the project. He said 'so, as a push from me on the side, you should be talking to Ministry people and seeing what they think what the development of indigenous curriculum means' (T3: 8/11/98, p. 2). As he also seemed to suggest that they may not be supportive of the project, I decided against doing this, as my status in what could be a political discussion could limit what was achievable for his school and for myself. Many community members, especially to begin with, asked my opinion on their curriculum. When queried, I would state that the mathematics curriculum that they developed had to suit their needs and not mine. My role was evaluating the Framework and trying to improve it.

Other roles were accepted or negotiated with the community. At the request of the principal, I produced graphs from questionnaires and wrote reports for some of the early meetings from the transcription of the tapes. After reading a transcript and then the report, however, my supervisor felt that my interests had controlled what was highlighted in the report. As a result, I wrote a codicil to the report, stating that someone else's interpretation of the transcription could have highlighted other issues. Several meetings later, the community included a large portion of this report as part of their policy document on mathematics. In one sense, I felt vindicated as my interpretation of the meeting was accepted by the community. However, their curriculum may have been influenced in a way that I had not expected.

In the second meeting, I ran a mathematics activity with the parents so as to raise the question of whether the intrinsic worth of an out-of-school activity could be destroyed if it was used as a vehicle for teaching a mathematical idea. Consequently, I decided to add activities to the resources that accompanied the Framework. In later

meetings, my role included typing sentences into the computer that community members wrote or dictated for their policy statement. This gave me a purpose for being at the meetings without being drawn into the discussions. As an insider, I was contributing to the project but remaining an outsider to the decision making process.

During the first few meetings, no one within the school community seemed prepared to be the organiser. It was inappropriate for me to have this role as too often I had seen outsiders run a project that stopped when they left. It also seemed from the interviews, that people were worried that the discussions were not leading to the production of anything concrete. In later interviews, various resolutions to these concerns were discussed and as a consequence I chaired the third meeting. In this meeting, the community decided what sort of mathematics curriculum they wanted and organised different people to take responsibility for the remaining meetings on particular topics. Unfortunately these decisions did not solve the problems as parents nominated as chairs failed to attend. As a consequence, fewer and fewer parents attended the meetings held in 1999.

As the community decided to write a policy document rather than a new curriculum from scratch, it became my job to remind the group about this decision in later meetings. I did this when they started discussing specific details. Sometimes this advice was ignored but more often it would focus the group back on the questions in the Framework and the discussion would become more general.

Occasionally, community members asked for clarification on some of the discussion questions in each issue of the Framework. This gave me an understanding of what was not clear and needed modifying in the Framework. I also gave unsolicited examples to illustrate topics that were discussed. This was done either by referring to the resources that accompanied the Framework or by giving examples from my own experiences in different indigenous communities.

Many of the roles that I performed during the curriculum development process seemed to be those of an objective outsider. I used my expertise to clarify or illustrate some areas but did not contribute to contentious discussions. My role also included trying to make discussions more general. At times, I did clerical tasks that consolidated my role as an insider to the project. However, other roles were declined. The negotiation, between myself and the community about the tasks, was often not done though explicit discussion but through nods and smiles. No hard data about these negotiations was obtained although information was often provided when the tape recorder was turned off. As I had told the community that only excerpts from the transcripts would be used in the analysis, it was difficult to discuss these negotiations within the thesis.

Generally the Framework seemed well regarded, but the community rarely modified it and other difficulties with the co-ordination of meetings meant that it was not adequately trialled. The research methodology allowed some problems with the Framework to be identified. However, the information, or lack of it, that was gathered from the community made me confront the importance of relationships. The relationships between myself and the community and those within the community affected not just the use of the Framework but also what I was told about the Framework and my role.

RELATIONSHIPS

From my previous work with indigenous people, it seemed important that the community knew who I was and where I was from. My introduction to them was through the principal whom I knew through a shared interest in mathematics education. At initial meetings, I tried to use my previous work as a way of legitimatising my inclusion as an insider to the project. I discovered later that it would have been better to have concentrated on how I would be related to other community members, in both the past and present (Bishop, 1996, p. 215). It also seemed important to be seen as being approachable by any community member and not to be seen as being on one person's side in any of the discussions. I, therefore, distanced myself from the principal and his wife who was also a teacher at the school. As a result, there was no one who could help me learn the protocols of this community. I often acted in a way that would have been appropriate with Aboriginal people but which I found out much later was not with Mäori people. In Aboriginal communities you wait to be asked to become part of an activity but in Mäori society, you are expected to just participate. This difference meant that I rarely attended anything at the school except the mathematics curriculum development meeting and so never had enough time to become close enough to one person for them to feel comfortable in criticising me. Pauses in responses to my questions became my only way of determining when there were problems with my contribution to the project.

As the relationships that I did form with community members were tentative, it was also difficult for me to critically analyse some events. In the second year, parent attendance at the meetings lessened. Reasons were given for this by various community members. A parent explained her hesitation in being involved by saying 'I don't know how the other parents feel but [...] it seems to be a bit over my head type thing, so like for me personally I'm quite happy to leave it with the teachers' (P8: 20/6/99, p. 3). One teacher said 'lot of parents in kura kaupapa who can't do that, so it goes back to those few to push things a little bit along' (T2: 20/6/99, p. 3). Another parent felt that parents were not committed enough in their support of the school

> from where I'm sitting, I've seen people come and want to be here that either don't know how to fit in or they have other work or other meetings that would take them out. To my way of thinking it's time to call for that commitment. (P5: 20/6/99, p. 3)

She felt that other meetings held at the school at this time were also poorly attended. It was difficult for me to ask questions about this issue which could be interpreted as being critical of the school community. Without their support, there would be no project and it would not be possible to trial the Framework. However, the information that I did get about the Framework was only from those people who were attending. Without appropriate standing in the community, I was unable to hear the views of those who withdrew.

It was in talking with Mäori colleagues outside the community, about how to discuss problems, that I discovered that I needed to be more involved in the community. By not living in the town where the school was and so not being

available at other times, by waiting to be invited to other activities, I had placed myself as an outsider to the community. Being an insider to the project did not make me an insider to the community. Yet to discover information about the progress of the project, I actually needed to be an insider to the community. My lack of cultural understanding meant that I was not discovering how well I did my job or how I could improve it. In her research, Chronaki (this volume) also found that relationships were important. Asking questions about participants' teaching although challenging was comfortable because of the relationship between them. In my interactions, I was not seen as an impartial learner about what the community did but rather as the instigator of the Framework. It was my feelings that community members were concerned about. Without more standing or knowledge, I was unable to convince them of the importance of what I wanted to know and my ability to cope with criticism. It may also be true that their unfamiliarity with mathematics curriculum development also made them hesitant to make comments about it. Unlike the teachers in Chronaki's study, community members were being asked to reflect on an activity they had not been involved in before and with which they had nothing to compare. Chronaki's teachers were commenting on what they did in the normal course of the working lives.

Trialling the Framework with a community, with whom I had not worked before, resulted in unforeseen repercussions. A lack of understanding of the impact of relationships both between community members and myself and within the community resulted in several problems. Community members agreed to participate in the project because it was recommended by the principal. It was difficult to find out about the usefulness of the Framework when people were unsure of what they were to do with it. The relationship between teachers and parents meant that parents believed that the teachers were the experts and so parents felt that they had little to offer to the process. In response to a question about whether the Framework was facilitating sharing between parents and teachers, one parent said 'so when you say sharing, well the only reason why I'm not really having any input is because I'm happy with the way my daughter's progressing with her maths' (P8: 20/6/99, p. 2). The parents saw attending meetings as a way that they could learn from the teachers. One parent said that the meetings were useful because

> It gives me an understanding of the curriculum and the *tamariki* [children] I'm going to be working with and if I understand it then I may be able to help them, because that's important too. I have to know what they're doing because if I don't understand it then I need to keep asking so that it becomes clear in my mind. (P5: 20/6/99, p. 1)

I had hoped that the Framework would support parents' contributing their knowledge of the children and the community. As the parents did not see this as part of the curriculum development process, it was difficult to determine how effective the Framework was in doing this.

The curriculum development project although not resulting in a new curriculum for the school did provide the school community with opportunities to share their beliefs about mathematics and why and how it should be taught. Dialogue was begun even if a consensus was not achieved. The school community and the teachers

in particular felt positive about a continuation of the dialogue after my involvement finished. Many of the questions that I had about my role were not answered, until I began to examine certain incidents. By pulling these incidents apart, I built up a description of what I did as a facilitator within an educational project. I then re-examined these incidents using Foucault's ideas about power (see Meaney, 2001; see also Cotton and Hardy, this volume). This was an emotionally-traumatic process as it caused me to consider the process by which truth can to be determined through interactions.

CONCLUSION

Social contexts both inside and outside the classroom have had a huge impact upon students' learning. In the past, research on social issues has concentrated on perceived problems of the students (Deyhle & Swisher, 1997, p. 115). The research discussed in this chapter examined the role of the mainstream educator within a mathematics education project involving indigenous people.

The research methods used in gaining information have potential but need to be tempered with other less formal interactions. I learnt some things about the Framework and my interactions and was able to make modifications to both (see Meaney, 1999). However, I felt that there were many things about the usefulness of the Framework that I never learnt. This caused me to focus more on the relationships between myself and the community. Although even in the initial stages of the research, I had been aware of the importance of these relationships, it was not until my involvement with the curriculum development process had almost finished that I realised that I could not evaluate my role without analysing these relationships. It was not so much what the community told me as what they did not tell me which forced me into these considerations.

By examining my work through the academic research lens, I have had to justify my actions, in ways that I would not have normally considered. At times I struggled to see the point of using this different lens. It seemed that I was just using a new language to describe what I already knew. At other times, this lens made me stop and discover some of the assumptions that I had been working under without really being cognisant of them. It has also connected my small study to many, much wider issues.

Working and researching in cross-cultural situations is extremely difficult. The decisions that I had made about setting up the research would probably have served me well if I had done my research in an Aboriginal community. I thought that I could learn the protocols for working in Mäori society while involved in the project. However, the other decisions that I made meant that I was actually not in a position where I could learn these protocols. By concentrating on my outsider role, I excluded myself from being the insider that I needed to be in order to do the research. The assumptions that I made about the research I was doing were not wrong, they just were not enough. There were other things which needed to be considered but my knowledge of this community was too limited for me to be aware of them. My timeline was adequate to do the project but inadequate to become an

insider. This is one of the issues related to trying to do this type of research within a Ph.D.

It is not sufficient to accept that indigenous students are less likely than non-indigenous students to do well in mathematics. However, in trying to research ways to improve the situation, it is not always easy to predict what should be focussed upon. By describing the journey covered while doing this research, some of the tensions between my background, the expectations of the community and the requirements for doing a Ph.D. are outlined. In order to improve the situation for indigenous students it is important to realise that part of the answer lies in the ways that these tensions are resolved. This is what I now believe is meant by researching the socio-political in mathematics education.

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FIFTH DIALOGIC UNIT

ADOPTING CRITICAL APPROACHES IN RESEARCH METHODOLOGY

In order to produce better educational outcomes for all students, it would be reasonable to expect that research should play an important role in understanding how education is integral to learning and life outcomes. However one of the major dilemmas confronting educational researchers is the impact of their work on the field of practice. In times of economic rationalism, funding bodies call into question the amounts of money being spent on educational research. Educational researchers already bemoan the minimalist funding allocations to research, particularly when compared with other fields. Hence, it becomes important to understand why research has little impact on the field of practice that it studies.

We can examine the relevance and appropriateness of research itself in relation to practice. Participative Action Research and other forms of critical investigation have questioned the assumptions, interests and ways of proceeding within research paradigms such as positivism and hermeneutics (see Atweh, this section). One of the conclusions of this critique has been that, if research is to have an impact on the field of practice, it is important to significantly change the relationship between researchers and research participants towards forms of collaborative inquiry, to place an interest in practice as a source of research problems, to privilege an interest in transformation, and, in general, to make research a socially and politically sensitive endeavour.

However, this proposition is not unproblematic since critical research models have not been widely accepted as valid and legitimate. The power game in academic environments about what is 'properly-produced' knowledge has effectively restricted the expansion and recognition of critical —some times even revolutionary— research and its results. One of the results of this political confrontation has been the scarce representation of, for example, action-research projects in academic publications in mathematics education, which contrasts with the wide acceptance of this type of inquiry carried out by practitioners wanting to improve their practice —but without 'academic' pretensions. Once again, 'professional research' is brought apart from 'practitioner research' in a provocative segregation between forms of knowing.

This dialogic unit continues the debate on methodological issues of socio-political research in mathematics education. Attention has already been paid to the

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visibility of the researcher as an agent influencing the research situation, to the necessity of researching 'with' instead of researching 'on', and to the evolution of power relationships among the different people involved in the research process. The three chapters here concentrate on complementary issues such as the resonance between critical methodological approaches and power, the qualities of critical research, and the challenges that the involvement in critical research pose to the researcher and to the process of knowing.

Bill Atweh's chapter highlights the difficulties that Participatory Action Research has faced for being considered legitimate knowledge. He argues in favour of PAR as a methodological approach that allows coping with the pragmatic, epistemological and political challenges that the field of mathematics education practices poses to research. He shows how action research has considerable potential in challenging and transforming both macro and micro contexts. By drawing on examples of projects with teachers in Brazil and Mexico, he demonstrates that action research can be an important tool for improving mathematics teachers' practices on a large scale. Atweh's discussion complements some of the points raised by Gorgorió, Planas and Bishop about action reseach projects.

In their chapter, Ole Skovsmose and Marcelo Borba refer to the process of critical research as the investigation of 'what is not the case', and define it in terms of movements between three types of situations: a current situation —that actually exists and can be observed—, an imagined situation and an arranged situation — which have a 'potential' existence or, in other words, 'are not the case' but have a potential to be realised. Their discussion concentrates on the qualities that the movement between those situations display and what it means for the researcher to be sensitive to as well as to promote those qualities. Notions such as pedagogical imagination, practical organisation and explorative reasoning emerge as key characteristics of critical inquiry.

Renuka Vithal draws on a teacher education project in South Africa where she introduced teacher students to diverse theoretical ideas related to critical mathematics education. She studied how these students made sense of the ideas and used them as inspiration in their teaching practicum. In the chapter she concentrates on a discussion of the way in which her own research evolved in an attempt to find coherence between the critical and socio-political theories illuminating both mathematics education practices and research practices. Vithal underlines issues such as choice, negotiation, reciprocity, reflexivity, the relationship between subjectivity and objectivity, change and instability in the research context, and concerns for emancipation, empowerment and hope. These issues represent points of struggle for a researcher concerned in achieving methodological and theoretical consistency in a socio-political research project.

The famous Colombian action researcher, Orlando Fals-Borda, argued that the best way to understand reality is to try and transform it. The chapters in this section illuminate the complexity of the link between socio-political research and social transformation, and offer tools to examine how power relations are realised in and through the very same practice of researching mathematics education.

BILL ATWEH

UNDERSTANDING FOR CHANGING AND CHANGING FOR UNDERSTANDING

Praxis between Practice and Theory through Action Research in Mathematics Education

The past twenty years of research in mathematics education has witnessed significant diversifications in research questions, theoretical stances, paradigms and methodologies (Bishop, 1992; Silver & Kilpatrick, 1994; Wiliam, 1998). First, there is a noted trend of diversification from the psychological and cognitive science informed research towards research using theories and constructs from sociology, anthropology and critical theory. Second, there is a diversification from quantitative, experimental and survey research, dominant in the early stages of research in mathematics education, to a wider acceptance of qualitative methodologies such as ethnography, case studies and, of course, action research (Atweh, Forgasz & Nebres, 2001).

Arguably, until recently, action research has not received the prevalence enjoyed by the other 'new' paradigms in the mainstream literature in mathematics education. A survey of published journals of education during the past ten years yielded fewer than 50 published articles dealing with mathematics education and action research. The vast majority of these articles appeared in what can be described as 'professional journals' rather than established 'research journals'. Let me hasten to add here that I do not accept this distinction as unproblematic. However, the fact that this distinction is often made in mathematics education discourse is an indication that teachers' knowledge and academic knowledge are constructed differently not only in the language they use and their level of abstraction but also, I argue, in their social status. Even though this figure can be a little higher since, as will be discussed below, the terminology used in referring to classroom-based and teacher participatory research is not standardised in the literature. This illustrates that action research is not yet a widely used research paradigm in mathematics education. However as an indication of the steadily increasing importance of action research in the field is its inclusion in several of the recent international books and handbooks on research in mathematics education. For example, the subject index of the internationally read *Handbook for Research on Mathematics Teaching and Learning* (Grouws, 1992) contains three references to action research. The more recent *International Handbook of Mathematic Education* (Bishop, Clements, Keitel,

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Kilpatrick & Laborde, 1996) and the *Research and Supervision in Mathematics and Science Education* (Malone, Atweh & Northfield, 1998) contain single chapters on teachers' research and action research respectively. The most recent *Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000) contains at least ten chapters on teaching experiments and classroom based research methodologies including action research. Naturally, not all these practice-based methodologies are synonymous with action research. The similarities and differences between these methodologies will be discussed further in this chapter. However, it is obvious that there is a noted trend towards increasing calls for and use of practice-based methods of research in the field.

Perhaps the past reluctance to use action research in mathematics education compared to other areas in education is at least partly due to some misconceptions of the nature of action research. Action research is sometimes associated with problem solving type of projects or professional development activities (Atweh & Arias, 2001) and hence, in the mind of many, falls short of being 'rigorous' research in the traditional sense of knowledge generation (Gitlin & Gore, 2000). Further, action research often involves the participation of practitioners and non-academics; hence, it is open to the criticism of being non-academic research (see Gorgorió & Bishop, this volume, for an illustration of this point). Similarly, because action research often stresses the importance of contextualised knowledge, it is sometimes perceived as very individualistic, localised and non-generalisable, and hence less worthwhile than other forms of research. While these criticisms are likely to arise about the use of action research in all fields of education, perhaps the concerns are more acute in mathematics education since the content area of mathematics has often been associated with objectivity, rigour and generality of its truths.

It is possible to identify another reason why action research has not been widely used in mathematics education. As Vithal argues in this volume, mathematics education has it roots in the fields of mathematics discipline studies and in psychology. Similarly in her chapter, (see also the chapter by Skovsmose and Borba) she discusses the concepts of resonance between the theoretical stances that the researcher has about teaching and learning, the research questions that they raise and the research methodologies that they employed. The research questions traditionally raised in mathematics education lend themselves to more traditional methods such as experimental designs and survey research. With the increased use of the sociocultural understandings of mathematics education (Atweh, Forgasz & Nebres, 2001), alternative qualitative methodologies were employed such as ethnography. In this chapter arguably, action research is consistent with critical understandings of mathematics education and, in particular, of critical mathematics.

The purpose of this chapter is to discuss the role of action research in mathematics education. The first section develops a case for the potential contribution of action research to the mathematics education research, classroom change and reform. The second section discusses some of the characteristics of action research according to one tradition known in the literature as *participatory action research* (PAR). The third section presents some examples of collaborative action research projects reported in the literature and conducted in diverse contexts. In keeping with the focus of this book, the emphasis in this chapter is on action

research as a valid and powerful methodology of researching the social context in mathematics education rather than on documenting the type of knowledge that action research studies may produce.

WHY ACTION RESEARCH IN MATHEMATICS EDUCATION?

This section develops the case for the use of action research in mathematics education based on pragmatic, epistemological, and political grounds.

Pragmatic argument

The past fifty years has witnessed a rapid increase in research in mathematics education around the world (Kilpatrick, 1992; Bishop, 1992). Such expansion in research activity has resulted in the publication of several national and international journals for research in the field, volumes on research summaries such as handbooks and a multitude of national, regional and international conferences devoted specifically to mathematics education. Similarly, during the same period, large-scale reform movements in curriculum content, pedagogy and assessment were implemented ranging from the 'New Math' movement in the 1960s to the more recent National Curriculum in the UK, the National Council of Teachers of Mathematics' (NCTM) Standards in the USA, and the National Statement on Mathematics in Australia, as in many countries around the world (Jacobsen, 1996).

However, and perhaps as a result of this rapid increase in knowledge and demand for change, there is an increasing separation between knowledge generation and knowledge application in mathematics education. This separation is in time —generate knowledge now and apply it later—, in personnel involved —academics vs. practising teachers—, and in language utilised —theoretical vs. applied. Research in mathematics education is mainly conducted by academic staff at universities and is published in journals mainly read by other researchers. In most Western countries professional associations, conferences and publications for researchers are distinct from those for teachers of mathematics. Perhaps one can identify curriculum developers as a third main group involved in mathematics education. These include personnel in education authorities and textbook writers who form a link between knowledge and theories generated by research and the practice of teaching in the schools. In many countries they are also responsible for professional development of teachers. Naturally, there are overlaps between the three groups; for example teachers undertaking postgraduate studies and doctoral degrees. However, communication between these three groups is not always smooth and unproblematic.

The effectiveness of the recent theoretical knowledge stemming from research and the large number of reform movements designed to change mathematics education in the school is still open to question. Sprinthall, Reiman and Thies-Sprinthall (1996) have argued that research on the gap between policy and practice has shown that often many innovations are seen by many teachers as external demands that 'force' teachers to change, and hence are resisted by teachers. The experience of teachers under the National Curriculum reform in the UK illustrates

the effect that sudden changes imposed from outside the classroom can have on demoralising and dis-empowering teachers (Hargreaves & Evans, 1997). Similarly, Kilpatrick (1999) argues that the USA reforms initiated by the NCTM have lead to a 'backlash' in some school districts in what has become to be known as 'maths wars'. In a book with the provocative title of *The Predictable Failure of Educational Reform,* Sarason (1990) identifies the piecemeal approach that many of these reforms take as responsible for their failure to change actual school practices. There are often separate reforms agendas for the curriculum, for teacher professional development, for school structures and organisations, and so on. Hargreaves (1994, p. 242) argues that 'significant change in curriculum, assessment or any other domain is unlikely to be successful unless serious attention is also paid to teacher development and the principles of professional judgement and discretion contained within it'. Sprinthall, Reiman and Thies-Sprinthall (1996, p. 666) argue that the 'massive failures of the [many] national curriculum projects of the 1960s' have raised interest in (re)investigating and (re)theorising the teachers' role in educational change.

This failure of research and curriculum reform to effect the classroom in mathematics is a *pragmatic* reason for exploring alternative research paradigms that may bridge the gap between research concerns and problems of the reality of the classroom. This chapter argues that action research is intrinsically is based on bridging this gap.

Epistemological argument

The second argument for the use of action research in mathematics education is an *epistemological* argument. A constructivist theory of learning (Davis, Maher & Noddings, 1990) asserts that learners develop their knowledge based upon previous knowledge and experience —and that this process is assisted by reflection and negotiation with others and not simply transmitted from expert to novice. Crawford and Adler (1996) have drawn some parallels between student learning of the mathematical content and teacher's learning about teaching. Using a neo-Vygotskian perspective they argue that knowledge or meaning is constituted rather than transposed through activity in a sociocultural context. They go on to assert that:

> Students taught and assessed in traditional ways, learn to demonstrate that they have encoded the culturally approved knowledge and can reproduce it. Those who learn about teaching through reading about education research develop knowledge of a similar kind. In neither case is the knowledge necessarily a basis for further action or a changing personal view of reality, (p. 1189)

Further, action research is based on the epistemological understanding that knowledge is never value free. Many action researchers have employed Habermas's theory of knowledge-constitutive interests (e.g., Carr & Kemmis, 1986; Grundy, 1987). Carr and Kemmis (1986) point out that the designation of this theory reflects its basic epistemological assertion that knowledge 'is always constituted on the basis of interests that have developed out of the natural needs of the human species and

that have been shaped by historical and social conditions' (p. 134). Habermas discusses three types of knowledge-constitutive interests: *technical, practical* and *emancipatory.*

Arguably, teachers in the conduct of their work have several interests and needs which can be classified as *technical.* For example, they need a huge repertoire of teaching and classroom management methods, knowledge of content, use of materials, resources and technologies, evaluation and assessment techniques and so on. Carr and Kemmis (1986) point out that the concern about this type of knowledge is not about its usefulness or validity, but in the mistaken assumption that 'it is the only type of legitimate knowledge' (p. 135). Likewise, undoubtedly, teachers have several needs that may be classified as *practical.* They need to understand students' motivations, beliefs, expectations as well as their previous knowledge to be able to assist them to develop their mathematical knowledge. Likewise, the teachers themselves have interests, beliefs and knowledge that undoubtedly influence the way in which change is enabled in the classroom (Fennema & Franke, 1992; Thompson, 1992). Teachers also need to understand the social context of the school and of their students in order to be effective professionals.

Yet, teachers also have *emancipatory* interest, which is intrinsically tied to autonomy and freedom from dogmatic dependence. Since technical knowledge is open to 'the tyranny of regulations', and practical knowledge is open to 'deceit and false consciousness' (Grundy, 1987, p. 17), a different type of knowledge is required for autonomy. Grundy argues that 'the emancipatory interest gives rise to autonomous, responsible action based upon prudent decisions informed by a certain kind of knowledge' (p. 18). While control and understanding are the motivating factors of the previous knowledge-constituted interests, empowerment, i.e., 'the ability of individuals and groups to take control of their own lives in autonomous and responsible ways' (p. 19) is the motivation for emancipatory knowledge. Since autonomy of one individual cannot be isolated from those of others in a social group, and since any practice is a social process that involves many others, there is more emphasis in this type of knowledge on the role of the social dimension of the practice. The development of such knowledge is enhanced by collaborating with other people inside and outside the practice. Also, this knowledge cannot arise simply from experiential processes, or be based on understanding, but develops through critical reflection.

Arguably, teacher's practical interests may be met by them learning about research findings conducted by other researchers either through in-service programs or reading the relevant literature. Practical interests may require a more direct engagement and reflection on the research knowledge. However, emancipatory interests are only achievable through teachers raising their own questions about their practice and collating evidence for the answers. Otherwise, they are forever subject to the agendas of others with 'better' knowledge than themselves. Here I argue that action research aims at empowering participants as a result of their involvement in their projects.

Political argument

The last but not least argument for advocating action research as a viable research methodology in mathematics education is *political.* The separation of roles between knowledge generation, knowledge translation and knowledge application may contribute to deprofessionalisation of teachers. The separation between the process of knowledge production and knowledge diminishes the responsibility for the teacher to understand and theorise their practice and to talk about their practice and defend it publicly. This separation between the role of the teacher and the academic is also reflected in the industrial conditions in which the two types of educators work under. Research is often part of the expectations and means of promotion for university academics but not always allowed for in the school systems.

In two investigations Romberg (1988) and Noddings (1992) have studied the status of mathematics teachers in the USA and concluded that they do not reach a professional status. Arguably, this situation applies to mathematics teachers in many countries around the world. Noddings (p. 206) notes that '[t]here are bright spots, however in this bleak picture. Reform movements are pressing for changes designed to professionalise teaching'. Nodding identifies several components of a profession, three of which are relevant to the discussion in this chapter.

First, professionals are acknowledged as having special knowledge in their field of expertise. This knowledge is essential for professionals to be effective in the provision of their service and in improving their practice. In the area of mathematics education there are many studies on teachers' content and pedagogical knowledge. Recently, few studies have addressed how teachers develop their knowledge and on how it is possible to change it. Evidence from the UK points to the fact that teacher change is most effective when teachers have explicit knowledge about their context and can articulate their personal theories on students' learning (Askew, Brown, Rhodes, Johnson & Wiliam, 1997). Through action research practitioners have a chance not only to improve their practice but also to develop an understanding of their practice and hence it enhances their professional status.

Second, professionals work within an atmosphere of collegiality. They work with others to produce professional knowledge and to improve their practice and the conditions of their practice. There is much evidence from research on school change in Australia that shows that more effective changes result when all the teachers in a school co-operate to change their practice in their individual classes (Baird & Northfield, 1992). Participatory action research stresses the collaboration between teachers within some practice and external critical friends. Collaborative action research not only contributes to the development of collegiality within the teachers themselves but also between teachers, curriculum developers and academics.

Third, professionals enjoy a degree of autonomy. There is some evidence that indicates teachers' sense of autonomy is correlated with their enthusiasm about their work and even their performance (Noddings, 1992). It can also be argued that changes can be more permanent and effective if initiated and controlled by the teachers themselves. Teachers do not resist reform when they choose aspects of their practice that need to be improved, and they control the direction of change. However, it is also true that total autonomy is not possible or desirable. There are

other people who have an interest and a right to be involved in educational decisions. They too must be involved in educational change. Through action research teachers can develop a sense of agency and control in raising their own questions for research and in generating knowledge about their practice and in reforming their own practice.

This section has argued that the relative failure of research knowledge as well as curriculum reform programs to drastically reform classroom teaching calls for the consideration of alternative research paradigms in mathematics education. Action research is posited here as such an alternative that aims at bridging the gap between theory and practice, and at the same time leads to professionalisation of teachers. Finally, it addresses not only the technical and practical needs of teachers but also their emancipatory needs. The following section discusses the characteristics of *participatory action research* as presented by Stephen Kemmis and his colleagues.

WHAT IS PARTICIPATORY ACTION RESEARCH?

It is not the intention here to present a comprehensive history of the action research movement both in education and general practitioner research. Such a history is reported by McTaggart (1991). Suffice to say that there are different sources of thought and practices that have led to the development and refinement of action research. Most noted is the contribution of the 'teacher as researcher' tradition often attributed to Stenhouse in the UK. Theoretical contributions to the development of action research have come mainly from the writings of Freire and Fals Borda in South America, and from the Frankfurt School of critical theory and its more recent development by Habermas. To a certain extent the writings of Schön (1983, 1992) about reflective practice as discussed below have impacted on action research theory and practice. Different traditions of action research have evolved in the United States, Latin America, United Kingdom and Australia. Kemmis (1999, pp. 156-7) points out how at times the differences between certain schools of action research have led to researchers 'relate[ing] to one another as 'external' critics of each others positions, rather than as 'internal;' critics who share broad agreements about the nature and conduct of action research'. It is beyond the scope of this chapter to discuss the characteristics of the various schools. The first section below adopts the theorisation of action research as proposed by Kemmis and his colleagues at Deakin University in the 1970s and 1980s. This is followed by an attempt to locate action research in relation to other emerging types of practice-based research discussed in the mathematics education literature.

Characteristics of Participatory Action Research

Perhaps the most widely known characteristic of action research is that it consists of cycles of action and reflection. There are various models illustrating this basic design of the methodology. Perhaps the model presented by Kemmis and Wilkinson (1998) is widely used. However, it is argued here that action research has some other important characteristics other than its cycles or actions and reflection that set it

aside from other research paradigms. Kemmis and Wilkinson have identified six characteristics of participatory action research (PAR).

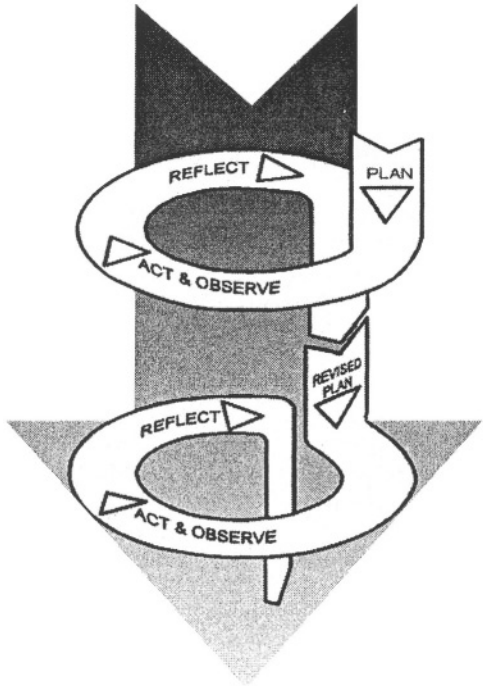

Figure 1: Model of Action Research

First, it is a *social* activity in that 'it deliberately explores the relationship between the realms of the individual and the social' (p. 23). It recognises that 'no individuation is possible without socialisation, and no socialisation is possible without individuation' (Habermas, 1992, in Kemmis & Wilkinson, p. 23), and that the processes of individuation and socialisation continue to shape individuals and social relationships in all the settings in which we find ourselves' (Kemmis & Wilkinson, p. 23). Here it is argued that action research is social in two meanings. First, it recognises that studying the practice of teaching should construct the classroom as part of a wider set of constraints, relationships and agendas. Many action research projects are based on socially-critical and socially-just practices of teaching and learning. Secondly, it problematises the process of research itself and critiques it in terms of power relationships between the participants, the representation of their voices and relative benefits each obtains from the research. As a social activity, action research is never neutral.

PAR is also *participatory* in that 'it engages people in examining their knowledge (understandings, skills and values and interpretative categories (the ways they interpret themselves and their action in the social and material world)'

(Kemmis & Wilkinson, 1998, p. 23). It is also participatory in the sense that people can only do action research 'on' themselves —individually or collectively. It is *not* research done 'on' others. Practitioners' involvement in research is not unproblematic. Teachers' time, work practices and initial training impose constraints on their involvement in research activities. Gitlin and Gore (2000) warn that unless these factors are taken into account, the commitment of education authorities to principles of teachers' research is mere lip service. Teachers need the support in both training and infrastructure to allow their participation in research toward improving their practice. Further, Clements and Ellerton (1996) warn about the dangers of imposing participation in research on teachers from above to serve predefined agendas,

PAR is *collaborative* in that '[a]ction researchers aim to work together in reconstructing their social interactions by reconstructing the acts that constitute them. It is a research done 'with' others' (Kemmis & Wilkinson, 1998, p. 23). Collaboration here means both collaboration between all people involved in the practice and between them and outsiders, e.g. staff from universities, who may have developed some experience with the processes of action research and/or wider knowledge of research in some problem area. Hargreaves (1994) argues that the concept of collaboration has become overused and is often ill defined and vague in educational discourse. He warns that forced or contrived collaboration may be used to impose special interest agendas on participants and may be used to prohibit originality and diversity. Grundy (1998) discusses the need to account for the different agendas of the partners, their different backgrounds and expertise and the different demands of their distinct context of doing research. She states that the elimination of hierarchy does not mean the elimination of different forms or sources of knowledge. However, the differences need to be delineated, indeed, they need to become the subject of research themselves. Of course 'parity of esteem' for all forms of expertise will be strongly dependent upon the principles of partnerships, trust, comparable rewards, and recognition of distinctive interests. It should not be assumed, however, that these will guarantee 'parity of esteem' for the expertise of the various partners. The question of expertise needs to be addressed explicitly.

PAR is *emancipatory* in that 'it aims to help people recover and unshackle themselves from the constraints of irrational, unproductive, unjust, and unsatisfying social *structures* which limit their self-development and self-determination' (Kemmis & Wilkinson, 1998, p. 24). While several authors have problematised the concept of emancipation and empowerment —e.g., Kemmis, 1995; Lankshear, 1994; Troyna, 1994— here I argue that action research helps teachers to analyse the social constraints on their practice and find ways to counteract these constraints and/or learn to work around them. Teachers seeking solutions to the problems identified by themselves gives them a sense of ownership of theses solutions and the knowledge generated through their action research projects.

Likewise, PAR is also *critical* in that '[i]t is a process in which people deliberately set out to contest and to reconstitute irrational, unproductive (or inefficient), unjust, and/or unsatisfying (alienating) ways of interpreting and describing their world (language/discourses), ways of working (work), and ways of relating to others (power)' (Kemmis & Wilkinson, 1998, p. 24). Action research is

critical in the sense of always raising questions about the conditions of practice and the assumptions and values of the various participants. Also Kemmis (1999) argues that action research is critical in the sense of attempting to develop an understanding of the history of the practice as it aims to look for insights about its possible change. Not only does it examine the practice under study, but it also examines itself as a social process with its own assumptions and aims.

Finally PAR is *recursive —reflexive, dialectical.* In this context I understand by this characteristic of action research two things. First, action research it aims to help people to investigate reality in order to change it (Fals-Borda, 1979 cited in Kemmis & Wilkinson, 1998, p. 24; 2002), and to change reality in order to investigate it. Hence, theory and practice are interrelated in dynamic ways rather than as posited in traditional research where knowledge is obtained first and then applied later. Second, this characteristic refers to the spiral of cycles of critical and self-critical action and reflection as discussed above.

This section detailed one definition of action research. It is essential to remember that aspects of these characteristics of action research are sometimes contested and relaxed by different traditions of action research. This definition is not presented here as a rigid criterion according to which a project may or may not warrant the label of action research. Rather, it is deemed to be a useful tool guiding self-critical reflection by the participants in action research projects about their individual and group commitments, understandings and achievements in the project.

Relationship of action research to other practice-based traditions

In this section I will briefly discuss the relationship between action research and other methodologies often discussed in the mathematics education literature. However, a discussion of the relationship between action research and the day-to-day practice of educators may be useful in order to establish the 'research' nature of action research in contrast to other activities that practitioners do. One can argue that teachers naturally do action research in their day-to-day teaching. After all, they learn from their practice when they reflect upon what did and what did not worked in the classrooms; they deliberately set out to try new practices in their classes; they involve themselves in observations and collection of evidence; and finally they discuss and justify their practice to other parties. There is no doubt that such activities are part of most practices of educators and there is no doubt about their usefulness. However, it is less clear whether they often provide explicit knowledge and understanding about a practice, whether they contain sufficient evidence to allow for defending that knowledge and whether they can provide for challenging rather than reproducing practices. Action research is often designed to take the familiar activities of teachers into a different dimension —that of research.

Action research is often contrasted with *reflective practice* popularised by Schön in the seventies. Schön (1983) commenced his development of a new 'epistemology of practice', by critiquing the technical rationality view of professional activity 'as consisting of instrumental problem solving made rigorous by the application of scientific theory and technique' (p. 21). Such a view of practice is based upon a

positivist epistemology and fails to account for many of the higher order functions of a professional. Perhaps Schön's most important legacy is the concept of *reflective practice.* Schön's view of reflective practice is based on the identification of different types of knowledge involved in professional practice. First, *knowing-in-action* is the tacit knowledge a professional possesses without necessarily being able to articulate it or justify it. While this knowledge is important for the everyday functioning of a practice, it fails to account for changes in the practice. To put the problem in simple terms: How can what one knows be changed if one is not aware of what one knows? To solve this problem, Schön introduces the concept of reflection. At this stage of his argument, he identifies two types of reflection: *reflection-in-action* and *reflection-on-action.* These reflections give rise to the second type of knowledge or *theory-in-practice.* A professional involved in generating this theory, according to Schön, is becoming a 'researcher in the practice context' (p. 68). Further, Schön identifies *reflection-upon-reflection-in-action,* the most complex component of reflective practice, as a 'process of getting in touch with the understandings we form spontaneously in the midst of action' (Schön, 1992, p. 126). This, according to Schön, is research in practice.

Action research, as articulated above, and reflective practice share a number of characteristics. They both relate theory and practice at an epistemological level. They both directly involve the practitioner in the development of a theory of their practice. Most importantly, they both give a prominent role to the processes of reflection. McMahon (1999, p. 167) raises the question as to whether a reflective practice is the same as action research. He argues that action research is 'distinguished by a deliberate and planned intent to solve a particular problem —or set of problems. By its nature action research involves *strategic action'.* Action research then can be distinguished from reflective practice as being a more systematic, deliberate and rigorous research activity.

The second movement often contrasted with action research, and perhaps more popular in mathematics education literature than reflective practice, is the *teachers as researchers.* The evolving role of teachers' research in mathematics education is exemplified by the establishment of the Teachers as Researchers Special Interest Group at the annual Psychology of Mathematics Education (PME) conference (Zack, Mousley & Breen, 1997). Undoubtedly, the work of Stenhouse (1975) and his colleagues in the Humanities Curriculum Project in the seventies has laid the theoretical and practical foundations of what has became a worldwide movement of teachers' research. Research is presented as the basis for teaching and curriculum development. Stenhouse's (1975) definition of research as a 'systematic enquiry made public' is a widely used definition of educational research even today.

What constitutes teachers' research and its principles and aims may not have been as well theorised in mathematics education. In reviewing teachers' research in mathematics education in Australasia between 1988 and 1991, Mousley (1992, p. 97) has asserted that '[i]t is difficult to decide when reflective or innovative practice becomes 'research''. Her review includes 'informal development, testing and publications of ideas by teachers' as research endeavours. These activities generate forms of knowledge with the common characteristic of 'what works for us' (p. 98). She claims that 'because it is seen as immediately applicable to classroom use, such

research can have a powerful impact on mathematical pedagogy' (p. 98). Breen (1997, p. 156) has reflected on his impressions on the wide range of views expressed at the meetings of the Teachers as Researchers Special Interest Group of PME.

> At one pole, people take the stance that one cannot teach properly without doing research, so that it goes without saying that anyone who teaches must inevitably be involved with research [...] At the other pole, people take the position that research is a clearly defined and understood activity in which it takes considerable expertise and training to become accomplished.

It is worthwhile to mention that other research reported by Mousley (1992) and Zack, Mousley and Breen (1997) involved the teachers in more systematic and rigorous research. Some authors (e.g., Adler, 1997) have argued for the use of the term 'teacher inquiry' to differentiate between such informal reflective activities and formal academic research. Adler uses inclusion and equity principles to argue for valuing teachers' inquiry in its own right or as a first step toward further involvement in academic research. The terms inquiry and research are often used interchangeable by some researchers involved in teachers' research.

In this chapter, action research may be conceived of as a subset of practitioner — or teachers'— research or inquiry. It shares the commitment of the teachers as researchers movement for the involvement of teachers in knowledge generation. However, it differs from other types or inquiry activities in its aim to improve some practice as well as to develop theory about that practice. It also shares other research tradition commitment to rigor and demonstration of its claims.

Another emerging methodology in mathematics education that may be contrasted with action research is the *teaching experiment.* The recent publication *Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000) devotes five chapters to this methodology. This emphasis on this methodology is based on the editors claim that '[t]here is no other type of research that more clearly illustrates distinctive characteristics of research in mathematics and science education' (p. 192). Steffe, Thompson and Glaserfeld (2000, p. 274) assert that:

> [a] teaching experiment involves a sequence of teaching episodes [...] A teaching episode includes teaching agent, one or more students, a witness of the teaching episodes, and a method of recording what transpires during the episode. These records, if available, can be used in preparing subsequent episodes as well as in conducting a retrospective conceptual analysis of the teaching experiment.

The aims of the various teaching experiments vary as well as the theoretical perspectives employed to analyse the data. However, most teaching experiments are conducted within the general constructivist understandings of teaching and learning and they aim at investigating students' conceptual understandings and the development of these understandings in a semi-naturalistic and controlled environment. They also vary in terms of the degree of involvement of and collaboration with the practising teachers.

Action research, as discussed above, shares some of the principles of teaching experiments, e.g. in their use of cycles of action and reflection. While teaching experiments seem to have a major focus on conceptual understanding and its

development in classroom, action research often concentrates on wider sociocultural aspects of the practice of teaching and learning. Further, while the theoretical tools employed by teaching experiments tend to come from constructivist theories, action research tend to utilises critical social science constructs. Also, while the degree of control in teaching experiment varies from one study to another, in action research, practice is studied and theorised in its naturalistic environment. Finally, action research places a greater emphasis on collaboration with the practitioners as researchers, which is found in some but not all teaching experiments.

Lastly, action research is often contrasted with *action learning,* a movement developed in 1930s and 1940s in the UK by Revans (1982). Action learning has developed mainly within business and other organisations as a way to improve conditions and productivity and to solve concrete problems encountered. Passfield (2001, p. 39) defines it as 'learning in and through action while collaborating with others on personal and organisational improvement. It typically involves a learning group (often called a "learning set"— focused on a project or work endeavour'). It is a form of learning from experience where groups of colleagues reflect on their past learning, make observations and reflections, form generalisations, and test their implications in cyclic fashion, not unlike the cycles employed in action research. In this, it shares some of the characteristics of action research discussed above. However, action learning projects tend to be less systematic, less theoretical and less dependent on collection of data than action research. Also, there is less of a tradition in action learning for making the results of learning public. Zuber-Skerritt (2001, p. 1) argues that 'the main difference between "Action Research" and "Action Learning" is the same as that between learning and research in general.'

EXAMPLES OF ACTION RESEARCH PROJECTS IN MATHEMATICS EDUCATION

This section presents two different examples of action research projects that differ in educational and geographical contexts as well as the participants involved in them. This discussion here is necessarily brief and is not intended to illustrate the learning about mathematics teaching that evolved from the projects. Rather they are presented here as illustrations of the wide range of action research projects reported in the literature and as illustrations of the characteristics of action research as discussed in this chapter.

The Critical Numeracy Project

Dias (1999) reported on a study using action research in the training of a group of 'lay' volunteers for teaching critical literacy and numeracy in Brazil. The project was funded by the Bank of Brazil and aimed at increasing literacy and numeracy of adults in both rural and urban communities in the country. The project was planned based on the theories of Brazilian educator Paulo Freire. It aimed at introducing literacy and numeracy skills in a holistic fashion and using real social context

examples of the participants. At the commencement of the project interesting, yet worrying patterns arose. While the volunteer teachers could follow the new critical pedagogy in teaching literacy they expressed great unease about applying the new principles to the teaching of mathematics. It was much harder for them to imagine teaching mathematics differently from the traditional abstract, meaningless contexts of the type that dominates traditional mathematics textbooks. Even when examples were taken from real life, they were situations that did not contribute to the understanding and critique of the students' social situation along the lines of 'the number of children' and 'the number of miles walked to get to class' (p. 18). The authors commented that

> while critical education unmasks teachers' 'apolitical' or 'neutral' position, taking a political positioning on the side of the *status quo,* this very 'apolitical' attitude seems to find its most resolute adherents among mathematics educators. This phenomenon has its roots in the historical and philosophical portrait of mathematics as a neutral and 'pure' discipline. (p. 17).

Through the process of action research the volunteered teachers came to examine their assumptions and personal philosophies about mathematics. The traditional workshops in the project involved the selection of a topic from the content and teachers brainstorming how best to make it relevant to the participants' social situation. This style of workshop did not seem to work in the case of mathematics. New workshops were planned that shifted the attention from the mathematics content to the philosophy of mathematics portrayed in the different styles of teaching. By examining pedagogies used by others, the group was able to critically identify the assumptions behind their teaching while they had failed to do that in reflecting on their own teaching.

The project itself was a product of negotiating the needs of the various participants. Not only was the structure of the project negotiated with the participants, the roles of the participants had to be negotiated also. Even though at the start of the project the external trainers were hesitant to give direct input to the discussion, it was felt that the participants needed to have a bigger picture of how the trainers could contribute to their practice. The external trainers then wrote articles about critical mathematics and addressed some of the volunteers' concerns as expressed in their responses to an earlier questionnaire. Relationships between the participants and trainers improved after that. At the conclusion of the paper, Dias (1999) argued that the process of reflection was not only used to challenge the teachers' assumptions but also to design and implement the action research project itself.

The TEBES Project[1]

The second story of an action research project discussed here comes from Mexico. In 1993, teachers from two primary schools in two rural communities, located about seven hours by road from the capital, Mexico City, assisted by a lecturer from the National Pedagogical University, formed a collective for transforming mathematics teaching in their schools. The teachers' deliberations commenced with their concerns about assessment practices in mathematics classes, in particular grades 3 and 4. After several attempts at changing their assessment practices, they realised that most of their practices in assessment centred around memorisation of facts and content knowledge, which they identified as the lowest level of students' learning, as it often failed to evaluate higher order learning such as analysis, synthesis, understanding, and reasoning. For them, assessment was a social justice issue in their classrooms. Without comprehensive assessment of student learning, it was not possible to be fair and equitable in their teaching. They realised that the achievement of this project required analysis of the higher order thinking skills, not only from psychological, but also from epistemological and philosophical aspects as well as theories of measurement and assessment.

However, their reflection on their assessment practices soon led the teachers to examine their teaching practices in the classroom as they related to the achievement of these higher order goals. The research problem was redefined as: how could the teachers change their teaching to best achieve these higher order aims of mathematics teaching? To achieve this, the teachers spent a considerable amount of time analysing and reflecting on their daily practice in mathematics classes. This analysis of their assessment and teaching practices was informed by a collection of articles that the cooperating lecturer from the university had provided.

The task that the teachers embarked upon was how to make mathematics teaching different in their classrooms in order to achieve higher order skills and at the same time make mathematics classes more interesting and varied for the students' needs. The mathematical topic under consideration was numbers, relations and operations on numbers. Classroom activities were sought on the basis of being interesting, challenging and having real world contexts. They believed that such activities would make learning more involved and lasting. Through these game-problems it was hoped also the students' perception of mathematics as a meaningless and useless subject might change.

At the conclusion of their first stage of the project, the teachers collaborated to write a 50-page proposal and analysis of their situation. The proposal identified aspects of the geographical and other contextual aspects of their schools, including the physical, historical, economical, cultural and educational features. It demonstrated that the teachers gained significantly from compiling and analysing this information. The proposal contained a very careful and detailed critique of the current problems encountered in their daily practices in teaching mathematics. It is

[1] TEBES stands for 'Transformación de la Educación Básica desde la Escuela' which translates into English as 'Transforming Compulsory Education from/within the School'. Further information about the project can be found in Atweh and Arias (2001).

interesting that the teachers did not identify the students' learning problems as environmental, or as lying in the students themselves, but in the very practices that the teachers had used in the classroom. Similarly, this proposal contained an extensive review of the literature on various aspects of students' learning and teachers' teaching in mathematics. Such literature covered topics such as the cognitive abilities that they intended to develop, theories of cognitive development in children, problem solving, and the role of games in education. The last section contained a detailed plan for changing their teaching so they could achieve their goals. Similarly, the proposal contained a selection of about 30 specific and general types of games that teachers might use in their classes and detailed criteria for the evaluation of these activities. The evaluation identified several dimensions of evaluation, levels at each dimension and indicators for the achievement of the levels. In doing so, the participants were able to realise their original aim to make their assessment more comprehensive.

However, the teachers were very candid about their failures and shortcomings in their projects. They realised that the problems of teaching mathematics were not always simple to solve. Nevertheless, they were convinced that they were on the right track in their endeavours and in their learning of how to become more effective mathematics teachers. Their proposal identified the history of the group and how it operated. It also outlined the procedures that they followed and the problems that they encountered working on the project. The difficulties encountered included differences in opinions between the participants, the lack of willingness of some to listen to other points of view, other demands on teachers' time, and a lack of system support. Finally, it is worth stressing that by performing this action research on their practice, the teachers aimed to improve their own practice, as well as to share their findings with other schools in compulsory education in their region and country. As with all collectives in TEBES, this group of teachers presented their findings and learning at various seminars for teachers organised by the network.

SUMMARY

This chapter discussed the role of action research as an emergent methodology in mathematics education. Action research as presented here is a methodology characterised by collaborative research conducted with the participation of the practitioners in a variety of contexts working towards the improvement of their practice *through* and *for* developing a deep understanding of their practice. It is a form of democratic, critical and empowering research that understands practice in a wider social meaning. As a research activity, it involves the collection of evidence, analysis or results, theory development and dissemination of results to a community of practice. However, unlike other research traditions prevalent in mathematics education, it aims at changing the world through understanding it and at understanding the world through changing it. Hence, it depends on the praxis between practice and knowledge.

Action research is an international movement that is slowly but steadily gaining a significant place in mathematics education. It falls into the general types of

research that are practice based. Some aspects of action research theory and practice are consistent with reflective practice, teaching experiments, action learning and with teachers as researchers movements. However, arguably it is one of the most theorised of all of these traditions and possibly most tested and trailed in general education.

This chapter has argued that calls for the use of action research in mathematics education are based on three principles. First, the failure of research in general to inform practice in mathematics classrooms gave rise to critiquing the rationale behind many traditional research paradigms that aimed to generate abstract generalisable knowledge that can be applied by practitioners to deal with their day-to-day practice. Second, action research is based on an epistemology that claims that knowledge can not be transmitted from one mind to another and that knowledge is always value laden. Action research advances not only the technical and practical interests of teachers but it contributes to their emancipatory needs. Last, if teachers are to enjoy the status of autonomous professionals they should feel that they are in control of the processes of knowledge generation as well as knowledge application.

Different traditions of action research have evolved around the world. Like all social practices action research is a dynamic methodology that will continue to evolve. In this chapter the conceptualisation of action research as portrayed by Kemmis and his colleagues was discussed in detail. This tradition is based upon the theories of critical social science and empowering research traditions. It places an emphasis on social/political aspects of practice and on collaboration and participation. Action research is a reflective and critical tradition that opens any practice to critique —including itself.

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OLE SKOVSMOSE AND MARCELO BORBA

RESEARCH METHODOLOGY AND CRITICAL MATHEMATICS EDUCATION

The label 'critical mathematics education' refers to a variety of perspectives and activities, which highlight some concerns.[1] Critical mathematics education is concerned with the social and political aspects of the learning of mathematics. It is concerned with providing access to mathematical ideas for everybody independent of colour of skin, gender and class. It is concerned with the use and function of mathematics in practice, being an advanced technological application or an everyday use. It is concerned with the life in the classroom, which should represent a democratic forum, where ideas are presented and negotiated.[2] It is concerned with the development of critical citizenship.

Keeping such concerns in mind, what would it mean to do research in mathematics education? Clearly enough it could mean to draw attention to certain problematic *issues* related to mathematics education, and many such issues have been pointed out as relevant for critical mathematics education: the social background of the children; the multilingual and the multicultural classroom; the pattern of communication in the classroom; the children's already established mathematical concepts; the organising of project work in mathematics education; the reliability of mathematics in practice; the ideology of certainty; the distribution of resources and the access to computers.

However, we want to concentrate on *methodological* aspects of doing research. Do some research methodologies 'resonate' with the concerns of critical mathematics education?[3] Does it make sense to talk about a research methodology —maybe some methodologies— as being in 'harmony' with the concerns of critical mathematics education? Going through the literature, we can find relevant studies sharing the concerns of critical mathematics education which embrace *action research, participatory research* and *participatory action research* as relevant.[4] Apparently, there seems to be established some connections between educational

[1] See, for instance, Alrø & Skovsmose (2002), Borba (1990), Borba & Skovsmose (1997), D'Ambrosio (1994), Frankenstein (1983, 1989), Gerdes (1996), Knijnik (1998), Powell & Frankenstein (1997), Mellin-Olsen (1987), Niss (1994), Skovsmose (1994), and Skovsmose & Valero (2001). For a discussion of the concerns characterising critical mathematics education, see Skovsmose & Nielsen (1996).

[2] For a discussion of mathematics education and democracy, see Valero (1999), Vithal (1999),Skovsmose (1998b), and Skovsmose and Valero (2000b).

[3] This question is raised by Vithal (this volume). See also Vithal (2000b).

[4] See Atweh (this volume) for a discussion of participatory action research.

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 207—226.

perspectives and methodological priorities. We will explore further this possible connection, when the educational perspective is expressed in terms of critical mathematics education.

Many of the ideas we are going to present in this chapter have been discussed in seminars in South Africa, Brazil and Denmark, organised as part of collaborative programmes between institutions in these countries.[5] A presentation and a further application of some of the ideas are provided by Vithal (2000b) in a study that leads to the development of a pedagogy of conflict and dialogue.[6]

RESONANCE

The notion of *resonance* has been developed by Lincoln and Guba (1985) in order to conceptualise the possibility of fit between theoretical perspective and methodology of a research approach. They discuss how a fit could provide 'resonant results'. As an example, they cite the idea that behaviourist research is in resonance with a basically quantitative approach for educational research which bases its conclusions on measurable results. Thus, the view that research should emphasise first of all external behaviour is in accordance with a methodology which concentrates on external outputs of tests.[7]

In *Competing Paradigms in Qualitative Research,* Guba and Lincoln (1994) describe four overall research paradigms in qualitative research: positivism, post-positivism, critical theory and constructivism. The critical paradigm is, among other things, characterised by the following aim:

> The aim of inquiry is the critique and the transformation of the social, political, cultural, economic, ethnic and gender structures that constrain and exploit humankind, by engagement in confrontation, even conflict. The criterion for progress is that over time, restitution and emancipation should occur and persists. Advocacy and activism are key concepts. The inquirer is cast in the role of instigator and facilitator, implying that the inquirer understands a priori what transformations are needed, (p. 113)

However, our conception of research methodology which fits the critical paradigm is not adequately captured by the above description. We do not accept the idea that the inquirer should understand *a priori* what transformations are needed. Nor do we accept that the researcher should play the role of instigator. We find such ideas devastating to the very notion of critique. If the inquirer should know *a priori* the transformations needed, then he or she should be informed by clearly formulated

[5] Cooperation between the University of Durban-Westville, South Africa, the Danish University of Education and Aalborg University, Denmark, has been set up in order to support research emphasising the importance of developing mathematics education with a concern for democracy. The cooperation between the Graduate Program in Mathematics Education at the State University of São Paulo at Rio Claro (UNESP), Brazil, and the Centre for Research in Learning Mathematics, Denmark, aims at developing new aspects of critical mathematics education and the ethnomathematical approach.

[6] See also Vithal (2000a) and Bopape (2002).

[7] This issue has been discussed in Borba (1993, 2000) with reference to the use of computers in mathematics education.

ideals. Maybe a classic interpretation of Marxism would imply the existence of an inquirer with such knowledge. But we do not assume this. Although we see research within a critical paradigm as connected to transformations, we do not link such research to *a priori* identified transformations. Transformation as well as critique includes uncertainty and doubt, they are rooted in collaboration, not in 'leadership'. So, we do not follow Guba and Lincoln (1994) in their characterisation of the nature of research that resonates with a critical perspective, but we agree with them that the issue of resonance is essential, also in this case.

The notion of 'resonance' is complex. We can, for instance, imagine a situation in which a large quantitative study has a tremendous political impact by drawing attention to unequal distribution of resources for schools within a certain country. Extensive descriptive research may reveal critical aspects of an educational system and provide recommendations for major changes. The notion of 'resonance' cannot represent a one-to-one relationship between a set of educational perspectives and a set of research methodologies. 'Resonance' refers to a much more complex relationship.

To simplify the discussion, we will deal only with educational research done in the classroom —thus we do not consider, for instance, overall studies of inequalities reproduced by mathematics education. As the territory is still vast, we concentrate on the following question: *Can we characterise a pattern of research methodology which resonates with the concerns of critical mathematics education, where the focus is changes in the classroom?* As resonance needs not designate a one-to-one correspondence, we expect that it is possible to identify different patterns of research methodology that resonate with the concerns of critical mathematics education. We will refer to several such patterns as *critical research.* In the following we shall discuss 'critical research', although a more adequate formulation is 'some patterns of critical research, which we find of particular relevance when we focus on changes in the classroom'.[8] As already indicated, critical research may include action research, participatory research as well as other patterns of research. We shall, however, be more specific about one such pattern.

In the rest of this chapter we will try, first, to specify in general terms what such a research might be addressing. What kind of situations is in fact considered in critical (and action like) research? Second, we want to describe the nature of some of the processes involved in critical research. How might one discuss the quality of theses processes?[9] What kind of analytical task is involved in such research? Third, we want to describe what is done by such research, and certainly we will be listening to the answer coming from action research: research makes a change! But does any kind of change count? By considering these three issues we hope to bring new aspects to the discussion of research-resonance within critical mathematics

[8] By this formulation we wish to acknowledge that research which is, say, not participatory and which is not action research might still serve the concerns of critical mathematics education.

[9] By 'qualities of research' we refer to some 'attractive features of research' as, for instance, have been highlighted in the discussion of reliability and validity. Vithal (this volume) provides a radical opening of the discussion of 'quality' by addressing issues of negotiation, reciprocity, reflexivity, etc.

education. The three issues we will associate to the three figures that are shown in the text.

RESEARCHING WHAT IS NOT THE CASE

Critical research could represent a mixture of proper research —maybe participatory research or action research— and activities of educational development.[10] We can find many descriptions of the researcher being engaged in developmental projects, acting together with teachers in a spirit of shared responsibility. Many formulations have been used to characterise this mixture: research-guided school developments, developmental projects, school-based research.[11] However, we believe that critical research has its own specificity. We consider critical research as proper research, and not as a conglomerate of different activities. We find that research, as such, can be political, and not that political concerns and processes of change have to be added to 'proper' research.

Many interpretations of doing research put a special emphasis on the collection of empirical material. Such material should meet certain standards, which has brought about a discussion of, among other things, reliability and validity. According to many interpretations, making observations represents the essence of the scientific enterprise. Naturally, observations can be distorted or fuzzy, but at least to some interpretations, a careful design of data-collection procedures can reduce disturbances and fuzziness. According to other interpretations, disturbances caused by the observer cannot be eliminated, and should not be eliminated. There is no 'data' without an interpretation. The important thing, then, is to become as explicit as possible about the perspective and subjectivity of the observer in order to produce a 'narrative' about what has taken place. Do we have to do with a study of interactions in the classroom, a main methodological concern then is to identify adequate ways of presenting observations or narrative about, say, what is said and how things are said. Videos have been used, and tape-recordings can be transcribed. The students' expressions can be registered, and the body language of the teacher can be noticed. In such cases, the research methodology concentrates on bringing about some empirical material from a situation, which has taken place.

But does it at all make sense to bring about some input to the empirical material from a situation which has *not* taken place? Classroom research is (normally) not considering what the teachers could have said instead of what he or she in fact did say. It is (normally) not considering what the students might have done, in case the teachers had said something different. Research is (normally) not referring to an imagined situation radically different form what is taking place in the classroom. In this sense research is (normally) addressing *what is,* i.e. what has taken place. In general, research is (normally) not exploring what is not taking place but what could have taken place. Research is (normally) not considering what we could imagine taking place.

[10] See, for instance, Crawford & Adler (1996) and Jaworski (1994). For a discussion of action research, see Atweh, Kemmis & Weeks (1998).

[11] See, for instance, Gravemeijer (1998).

As a first simplified characteristic, we will suggest that doing critical research means not only to consider what is taking place but also to consider what could have taken place and what could be imagined as possible alternatives to what is taking place. Doing critical research also means to explore *what is not there* and *what is not actual.* To research also what is *not* there and what is *not* actual means to investigate *what could be.* Critical research pays special attention to hypothetical situations, although still considering what is actual. Critical research investigates alternatives. By 'investigating alternatives' we do not simply mean 'suggesting alternatives'. A crucial task for critical research is to investigate alternatives in such detail that they can confront what might be conceived as a given. Critical research points out that something could be different. The statement 'something could be different' refers to both what is considered as a given and what is investigated as a possibility. Critical research tries to confront what is actual with what could be. This idea we consider an important aspect of doing critical research.

Let us imagine a classroom where the children are sitting in rows and the teacher asks questions. The students raise their hands; the teacher asks a student; the student answers the question; the answer appears to be correct. We could just describe and interpret what takes place and what goes on in the classroom and leave it like that. Alternatively, we could associate our observations of the activities in the classroom with critical comments. We could describe how the classroom communication reflects a dubious idea about teaching and learning. Still, in both cases, what is happening in the classroom is left untouched by the research, although in the second case it is accompanied by critical comments. However, when critical research is concerned about changes, this also means researching *what is not* the case —but what could be a possibility.

Studying alternatives to an actual practice, naturally links closely to the idea of action research. Hitchcock and Hughes (1995, p. 27) provide a summary of action research:

> The principal features of an action research approach are change (action) or collaboration between researchers and researched. Action researchers are concerned to improve a situation through active intervention and in collaboration with the parties involved. This gives action research a very particular character.

They emphasise that action research is a cyclic process that takes the form of acting-observing-reflecting-change-planning-acting. We certainly regard critical research as being dynamic, but we want to represent this dynamics in a different way.

Changes do not guarantee the quality of research, not even of action research. The circle acting-observing-reflecting-change-planning-acting may express the nature of action research, but it does not express the quality of such research. We need a terminology which, on the one hand, can characterise the nature of doing critical research in general, and which, on the other hand, can help to clarify relevant issues when addressing the *qualities* of critical research. In search of such a terminology, let us present an example that we can refer to later in the discussion.

AN EXAMPLE

The purpose of introducing the following example is to illustrate some of the aspects of doing critical research. We do not take the following example as exemplary for critical research. However, it represents a form of co-operation between teacher and researchers as a response to some particular educational problems.

In Rio Claro, Sao Paulo, Brazil, the GPIMEM research group focuses on issues related to new technologies and mathematics education.[12] This research group works with students at different levels and with teachers. A teacher, Stela, from a nearby school, approached the group through her supervisor expressing an interest in using computers with her students. Her ideas were not clearly specified, and one of the members of the research group, Miriam Penteado, decided to work with her. From the standpoint of the researcher, there was the interest in discovering how a teacher, who was not familiar with the use of computers, would appropriate this new technology. There was also an interest in supporting the teacher. From the teacher's perspective, she had some tough problems to face. However, she foresaw that computers would be able to help her. The details of the co-operation are presented in Penteado (2001). Here we shall only outline the overall situation trying to illustrate the terminology we are going to suggest.

Stela was teaching what was considered a difficult class of 5th graders. The children were older —15 years average— than the expected age for this grade, which is 11. They felt humiliated, as they were put in a school with much younger students, and they had failed many times. Several times they had to repeat all the subjects of a given school year because of their 'failure in mathematics'. The students transformed this humiliation into violence in class. The teacher was in fact considering the possibility of just quitting the job, since she could not work with these students in a way she found effective.

The teacher saw the computer as a way of trying to engage the students in mathematical thinking. The teacher and the researcher started meeting regularly so that they could agree on pedagogical issues regarding the use of computers. In these meetings there was teaching and learning on both sides. A problem arose, though: the computers, which had been promised by the state government, had not arrived by the time the teacher and the researcher had agreed to start using them in the classroom. The school had only four computers and they were placed in a room that would only hold eight people. Thus, there were no computer facilities for the 25 students in the class. The researcher suggested to the teacher that she might use the university laboratory for the lessons in which she wanted to use the computer. The teacher agreed to this idea, as she believed that the students could easily walk to the laboratory.

Stela was enthusiastic about the software programme, *Fracionando* (Bordin, Milani, Grunkraut & Ricotta, 1995), which deals with rational numbers. Stela took

[12] GPIMEM stands, in Portuguese, for Grupo de Pesquisa em Informática, outras Mídias e Educação Matemática (Research Group in Technology and Mathematics Education) (http://www.igce.unesp.br/igce/pgem/gpimem.html). This group is chaired by the second author of this paper. The first author of this paper has collaborated with this research group in the recent years. This group is based at UNESP (State University of São Paulo). See, for instance, Borba (1997, 2000).

the lead in the choice of the program and in the design of the tasks that were presented to the students. Although the GPIMEM research group found that this software was full of rote learning tasks, they agreed to use the software in Stela's class. They felt that this might ease the transition of the teacher into a classroom with computers. Besides, both researchers and teacher believed that the computer might have a positive effect in this class and might enable the students to pass. However, how might one cast this form of co-operation in terms of research?

THE CURRENT, THE IMAGINED AND THE ARRANGED SITUATIONS

We shall now try to be more specific about what critical research might be addressing. The situation before the educational experiment took place, we call the *current situation* (CS). This situation contains problematic features. In the example the teacher faced behavioural problems in the classroom, and the students experienced the humiliating position of being 'outcasts'.

In this classroom a researcher, concentrating on data collection, would certainly find much to observe and to make notes about. This research could reveal many critical aspects of the current situation. Stela experienced difficulties in her classroom. And, naturally, based on the observations, the researcher could possibly suggest new interpretations of the difficulties. The research might be participatory, and findings might be discussed with Stela. Alternative interpretations of what actually happened might be created as well. But although observations of the current situation are important, our point is that it is vital to critical research to study something different as well. It is important to research alternatives. Critical research cannot be absorbed by the current situation, although it certainly pays a special attention to the educational problems as they emerge in the current situation.

The current situation can be imagined to be different. In the present case, we can imagine alternatives that might solve the problem of lack of motivation. The students might be provided with a different learning environment. Naturally, the imagination can be related to the expectation and the hope of the teacher. It can also be supported by the experience of the researcher. We call this vision about the possibilities of alternatives an *imagined situation* (IS). Stela has formulated such an imagination.

What would happen if the students got access to computers? What would happen to their self-esteem? Within the current situation the students had no way of escaping the feeling of being the 'outcasts of the school'. From a research point of view, it would be important to research these students in an imagined situation occupied with a technology representing life outside school; a technology which could provide a new form of prestige. The students might develop a different attitude towards schooling.

In the present case, the students had no possibility of using computers at school for the simple fact that the school did not have adequate computer facilities. There were no possibilities for actually studying the imagined situation —an ongoing study of the current situation no longer seemed satisfactory. But some arrangements were possible. The students were invited to get to the nearby university in order to

get their hands on a keyboard. This illustrates that the *arranged situation* (AS), which is certainly an alternative to the current situation, is also different from the imagined situation.[13] In general, an arranged situation is a practical alternative that emerges from a negotiation involving the researchers and teachers, and possibly also students, parents and administrators. The arranged situation may be limited by different kinds of structural and practical constraints.[14] But it has been arranged with the imagined situation in mind.

In this way, three different situations become part of the research perspective in which classroom change plays a crucial role.[15] The distinction between the current situation, the imagined situation and the arranged situation is analytic. The distinction is illustrated in Figure 1. The distinction emphasises that critical research is not located within a descriptive paradigm. Such research cannot only concentrate on the actual situation and on what is the case. Critical research may also address possibilities that can be imagined and alternatives that can be realised. It confronts what is the case with what is not the case but what could become the case. Critical research must consider not only the actual situation but also an imagined situation. Grasping details of the imagination can be done by considering an arranged situation. In Figure 1, we indicate what critical research is addressing. We see the actual situation, the arranged situation as well as the imagined situation as relevant resources for observations, narratives or other inputs to the research process —still keeping in mind that we refer to some patterns of critical research.

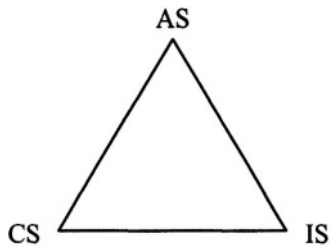

Figure 1: Model of critical research indicating what such research is addressing

Could the arranged situation be identical with the situation the researcher and the teacher in fact want to investigate? Hardly. Let us return to the example. The research perspective could be about the students' activities with computers, and certainly not with what is happening to students when they have to walk to a nearby

[13] Instead of arranged situation, we could also talk about an adapted situation.

[14] An alternative situation is different from a laboratory situation. A laboratory constitutes a particular set-up, the aim of which is to make it possible to study some well-specified research questions and to give the researcher more control over the situation. By contrast, an arranged situation includes the complexity of any educational situation.

[15] Vithal (2000a, 2000b) uses a slightly different terminology in her presentation of these ideas. She refers to the actual, the arranged and the hypothetical situation.

university to work with computers, etc. The arranged situation and the imagined situation are different. The imagined situation exists only as a conception established by different hypotheses and ideas. It is partial and fluctuating. It has been established as part of an educational discourse, but it plays a crucial role in critical research.

The imagined situation differs from both the current situation and the arranged situation, as it refers to ideas and ideals. Certainly the imagined situation is a fuzzy notion. This indicates that it might be more adequate to concentrate on processes instead of on 'situations' in our attempt to clarify the notion of critical research.

QUALITIES OF RESEARCH

Critical research does not just address what is the case. It also deals with what is not the case but could be brought about. Figure 1 might help to emphasise this by illustrating the focus of critical research as consisting of the current, the imagined as well as the arranged situation. However the analytical distinction also provides a false picture. The situations are far from static, as the figure may falsely indicate. For instance, it does not make sense to interpret the imagined situation as a ready-made picture of an intended reality. Research is a process, and in Figure 2, we try to illustrate what processes critical research might include.

The following considerations are more specific than the previous ones. They might address only more particular patterns of critical research. As mentioned, we have in mind situations that include co-operation between researchers and teachers addressing changes in classroom practices. Such research can include different forms of co-operation. One research paradigm would claim the importance of collecting as 'objective' data as possible, which could bring forward the idea that the researcher should be as invisible as possible in the classroom practice, just making observations and not interfering with what is happening. The researcher could collect data *on* the teachers as well as *on* the students and their interaction. Participatory research and participatory action research have emphasised the importance of collecting data *with* the teacher and *with* the students. In many cases this is presented as an attractive feature of a co-operation. We agree. However, we shall try to discuss the aspects of co-operation in terms of qualities of research. What qualities does such a co-operation bring to critical research?

We want to highlight different processes related to critical research (see Figure 2). *Pedagogical imagination* (PI) concerns the relationship between the current situation and the imagined situation. We can interpret pedagogical imagination as the processes that help us create imagined situations. It represents the complex process of conceptualising that things could be done in a different way. The relationship between the current situation and the arranged situation is established by a *practical organisation* (PO). This organisation consists of such practical planning activities as are necessary to establish a situation —an arranged situation— that shows some similarity to the imagined situation. Finally, *explorative reasoning* (ER) means the analytic process of reconsidering the imagined situation in the light of experiences related to the arranged situation. Explorative reasoning is a process in

which the feasibility of pedagogical imagination is considered as well as the innovative elements of practical organisation. An explorative reasoning represents a critical interaction between pedagogical imagination and practical organisation.

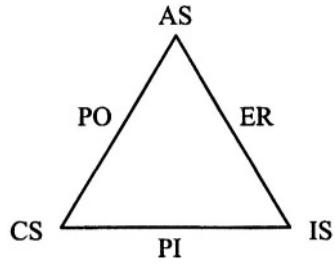

Figure 2: Model of critical research indicating what processes such research might include

We will try to discuss the qualities of critical research in terms of the qualities of the processes described above: pedagogical imagination, practical organisation and explorative reasoning. Still we must keep in mind that in our characterisation of critical research we have focussed on changes in the classroom.

Pedagogical imagination

To identify an alternative to a current situation is a creative act that we call *pedagogical imagination.* This imagination is related to 'sociological imagination' as described by Mills (1959). Sociological imagination is not directed towards a social situation, understood as a given fact. Instead, sociological imagination tries to identify alternatives to the actual situation. In this way it provides possibilities for change, because the existence of alternatives shows that the actual situation is not 'a necessity'.

In his introduction to sociology, Giddens (1986) uses sociological imagination as encompassing three kinds of sensitivity: the historical, which has to do with 'recovering our own immediate past' (p. 14); the anthropological, 'which allows us to appreciate the diversity of modes of human existence which have been followed on this earth' (p. 20); and the critical, which 'makes it possible to break free from the straitjacket of thinking only in terms of the type of society we know here and now' (p. 22). Giddens concludes that 'the sociological imagination fuses with the task of sociology in contributing to the critique of existing forms of society' (p. 22).

Similar to this, we see pedagogical imagination as having to do with conceptually exploring educational alternatives to the current educational situation. Pedagogical imagination may express a historical sensitivity, acknowledging what has happened in education; an anthropological sensitivity, acknowledging what else has been done in education; and a critical sensitivity which means not taking the current situation as a given. A pedagogical imagination with respect to mathematics

education means being involved in a process of conceptualising new possibilities by acknowledging critical features of the current situation.

Positivist research, i.e., research which investigates what is given, is based on a weak imagination, something we may call 'imagination zero'. The negotiated constructions of an alternative to the given, worth to be investigated, are not elaborated. To invent alternatives would designate an illegitimate research. Naturally, positivist research can provide resources for making changes, but such activities cannot be part of any positivist research scheme. Imaging changes would express a non-scientific and metaphysical element of the process. According to positivism, such elements have to be eliminated. However, traditional qualities of positivist research, not relating studies to imagination but to facts and facts only, represents in our terminology a lack of critical quality.

An imaginative construction of new alternatives has many sources, one being theoretical and philosophic resources as, for instance, expressed in the philosophy of critical mathematics education (Skovsmose, 1994). Another source for identifying alternatives in education is, naturally, the practical knowledge of the teacher. As a consequence, we see negotiating as supporting a pedagogical imagination. In the example we can argue that within the research group, GPIMEM, there is a firm belief in computers being able to support students in their own investigation. It is believed that pedagogical approaches such as modelling (Borba, Meneghetti & Hermini, 1997; Borba & Bovo, 2002) may be facilitated by this new technology, as students will have different paths in searching for information to develop mathematical investigation of their own interest. This can be helped in particular if 'open and flexible' software is available together with access to the internet.[16] Such ideas were negotiated with the teacher, and it also became clear that violent students may not have been considered in GPIMEM's belief about the positive impact of computers.

Pedagogical imagination has the current situation as point of departure. However, this imagination is not only based on the ideas of the researcher and of the teacher. It is also linked to the co-operation between researchers and teachers. This co-operation includes negotiation and deliberation. Deliberation is based on the idea that nobody has access to unquestionable knowledge. The process of decision-making must always include a variety of ideas.[17] This is one reason to dismiss Guba and Lincoln's description of doing critical research as including an a priori understanding of what has to be done, and as presupposing the researcher playing the role of an instigator. We do not exclude that ideals can be established about, say, equity and justice in mathematics education, and that a possible classroom practice may provide a reaction to sexism, racism and class conflicts. However, the particular interpretation of such ideals we regard as established in a process of negotiation.

[16] We refer to software as flexible, if it is suitable to open-ended tasks and activities that can provide new insight in mathematics.

[17] Valero (1999) has emphasised the importance of the notion of deliberation for the discussion of the democratic aspects of mathematics education. Davis & Cooke (1998) present, as the first step in their projects of including parents as partners for educational change, the creation of a 'shared vision of a healthy school environment'. This idea of a shared vision can naturally be related to the idea of pedagogical imagination.

Of course, not every discussion between teachers and researchers promotes pedagogical imagination, but this does not change the idea that *critical research includes processes of pedagogical imagination,* and that *qualities of this imagination can be discussed in terms of co-operation.* This brings us to emphasise the importance of doing research with teachers —and not on teachers. Naturally, the process of developing a pedagogical imagination can also include students. This appears obvious, for instance, in contexts of doing project work, where both organisational and content issues are dealt with in co-operation with students.[18]

Practical organisation

The educational context constrains the pedagogical imagination. At one extreme, we might witness a situation with only a few structural limitations and, as a consequence, it becomes easy to arrange an alternative educational practice. The other extreme excludes any experimental work from taking place in school, maybe due to curriculum demands.

But in most cases we can ask the question: How to ensure the quality of the practical organisation? We find this quality to be closely linked to the quality of the co-operation between teacher, researcher, students and administrators. To organise an arranged situation means to negotiate a specific situation within specific constraints. For instance, it was possible to use the laboratory of GPIMEM only because all parts involved agreed upon it. If there were no consensus, there would be no way that the arranged situation could happen.

Critical research also includes processes of practical organisation. Stela chose a non-flexible software for her students. From a research point of view, a flexible software might have provided different opportunities. However, the step the teacher took of making it possible for students to work with computers was a big change to the ecology of the classroom. In addition, the possibilities of an open software might have caused so much uncertainty that it may not have been possible to manage the situation. It was not possible to predict the students' behaviour in the university laboratory. Only guesses could be made. Making an informed guess would presuppose that you know the students, the laboratory, the software, the possibilities the software provides, and certainly many other things. In other words: the practical organisation presupposes negotiation.

Practical organisation uses the current situation as the point of departure. The practical organisation represents a 'realistic', maybe 'pragmatic', version of the pedagogical imagination. As pedagogical imagination represents one aspect of doing critical research, so practical organisation represents another. Furthermore, we find that the *qualities of the practical organisation can be discussed in terms of co-operation.* This differs from much experimental work, where the teachers are not part of the negotiations of the organisational set up. Much action research has concentrated on the practical organisation of the arranged situation, as this clearly represents an alternative to the current situation.

[18] In Alrø & Skovsmose (2002) it is also indicated that co-operation with students plays a role in the developing conceptions of possible educational practices.

Explorative reasoning

As we cannot expect an arranged and an imagined situation to be identical, a particular analytical approach has to be considered as part of critical research. We will refer to this as *explorative reasoning.*

To some extent, we want to draw conclusions about educational ideas, as expressed in the imagined situation, and not only to theorise about the arranged situation itself. For instance, with reference to the example, we want to draw conclusions about the use of computers in a certain classroom situation and with respect to a certain group of students. But we do not want to draw conclusions about the use of computers when the students have to walk to a nearby university. Many limitations that are part of the arranged situation are irrelevant to our study as such. In order to draw conclusions about the feasibility of the imagined situation, we have to analyse our observations in a particular way. This is the idea of introducing the notion of explorative reasoning.

Our observations are linked to the arranged situation and also limited by this situation, but some of our analysis concern the imagined situation. We are not simply interested in, say, generalising from data representing the arranged situation. We are also interested in *looking through* such data. In particular, it is relevant to make conclusions about the imagined situation based on what we have observed with respect to the arranged situation. In this way this later situation turns into a window through which we might better grasp and qualify the imagined situation

Explorative reasoning designates the analytical strategy of investigating imagined educational situations based on observations of particular arranged situations. Thus, *critical research also includes processes of explorative reasoning.* By explorative reasoning we try to qualify a pedagogical imagination from evidence gathered in the arranged situation. Explorative reasoning represents a reflection on pedagogical imagination based on practical organisation. Explorative reasoning, therefore, constitutes a particular analytical process.

Let us illustrate this in terms of 'expanding data', a process which has been suggested by Blomhøj (1998) with reference to mathematics education, and by Alrø and Kristiansen (1999) with reference to supervision. The analysed 'data' are not the observed data. Direct observations concerning communication in the classroom might be presented in forms of transcripts. But it makes sense to ask what would happened if, say, the teacher asked the students a different question and the communication thus took a different route from what is seen in the transcript. It makes sense to ask what would happen if students, working in a dynamic geometry environment, drag a different point of an object than the one they actually did. The not-experienced alternatives can be specified and presented in the form of an invented dialogue. Such 'narratives' have been presented by Blomhøj. His narratives are not unrealistic. To a great extent they follow actual dialogues from the classroom, but some parts of the narratives demonstrate possibilities which nobody in the actual classroom made use of. What does it mean to produce and to analyse such narratives? They represent possibilities. In this sense we are investigating aspects of the imagined situation 'through' observations of the arranged situation, as

narratives are produced by experiences from the arranged situation. But how to carry out such investigations in a qualified way?

According to, say, the positivist paradigm, it is essential to maintain a distinction between the researcher and the researched in order to ensure objectivity, generalisability and validity. However, we find that a negotiation of data is important, in particular when this negotiation includes those who are part of the research process. The data analysis does not simply concern the arranged situation but also the imagined situation. Expanding data is a way of opening the arranged situation and to look for possibilities. We see explorative reasoning as an analysis of observations that might provide an understanding of alternatives and of a pedagogical imagination.

In explorative reasoning, we want to draw conclusions about a situation that is not the one in which the empirical material is gathered. This is a basic characteristic of doing critical research. Explorative reasoning is a strategy for looking 'through' the arranged situation in order to provide a better understanding of the imagined situation. Explorative reasoning is a strategy for analysing possibilities that have not been acted out. This analytical aspect of critical research has not been particularly illuminated in the discussion of action research. However, we find this extremely relevant, as it is by means of explorative reasoning that pedagogical imagination can be qualified and developed. This analytical task is also dispensed, in case the researcher should know in advance the nature of changes to be made. As already mentioned we do not believe this to be the case, and as a consequence we emphasise the importance of explorative reasoning.

Explorative reasoning can be supported by a negotiation among the people involved: teachers, students, and researchers. As a consequence we find that *qualities of explorative reasoning can be discussed in terms of co-operation.* Also in this case, doing research with teachers, and not on teachers, qualify the research. Here we come to an essential aspect of doing participatory research.

It makes sense to compare explorative reasoning with those analyses presented by Lakatos in *Proofs and Refutations.* Lakatos provides a narrative in the form of a dialogue that never took place. This narrative is composed with many references to events in the history of mathematics. In this way, an analysis of the narrative might provide a new understanding of the logic of mathematical discovery. In a similar way, an analysis of a narrative with reference to real classroom situations may provide a new understanding of educational possibilities.

THE NATURE OF TRANSFORMATION

In our view, qualities of some patterns of critical research can be discussed in terms of the qualities of three interacting processes: pedagogical imagination, practical organisation and explorative reasoning. Furthermore, these processes can be discussed in terms of co-operation. In other words, researching possibilities becomes linked with researching *with,* and not *on* teachers and students. Thus, we do not find it surprising that action research and participatory research have been associated with critical mathematics education and to critical education in general. We find that

there are strong connections between these approaches. We have tried to clarify this connection by outlining the involved processes, and the basis for providing these processes with quality.

In particular, in order to do critical research it does not make sense to turn people involved in the research project into 'research objects'. If, say, a teacher becomes 'objectified', then, for instance, the quality of pedagogical imagination may be reduced because the process of considering what could be an attractive educational possibility includes co-operation. Of course, it makes sense to observe a particular situation to make interpretations of what exists. Thus, it might be useful to research the classroom that Stela was trying to manage in order to identify and describe the difficulties. However, the quality in doing critical research means involving Stela in the research processes as a participant and not as an 'object'. Teachers and students cannot remain mere objects of critical research.

Educational research can prepare for curriculum changes.[19] Implementation of such changes can be understood as top-down process. Curriculum developers identify a new curriculum, which is then put into practice, and the teachers are instructed to follow this new curriculum. This understanding of implementation is problematic for researchers who embrace the concerns of critical mathematics education.

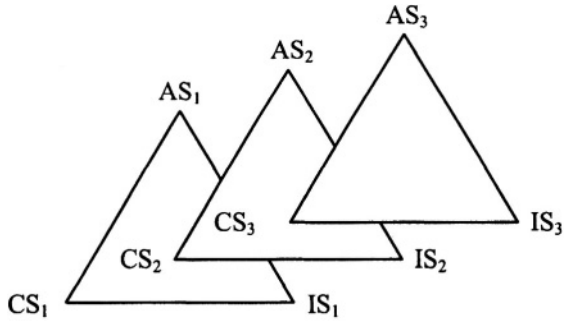

Figure 3: Model of critical research indicating what transformations such research might be associated with

Critical research processes refer to changes, i.e., the current situation starts to be altered. It is likely that the pedagogical imagination will develop and, therefore, the imagined situation will be modified as well. The same is the case with the arranged situations. This can be illustrated in Figure 3. The actual development of school practice is illustrated by the trajectory $CS_1, CS_2, CS_3,\ldots$ The development of the arranged situation is illustrated by the trajectory $AS_1, AS_2, AS_3,\ldots$ Finally, the

[19] For a discussion of the mathematics teacher and curriculum development, see Clarke, Clarke & Sullivan (1996).

trajectory IS_1, IS_2, IS_3,... illustrates the development of the imagined situation. In this way, Figure 3 tries to illustrate what critical research might be doing. Such research includes changes as part of the research process. This not only means changes in a classroom practice but also a change in the pedagogical imagination. In fact, all aspects included in our description of critical research undergo change.

This indicates that a development of classroom practice can be discussed in the same terms in which we discuss the quality of critical research. This is the reason why we regard pedagogical development as an integrated part of critical research. More generally, critical research has to do with making change and transformation, although not with making a priori identified transformations. And certainly not all kinds of transformation represent attractive qualities of critical research. As presented, critical research can be associated with action research, but the quality of such research cannot simply be described in terms of the cycle acting-observing-reflecting-change-planning-acting. We find that the qualities of critical research are linked to the qualities of pedagogical imagination, practical organisation and explorative reasoning, and that co-operation suggests a way of ensuring the quality of these processes.

DISCUSSION

Although Figure 1 presents a seemingly static picture, it was intended to illustrate that critical research does not, first and foremost, focus on what is the case, the current situation, but also on an imagined and on an arranged situation. The figure tries to illustrate what many patterns of critical research are addressing. Figure 2 illustrates that some patterns of critical research can be discussed in terms of the processes of pedagogical imagination, practical organisation and explorative reasoning. The figure tries to illustrate what processes are included in critical research. Figure 3 emphasises that critical research represents changes, illustrated by the trajectories of the current, the arranged and the imagined situation. This figure tries to show what critical research might be doing. The three trajectories indicate that pedagogical ideals are not fixed but develop as part of doing critical research; that pedagogical experimental work changes as well during the research process; and, not to forget, that critical research changes the current situation.

Freire suggested that critique presupposes equal partnership and co-operation, and the idea of equity prevents critical research from turning involved persons into 'research objects'. We have tried to argue that this equity is basic to the quality of critical research. We have related the qualities of pedagogical imagination, practical organisation and explorative reasoning to the notion of co-operation, which establishes 'research subjects'. The relevance of participation in research is not only to provide a friendly climate or to promote changes. Participation is related directly to the qualities of the processes included in critical research: pedagogical imagination, practical organisation and explorative reasoning.

In his discussion of different approaches to participatory inquiry, Reason (1994, p. 325) emphasises that 'in co-operative inquiry all those involved in the research are both co-researchers, whose thinking and decision making contribute to

generating ideas, designing and managing the project, and drawing conclusions from the experience, and also co-subject, participating in the activity being researched'. 'Generating ideas', 'managing the project', and 'drawing conclusions' can be understood as the processes of pedagogical imagination, practical organisation and explorative reasoning. As a consequence, co-operation between the participants is essential in order to explore what does not yet exist.

As mentioned in Guba and Lincoln's description of critical research, making transformations is essential. We concur that making changes is an essential aspect of critical research. Making transformations in the classroom, as conceptualised and illustrated on Figure 3, is associated to pedagogical imagination, practical organisation and explorative reasoning, all of them requiring co-operation between the persons involved. To put the understanding of what transformations are needed in the hands of the researcher easily comes to represent dogmatism in the form of we-know-better. What had to be done is not possible to identify a priori.

When research has to do with, not only what is the case, but also with opening and exploring possibilities, then doing research *on* somebody becomes problematic. Researching possibilities means researching *with* somebody. The qualities of the processes of pedagogical imagination, practical organisation and explorative reasoning depend on co-operation. In fact, it does not really make sense do engage in, say, pedagogical imagination *on* somebody. Therefore, co-operation becomes important for doing critical research. Making a critique means to specify that 'something could be different'. Similarly, doing critical research means to specify why and how 'something could be different'. This is also the case in doing critical research in mathematics education.

Let us finally remind ourselves that resonance in not a one-to-one correspondence. Other patterns of research than the one we have discussed here may resonate with critical mathematics education, and with critical pedagogy in general. We have concentrated on research related to making changes in the classroom. Such research is addressing not only what is, but also what is not. It is researching possibilities.

ACKNOWLEDGEMENTS

The ideas included in this paper have been developed in collaboration with a research group in South Africa consisting of Mathume Bopape, Herbert Khuzwayo, Manikam Moodley, Anandhavelli Naidoo, Nomsa Sibisi and Renuka Vithal. They have been further developed by Lene Nielsen, Aalborg University. Many of these ideas have also been discussed with Alexandrina Monteiro, Ana Maria Pires, Antonio Sylvio de Oliveira, Elizabeth de Azevedo, Heloisa Silva, Margarida de Mendonça, Maria Deusa da Silva, Miriam Goody Penteado, Mónica Villarreal, Odesnei P. Gustineli, and Sylvia de Azevedo, joining the Graduate program of Mathematics Education at UNESP, Rio Claro, São Paulo. Drafts of this paper have also been discussed with Pernille Pind, Jeppe Skott, Allan Tarp and Paola Valero, from the Danish University of Education. We have received additional comments from Helle Alrø, Rubia Barcelos do Amaral, Pedro Gómez, Arne Juul, John Mason,

and Robyn Zevenbergen. We would like to thank all of them for their comments and suggestions for this paper.

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RENUKA VITHAL

METHODOLOGICAL CHALLENGES FOR MATHEMATICS EDUCATION RESEARCH FROM A CRITICAL PERSPECTIVE[1]

The idea that mathematics and mathematics education could or should somehow be quite directly and explicitly connected to issues in society is now well established. It is possible to sketch a wide theoretical landscape of the social, cultural, and political dimensions of teaching and learning mathematics. Such a landscape comprises at least four developments in mathematics education that have grown substantially in the last two decades: a political or critical mathematics education (e.g., Frankenstein, 1987; Mellin-Olsen, 1987; Skovsmose, 1994); ethnomathematics (e.g., D'Ambrosio, 1985, 1990; Gerdes, 1996; Powell & Frankenstein, 1997); concerns about diversity including race, class and gender (e.g., Dowling, 1998; Keitel, 1998; Parker, Rennie & Fraser, 1996; Rogers & Kaiser, 1995; Secada, Fennema & Adajian, 1995; Shan & Bailey, 1991; Walkerdine, 1998); and people's mathematics in people's education for people's power from the Apartheid era (e.g., Adler, 1988; Breen, 1986; Julie, 1991, 1993). Taken together these may be interpreted as offering a critical perspective in mathematics education[2]. Arguably, these ideas that could be said to constitute a social, cultural, political approach to a mathematics curriculum, are far more developed in theory than in practice. Not discounting some excellent efforts to work with such ideas in classrooms, hard research evidence to support —or counter— theoretical propositions and associated practices seeking to realise notions of empowerment, emancipation, democracy, social justice, equity and so on through mathematics education, is still rather thin. It is with this concern and a commitment to making these kinds of theoretical ideas and related practices more widely available to practitioners for interpretation and critique that I explored the research question: What happens in a mathematics classroom when student teachers, who have been introduced to a social, cultural, political approach to a school mathematics curriculum that integrates a critical perspective, attempt to realise such an approach.

Student teachers are typically introduced to new ideas in teacher education curricula with the implicit assumption —or hope— that they will somehow integrate these ideas into their teaching. A co-operative effort with them, during that time they

[1] This chapter is a shorter version of a plenary paper presented at the Second International Mathematics Education and Society conference (Vithal, 2000c) adapted from my doctoral dissertation (Vithal, 2000a)

[2] See Vithal (2000a) for a detailed elaboration of these.

P. Valero and R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology,* 227—248.

go to schools for practice teaching, offered an opportunity to take a closer look inside classrooms, and to examine what meaning is given to these theoretical ideas in practice. In this process, the student teachers would learn to teach by teaching in a new and different way. This opportunity created the means for myself, as the teacher educator, the student teachers and the class teacher to reflect on both the theoretical ideas themselves and the forms they take in practice.

In this chapter I focus not on the findings of the study but rather on the methodological challenges that arose in conceptualising and undertaking the study. In essence this was a study in theory-practice relations in mathematics education, but here the focus is on exploring that relation within the research itself. The problem of methodology, from the perspective of theory, is two-fold. The first is that of making a deliberate intervention when researching particular theory-practice relations, especially in contexts in which practices associated with those theoretical ideas are not widely available in the existing (mathematics) education system. This challenge has been addressed (see Skovsmose & Borba, this volume; Vithal 2000a, 2000c) by referring to *three situations* that offer methodological theoretical tools for thinking, doing and talking about researching theory-practice relations in mathematics education from a critical perspective: a *current actual situation* which refers to the situation as it exists before making any intervention for the purposes of research; a *hypothetical imagined situation* as a space for teachers, researchers — and others— to jointly think, discuss and dream about possibilities as theoretical ideas are recontexualised and produced as practice; and an *arranged situation* which is created, with reference to, but different from both the actual and imagined situations, in which a particular reality is played out and that provides the means for a researcher to study both the theoretical ideas and related practices.

The second challenge, addressed in the rest of this chapter, is that of finding a research methodology and approach embedding research relationships and processes that itself preserves the main concerns of the theoretical landscape and practices of a mathematics education that integrates a critical perspective. To do this, I refer to the three broad well-known positivist, interpretivist and critical paradigms in research, which serve as a map for the discussion and for locating my research concerns. I discuss what may be considered a serious difficulty in researching a critical perspective in mathematics education —that of a researcher trying to find resonance between her research approach and her educational approach. The search for a research methodology that integrates a critical perspective takes two routes: one into mathematics education, and the other outside it. I then raise issues that I consider to be necessary —but by no means sufficient— in developing such a methodology, particularly in the relationships between the researcher, the research participants and the research process.

A CRITICAL APPROACH TO MATHEMATICS EDUCATION VERSUS A CRITICAL APPROACH TO RESEARCH

A distinction quite commonly made between different research approaches in the literature, drawing from Habermas' original framework of knowledge-constitutive

interests, and variously labeled and described is: a) the empirical-analytical, logical positivist or behaviourist paradigm; b) the interpretive, hermeneutic, phenomenological or symbolic paradigm; and c) the critical paradigm. This classification has often been imported into research discussions in mathematics education in various ways. It is one of many, and not in any way exhaustive; we need only refer to the growing research debates related to postmodernism and feminism that have emerged more recently and which defy any neat categorisation. Nevertheless, arguably, the first paradigm has dominated mathematics education research; and in recent years, the second paradigm has also gained much ground (see also Atweh, this volume). However, if the research journals and the recent handbooks published in mathematics education (e.g., Bishop, Clements, Keitel, Kilpatrick & Laborde, 1996; Grouws, 1992; Kelly & Lesh, 2000; Sierpinska & Kilpatrick, 1998) are taken as indicating the state of the art in research in mathematics education then it is reasonable to conclude that the critical paradigm is significantly under-explored and under-represented in mathematics education research.

What exactly is a critical research paradigm? And what relation, if any, could it have to a critical perspective in mathematics education and to how I was construing my research? To grapple with this issue it became necessary to consider the research focus, theory and methodology of the study. A main purpose of my study was to explore the relation between a particular theoretical educational approach and its recontextualisation into practice by student teachers, that is, to talk back to the theory and related practices —both often imported— through research. No doubt the theoretical commitments of the study were to concerns of equity, empowerment and social justice found in the landscape of the mathematics educational approach described as a social, cultural, political one foregrounding a critical perspective. But the dilemma that became apparent is: what of the methodological commitments to these same key concerns as I engaged a particular research process and research relationships, collected data, began analysis and writing. A question that arose from the description of the research design is, what is the underlying theoretical base supporting the empirical work and methodology? Further, what is the relation between these theoretical assumptions upon which the research methods are based, and the educational theory being examined in its interpretation into practice? Indeed, what could or should be the nature of the relation between the educational theory being investigated and the 'research theory' —the theoretical underpinnings that inform the construction and enactment of the range of research practices within an empirical study?

Typically, the theoretical framework set out in a study provides the theoretical tools by which the data will be analysed. But is it possible to explore a deeper, more broader link between the theory underpinning the educational practice and that upon which the research itself rested, in all its facets —such as in the nature of the question asked, the relation between the researcher and the researched, the involvement of the research participants in the activity of data generation, analysis and writing, and in the criteria for verification and evaluating the study? The assumption being made here is that just as there is no neutral and value-free mathematics education, the research enterprise is also neither neutral nor value-free.

The problem then is to not only understand the assumptions which (dis)connect theory and practice in education but also the theoretical assumptions that underpin the research methodology through which that link is explored and understood —the theory and practice of the research itself.

In the way in which the above questions are framed, one could posit a separation within the theoretical considerations of an empirical study. That is, a possible disjuncture, even a conflict, between the educational theory and its practices and processes on the one hand, and the research theory and its practices and processes on the other. The research paradigms distinguished above can assist in making visible the theoretical assumptions in the research process and the fundamental ideological differences in how research is understood, engaged and its goals achieved. The theory-practice relation in the different research paradigms, for instance as Carr and Kemmis (1986, p. 152) state, is understood differently. In positivist research 'theory is regarded as a source of disinterested principles which [...] may be taken to prescribe for action'. In interpretative approaches to research, interpretations do not prescribe action but 'merely informs teachers about the nature, consequences and contexts of past actions, and require that practitioners use their own practical judgement in deciding how to act'. From a critical perspective, the relation between theory and practice 'is seen to require the active participation of the practitioners in collaborative articulation and formulation of the theories imminent in their practices and the development of these theories through continuing action and reflection'

To explore further the issue of theoretical considerations in research methodology and its link or disjuncture from theoretical considerations in education consider the following from my research. Perhaps an approach to the research could have been to develop a set of criteria or prescriptions from the theoretical landscape to guide the student teachers, to follow this with classroom observations and interviews with them, and then to analyse the data against a predetermined set of indicators of this critical perspective to mathematics education. The idea that a set of criteria or indicators can be found and applied in the research process comes into a serious and significant contradiction with the theoretical positions within a critical mathematics education and conflicts with the educational process. That this is observed as a conflict, of course, depends on how a critical perspective in mathematics education is understood. Taking a critical perspective in mathematics education cannot be equated to, as Skovsmose and Nielsen (1996, p. 1260) point out, 'a sort of methodological principle'. Critical mathematics education does not refer to a particular form of mathematics education but rather to a perspective in an educational landscape that involves mathematics. As such, it cannot be outlined as a set of rules for action and content and then followed in order to realise a 'critical mathematics education'. The problem can be concretised more sharply. In the educational theory, a particular educational relationship is argued for between teachers and pupils, for instance, pupils cannot be 'forced' to learn or become critical. The question to be considered is then similarly, what should the research relationship be between researcher and teacher —or pupils—, for instance; teachers too cannot be 'forced' to take a critical perspective in mathematics education. Hence, the difficulty is that whilst the educational theory is located in a critical paradigm, the theoretical underpinnings in the research could become lodged in a

positivist paradigm. My experience in trying to investigate a critical perspective in mathematics education is that this conflict arises quite easily if the theoretical assumptions on which the research is based are not explicitly considered by the researcher in the research processes, and its connection to the educational theory is not explored and maintained. What needs to be understood is how and what mediates the way in which a researcher understands the theory-practice relation and chooses to act in particular ways as a researcher —in constructing research relationships, in deciding who collects data and what is construed as data, and in how research participants are to be involved in the analysis and writing. In this research it has to do with my views about what constitutes a critical perspective in mathematics education versus the research paradigm in which I locate myself as a researcher.

One difficulty in grappling with these problems is that the literature expounding a social, cultural, political approach to mathematics education seldom makes its research methodology explicit nor explores issues of methodology beyond the descriptive level. In the review of research and literature on ethnomathematics Gerdes (1996, p. 927) admits that 'Ethnomathematical - educational research, including the study of possible educational implications of ethnomathematical research, is still in its infancy'. Much of the attention, as can be seen in the work of Frankenstein on a critical mathematics literacy, is focused on advocating a critical approach to mathematics education, and developing a theoretical base for the approach and related educational practices. Skovsmose (1994) develops his theory of a critical mathematics education by referring to teachers' descriptions and not his observations. More recently critical mathematics education is used to refer to both 'educational practices as well as to research on this practice' (Skovsmose & Nielsen, 1996, p. 1260) and it is suggested that research in critical mathematics education can be largely identified with ethnography and action research. This is not to say that descriptions of research processes do not exist. What has still not been adequately developed are a set of reflections at a meta-level in research that begin to put forward a coherent and comprehensive theoretical framework for doing research in this mathematics educational landscape. In the next section I attempt to initiate such a discussion by tentatively marking out some means for making these reflections and then identifying some issues for consideration, particularly as they arose in my research.

SOURCES FOR DEVELOPING A RESEARCH METHODOLOGY FOR A CRITICAL PERSPECTIVE IN MATHEMATICS EDUCATION

There appears to be at least two sources for developing a research methodology for a critical perspective in mathematics education: one from inside mathematics education; and another from outside it.

The search inside mathematics education

A source for developing a methodological base for researching a critical perspective in mathematics education is the educational landscape itself, that is, to draw on theoretical, critical formulations within mathematics education that could inform the research process. These may be sought in the work of D'Ambrosio (1990), Frankenstein (1987), Skovsmose (1994) and others who have developed critical perspectives in mathematics education. For instance, Skovsmose and Nielsen (1996, p. 1257) identify several 'concerns' of a critical mathematics education such as 'Critical mathematics education concentrates on life in the classroom to the extent that the communications between teacher and student can reflect power relations'. This could also become a concern in a critical mathematics education research methodology in the relation between the researcher and the research participants — as I will elaborate later. Concepts in critical mathematics education (Skovsmose, 1994) such as: 'reflective knowing'/'knowing as an open concept' may have parallel interpretations in research as reflexivity; 'intentionality' in learning may give insights into understanding the interests of the research participants in the research process linking to the concern with reciprocity in research; and the notion of 'exemplarity' could provide alternative or complementary meaning to the issue of generalisation in this study (Vithal & Valero, 2003; Vithal, 2000a). What may be observed here is how concerns and concepts in the educational landscape could be recontextualised in the research landscape. This means that the imagined hypothetical situation serves both the researcher and the practitioner in recontextualing ideas from theory, both for practice and for research.

A counter to this proposition may be that the task, goals and discourse of education must be distinguished from those of research —there may be overlap but they are essentially different human activities or practices. The dilemma, however, is that in preserving this necessary separation, the researcher runs the risk of seriously contradicting the democratic project at the heart of her study. This correspondence between research and educational theory is not only desirable but is essential if a mathematics education theory that is concerned with questions such as, 'Does mathematics education reproduce inequalities [… that] are reinforced by educational practice?' (Skovsmose & Nielsen, 1996, p. 1261) is not to become implicated in reproducing or reinforcing forms of inequality —and oppression— in the research methodology employed to study those very theories and related practices. The main thesis here is that any study that puts issues of democracy and power in the centre of an educational theory must equally be concerned with issues of democracy and power within the research process itself. A theory, which draws attention to the politics of mathematical knowledge as an integral part of mathematics education, must concern itself with the politics of this knowledge production within the research enterprise that seeks to investigate such ideas. Indeed, the same way that mathematics educational theories try to unmask injustices and oppression, researching those theories should also open possibilities for research participants to deal with forms of identified oppression and inequalities, also within the research enterprise. This research challenge may be characterised as seeking *resonance* in the

methodological perspectives employed to accompany a critical mathematics education (see Skovsmose & Borba, this volume)

The search outside mathematics education

A second source comes from the progress that has been made in developing and using such research approaches outside mathematics education, in response to similar kinds of concerns. Drawing on the advances made in methodological issues from outside mathematics education may be justified on the basis that critical perspectives in mathematics education are inspired by and rooted in a critical paradigm, and draw on the work of those same theorists/theories outside mathematics education. Ethnomathematics, feminist and critical mathematics educators draw on the work of theorists such as Freire, Giroux and others who are proponents of critical theory and related perspectives. This means that further elaborations can be made by examining the relation between research and educational theory as developed in general education and social science research. There seems to be agreement that (educational) theory and practices that locate themselves within in a critical paradigm must be investigated through methodologies which are themselves located in a critical paradigm.

Critical educators have indeed recognized this problem of resonance in research. For example, Young (1990, p. 138-9) refers to the need to 're-theorise or reconstruct general methodological understandings in educational research' and the problem of employing research methodology informed by positivist ideas in studies to investigate critical education:

> The existing literature theorises the activity of researchers in epistemological terms and not as social agents. That is, for the most part, educational researchers are theorised as privileged epistemological actors within a theoretical model which is conceptually quite distinct from the theory in which the behaviour of teachers and pupils is theorised.

The clear implication is that there should be some kind of harmony between the educational relationships advocated in a particular theory and the research relationships constructed in the research process. But what does such a research methodology look like and how does one create some kind of correspondence between the theoretical or epistemological base of the educational and research processes and practices?

Going outside education, this issue is perhaps most directly and succinctly discussed by Lather (1986, 1991) in which she explores 'the methodological implications of the search for an emancipatory social science'. She writes (1986, p. 258):

> The essence of my argument, then, is that we who do empirical research in the name of emancipatory politics must discover ways to connect our research methodology to our theoretical concerns and commitments. At its simplest, this is a call for critical enquirers to practice in their empirical endeavours what they preach in their theoretical formulations.

A key aspect of what is described as an 'emancipatory approach to research', an approach that is 'explicitly committed to critiquing the status quo and building a more just society' (p. 258), is the relation between the researcher and the research participants and the form in which this relation gets expressed through the research process in the generation of the research question, the data, the analysis and theory and even the writing of the research report. The main point to be gleaned from this for the discussion at hand, is that whatever the understanding of democracy, creating a research process that is characterised by democratic concerns is essential in a study that itself puts issues of democracy in the centre of educational theory and practice. Researchers who involve the research participants in a democratised process of inquiry, according to Lather, engage in research that features negotiation, reciprocity and empowerment. In this way she argues, empowering approaches may be developed to the generation of knowledge. But as she also points out, there are few clear strategies for linking critical theory and empirical research. One approach is to build in opportunities for reflexivity into a critical enquiry at all levels in the research process not only for the researcher but also for research participants.

It is possible to observe, across writers, that to take a critical perspective in theory and practice, means also to take a critical perspective in research. They argue that ways must be found to make this connection and refer to the growing phenomenon of critical research (e.g., Carspecken & Apple, 1992; Cherryholmes, 1991; Skovsmose & Borba, this volume). Examples of methodologies linked to critical research are critical ethnography (e.g., Carspecken, 1996; Kinchloe & McLaren, 1994; Quantz, 1992) and some forms of action research (e.g., Atweh, this volume; Carr & Kemis, 1986).

TOWARD A CRITICAL RESEARCH METHODOLOGY: SOME KEY ISSUES

The main challenge I set out for researching a critical perspective in mathematics education earlier, I now illustrate with respect to my own study. In what ways, and to what extent have I dealt with integrating a critical perspective in my research? I do so by examining different facets of the research process and forms of participation. Several key issues are identified and discussed in an attempt to connect concerns of practice and theory in research on the one hand, and in mathematics education on the other; and for the different research participants: student teachers, class teacher and pupils— and the researcher.

Choice

The choice to participate in the research recognises the aspect of agency, the freedom and the capacity to decide to act, which must be considered, given its central role in a critical education setting. In the research, student teachers were invited to participate as co-researchers to investigate what for them was an innovative, even radical approach to teaching and learning mathematics to which they were introduced as part of their coursework (see Vithal, 1997). They also chose

the schools in which they wanted to work. Several additional preparation sessions were held prior to the student teachers' entry into schools that provided them with information about the study, so that the choice for their involvement could be informed and based on their willingness, interest, understanding and commitment to the educational ideas and the research process. The choice to participate is followed by the choice to shape that participation. This is observed in the choice exercised in the interpretation of the approach within the arranged situation. Although at the beginning of these preparation sessions I had left open the educational tasks and ideas for implementation of a social, cultural, political approach to the mathematics curriculum (e.g., critical literacy tasks involving newspapers —see Vithal, 1997), the student teachers focussed on project work as its main interpretation in practice. Just as I created an opportunity for joint ownership of the research question, some student teachers created similar opportunities for their learners to own their project problems. So learner interest, a key idea in a critical approach to education, paralleled the student teachers' interest in the research. Notwithstanding, interests themselves differed and were vested in different ways in different parts of the educational and research processes. Within project work, further choices were made, for example in handling the teaching and leaning of mathematics —should it be taught first and then learners apply it, or could it be learned as a part of the process of working in the project? In dealing with such dilemmas, my relationship to the student teachers reflected the role of a teacher-supervisor in project work that they were attempting to model in their projects. For instance, they give the learners a relevant chapter from a textbook to deal with the mathematical issues that arose in the projects. Such a radical deviation from the conventional approach was negotiated and tried. But was it also in some sense imposed? After all I was the teacher educator making the relationship inherently unequal.

There are also the choices that researchers make throughout the research. By the time teaching practice ended I was not able to be present in the classes for three of the projects. Even though I explained my lack of equal physical presence and the way in which I was making the choices, this was not without consequences, especially for my relationship with the student teachers concerned and the projects not visited regularly, and for those student teachers' lack of participation in later activities. Within the research process itself student teachers chose to collect data and participate in an initial analysis in order to produce a paper for a conference. Drawing on Skovsmose's (1994) notion of intentionality and learning as action, it may be argued that students learn when they own the reasons and goals to learn. Similarly, student teachers' engagement in the research process was also mediated by their own interest to become researchers and/or practitioners. Student teachers who expressed a strong interest in doing research, collected far more classroom data than other student teachers. After teaching practice, it was these student teachers who drove the process for writing and presenting the paper at a national conference (Vithal, Paras, Desai, Zuma, Samsukal, Ramdass & Gcashbe, 1997) which came to be a powerful means to see themselves as intellectuals and generators of knowledge.

Choice is an important element in researching a critical educational perspective and essential for participants in the research process because it allows them to regulate their own participation and the possibility to act, bringing into the research

fold the concept of *'Mündigkeit',* a key concept in a critical mathematics education (Skovsmose, 1994). Should their experience of the research process become in any way exploitative they could exercise the capacity to withdraw or change the nature of their participation. This freedom to choose, however, is bounded since it is enacted within other constraints, such as the requirements of their teaching practice course for their degree and the commitment to the school. Choice assists the researcher to not fall foul of practices that are contradictory to the theory on which the research rests. It serves to counter, in part, imposition that any intervention, even if democratically negotiated, might lead to especially when the researcher is in an inherently more powerful position than the participants because of the unequal balance of knowledge and skills specific to the situation and to research itself. Choice is also essential if participants are to maximise their participation in the research especially in terms of the effort and commitment that any successful research project requires. The postulating of a hypothetical imagined situation, which offers a space for mediating between theory and practice, supports the element of choice in that the recontextualisation of theoretical ideas need not be uniform or consistent within or across research sites, though they may be negotiated. Choice is recognized and activated through pedagogical imagination as research participants attempt to try out alternatives. Choices are shaped by critical reasoning to anticipate actions and their consequences in the arranged situation (see Skovsmose & Borba, this volume).

Negotiation

Once the choice to participate is made, there are different kinds of negotiations related to different aspects of the research endeavour that need to be made. First there is the negotiation of research relationships and identity of the different participants: student teachers, learners, teacher and researcher. These must be managed both with reference to the research situation and the educational setting which comprise the arranged situation. My relationship to the student teachers is foregrounded in the study and it is different for each one, not least because our histories are both similar and different given our multiple identities in terms of race, class, gender, age, language, etc. From the outset of the research project I emphasised a relationship of colleagues jointly interested in the same research question but I was aware through their reflections and course evaluations that they identified with me in different ways at different times as a teacher educator, a fellow teacher, a researcher, a woman, a mentor and as a friend. The research process allowed greater closeness than usual during teaching practice supervision. This relationship underwent further change during the course of the study with a closer relationship developing with those students in whose project I was present a great deal more whilst a distance emerged between those students for whom I was not there. Students who did not participate in writing the paper, were almost all students whose projects were on the margin.

Once the choice to participate is made, participation has to be negotiated given the different vested interests and identities of the researcher and research

participants in the research that must co-exist with the educational requirements of the school and the university programmes. Although I construed the student teachers as co-researchers this belies the differential knowledge, skills, interest and participation in the research. I declared my dual role of researcher/teacher educator and discussed how these could be managed. We shared a joint interest to know the outcomes in real classrooms of these new theoretical ideas and practices developed in other countries discussed in their teacher education course, but, as their teaching practice supervisor I was required to give an assessment during the research process. Hence, assessments were opened to negotiation. Some student teachers showed extraordinary commitment to the research project. They were very enthusiastic and constantly talked to each other and to me about their ideas, especially those who showed a strong interest in being involved in research themselves. But some student teachers were also skeptical because the ideas were considered to be radical, but they were curious about its possibilities and therefore chose to participate.

The student teachers in turn, entered into negotiations with the teacher and with learners in school for a chance to work with a different approach. The preparation sessions held prior to meeting the teachers and learners, and continuous post lesson reflections during the research helped student teachers in their negotiations and confrontation with the actual existing situation and to arrange a situation in a school for experimentation during teaching practice. These actual and arranged situations give an indication of the student teachers' and my imagined hypothetical understandings and interpretations of the theory and practice under investigation, and a space for a continuous process of negotiation of ideas from theory for practice. Negotiation of participation involves also a negotiation of practice. Despite drawing student teachers attention to several alternative forms of practice, they focussed on project work. However, the project work itself took different forms and evolved through negotiations in the arranged situation, referring to both the actual and imagined situations, and educational and research considerations.

Negotiation is essential and central to the relationships between the imagined, actual and arranged situation. It is also the key to creating the possibility for transforming the situations (see Skovsmose & Borba, this volume). Negotiation allows spaces in which, whatever the idea that is put forward by the researcher and by the participants, it has the status that it can be challenged, critiqued, discarded, reformed or transformed. This means that the quality of reasoning both in the practice and in the theory may be improved. Between the ideals of the hypothetical and the reality of the actual situation, the researcher and research participants negotiate their creative pedagogical imagination to develop possibilities for action in practice. But for a workable and realistic interpretation of the imagined situation into the arranged situation, a collaborative transformation of the actual into the arranged is needed that takes account of a myriad of concerns relating to the broader educational context and community which both constrains and enables any intervention, for example, requiring also the involvement of school administrators, parents or the community. Negotiation enables theorising to occur from the ground in the arranged situation, back into the imagined situation, and to the *a priori* theoretical landscape. Throughout these relations, negotiations serve to enhance the quality of the participation of the research participants and therefore of the research.

However, negotiation is not without its problems. Given the inherently unequal power in research relations, negotiation itself can dilute different perspectives and contradictions in seeking consensus to act in a particular situation, though it serves to counter the problem of imposition.

Reciprocity

Reciprocity ensures that the goals and outcomes of the research process will meet the needs and interests of both the researcher and the research participants. Given the availability of choice and negotiation, reciprocity keeps at bay the possibility for the research process to collapse by helping to secure commitment and participation of the research participants in the arranged situation. It assists in bringing equity to the research partnerships since both are seen as needing something the other can offer which in turn contributes to investment of effort. All involved participants should have a clear idea about what is in it for them in the research process. Through reciprocal partnerships, participants are made accountable to each other even if that accountability lies in different domains and interests. It is in reciprocity that the ethics and politics of research are reified and the aspect of rewards and reasons for participation need to be dealt with. Unequal power and differing vested interests make reciprocity crucial in critical approaches to research. In my research, student teachers negotiated and chose to jointly write a paper about their experience in the project and to present it at a conference, rather than receive material rewards. Nevertheless, I still had to recognise that the power relations operated in my favour by virtue of my status as researcher and teacher educator; and nor was I able to apply these concerns in equal measure to the teachers in the school and especially to the pupils in the study. Even offering rewards are not without their difficulties since, in themselves, they constitute a power-induced intervention mediated between the researcher who has the authority and power to reward, and those who receive that reward.

Despite having employed many of the strategies similar to those described by critical researchers, the inherent hierarchical nature of the relation with the student teachers had to be acknowledged and worked with. As researcher/teacher educator I was often construed as the one who should know. Having discussed at length various alternatives for a particular classroom activity in the project work, student teachers still asked for my opinion as an 'expert', the one with resources. Thus, in the teaching and research process, where student teachers are learning to become teachers of a critical approach to mathematics education and are also participating as novice researchers, the power relations operate in favour of the teacher educator and researcher. However, there are spaces where this is reversed or equalised, for instance, during discussions of practice related to knowledge about the learners and classroom organisation issues —such as how to arrange the groups in a particular class or what to do with a specific group or student. These shifting power relations are at work throughout interactions between the researcher and the research participants. It is important to recognise this because it influences what data are

produced and how these are analysed, especially when the student teachers are also involved in the analysis and writing, and possibly expressing dissenting claims.

The question is to what extent were my suggestions, in fact experienced by student teachers as impositions. There appears to be an inherent paradox in critical research described by Lather (1986, p. 269) as follows: 'The potential to create reciprocal, dialogic research designs is rooted in the intersection between people's self understandings and the researcher's efforts to provide a change enhancing context'. The problem is that whilst the researcher and the research process seek to avoid being impositions —but inevitably are— the research participants simultaneously need to be empowered to think and act in new and transformed ways. For the student teachers it was a constant struggle between traditional ways of thinking, acting and being mathematics teachers, and their new role as supervisors or facilitators in project work. And this struggle played out within the constraints of how the school views and organises mathematics teaching, learning, assessments etc., as well as their role as student teachers in subject matter areas other than mathematics.

Throughout the research process I sought to respect and took seriously the views of the research participants no matter how much they differed from my own as opportunities were created for the ideas in the theoretical landscape to find expression. But this also meant being confronted with racist and sexist views and views that condone corporal punishment. The research process required staying in dialogue with the student teachers, challenging them to be innovative, supporting and building on their own ideas, yet also developing their capacity to critique not only what was happening in the classroom but also their own deeply held beliefs and values. In return, I remained open to the possibility that my deeply held beliefs and values and the heart of my research concern that mathematics education has a role to play in building a democratic and just society and that cultural, social, political issues can and should be discussed in mathematics classrooms, may be seriously challenged and completely shattered when faced with the reality of mathematics classrooms and schools. Choice, negotiation and reciprocity are important features not only in methodology but also in criteria for evaluating such research, such as in democratic participatory validity, which requires the researcher to make visible the extent to which the research participants participate in the research (see Vithal 2000a, 2000b).

Reflexivity

Reflexivity opens for constant critique in research and in the educational setting on the part of both researchers and research participants. Opportunities for reflecting on the arranged situation were made available through: various interviews and conversations with student teachers and the class teacher; student teachers' journals, lesson and project materials; classroom videos; data generated by the pupils which included their diaries, work done on projects; presentations made by the student teachers to a Ph.D. reference group; a faculty research seminar; and the production and presentation of a joint paper at a national conference for mathematics education.

Reflections on the arranged situation are important because they provide the means by which to connect back to the imagined and actual situations and to seek shifts in these. Involving multiple levels and points for reflections and involving different participants both in the centre and at the margins in the research and educational processes allows for a reflexivity which brings both an insider-outsider perspective, and opens opportunity for self-critique. Student teachers responded differently to critique from me, the class teacher and/or their peers who were assisting in the project work. The student teachers' involvement in these research activities demonstrated how spaces for reflexivity may be created in the arranged situation and as they leave it to return to the actual exiting situation. These spaces also have methodological implications because they contributed to an early and initial analysis of the data. The issues they identified in these reflections were debated and discussed and through these processes became part of my analysis in the research. They showed how imagined situations were changing and therefore also how they might act beyond the research setting or even in a newly created arrange situation.

Reflexivity is needed for managing the multiple identities of the researcher and research participants. Throughout, I was acutely aware of my shifting framework for observing and interacting as I grappled with and tried to understand what it meant to be both the researcher and the teacher educator. For instance, when standing at the back of the classroom with the video camera, I was to all intents and purposes a non-participating observer. That immediately changed when the teacher figures asked my opinion or if the pupils drew me in with a direct question. Simultaneously reflecting on and understanding what was going on as a researcher while acting as a participant in the process as a teacher/teacher educator was challenging to manage. I participated when invited but also made inputs. For instance, in the post-lesson discussions I presented alternative ideas from the ones the student teachers were considering which were usually informed by a traditional mathematics pedagogy. Reflexivity serves to flatten the hierarchy of relationships in research and in the educational environment because all reflections are considered and valued.

By making more equitable theory-practice relations possible, reflexivity allows for the development of both theory and practice through shared reflection and critique. That is, critical reasoning is given an essential role as the relation between the imagined and arranged situation comes under scrutiny. The post-lesson discussions provided strong opportunities for this reflexivity and joint analysis, on a regular basis, where alternatives were discussed and decisions negotiated for the day to day running of the project. In writing up their projects, began a preliminary data analysis process and overview of the project for each student teacher. The process of producing the joint paper provided a space to reflect and more closely resembled what researchers do —having collected data, doing an analysis and writing. It was during these times that students could reflect on what had happened in the different projects across contexts, which led to multiple explanations for events observed in their own projects and affirmed or showed gaps in the theory.

This brings us to another important, but related concern about how the relation between theory and practice is constructed in the research situation and the role of the researcher and the research participants in the critique and generation of ideas for both theory and practice. Typically theory is privileged and this privilege is

extended to the researcher —theory speaks to practice. Reflexivity in critical research gives practice the opportunity to speak back to theory. Both theory and practice become matters for negotiation between the researcher and the research participants. It is also for this reason that the recognition of a hypothetical imagined situation, which mediates how theory and practice are recontextualised during the study, is important. Moreover, a critical perspective positions the researcher not only as someone who seeks to understand the research situation, but legitimates the researcher's active involvement in, and on the research situation. This means that while the researcher has an important role in sharing reflections on practice, research participants such as student teachers have equally a space to reflect on their experience with respect to the theory.

This equity in reflexivity is also essential for contexts in which there is theory and practice importation. The main focus of my research was to give meaning to a particular theoretical landscape in a context vastly different from that in which it was conceived and then to see how the process could yield new insights both for theory and practice. Thus, while the theoretical landscape led to a particular practice that generated specific data, the data in turn would come to inform the theory. Lather (1986) brings this issue to the fore by examining theory building versus theory imposition as dialectic. This dialectic played itself out in my study, in a sense, as theory imposition creating opportunities for theory building by both the researcher and the student teachers. Seeing theory and practice as dialectical (Roman & Apple, 1990) is important because it recognises that practitioners or research participants have an active role to play in theory building. Extending reflexivity or critical subjectivity (Lincoln, 1995) to a negotiation between researcher and research participants to this end, however, is easier said than done because what needs to be considered is the research participants' (lower) interest in theory building. For most of the student teachers, their main interest was in improving practice rather than critical reflections on theory.

Subjectivity-Objectivity

The relationship of the researcher to the research process takes different forms in each of the positivist, interpretivist and critical approaches. In positivist educational research the researcher is the 'instrument by which research is undertaken' as an objective observer. The interpretivist researcher reconstructs and interprets events for greater understanding which 'become part of the language of their time and influence(s) the decisions made by others'. However, in a critical approach to educational research the researchers' 'participation in the development of knowledge is comprehended as social and political action which must be understood and justified as such' (Carr & Kemmis, 1986). This does not mean that these are discrete relationships. Although the relationship of the researcher to the research process may be driven by one paradigm, there are instances during which other relationships do manifest. That is, in the practice of research, there are times when the critical researcher might be positioned as the 'objective observer'. However, what is being argued here is that the overall research process must itself be guided

and grounded in methodology that resonates with the theoretical commitments invoked in the research by the researcher.

The relationship of the researcher to the research process and the research participants has often been discussed through the objectivity-subjectivity debate in research methodology. A brief tracing of the history of this debate seems to suggest a shift from a preoccupation with establishing objectivity in research in the positivistic paradigm to a recognition of the importance of subjectivity and a concern for it to be understood and worked with in a multiplicity of ways in research (e.g., Eisner & Peshkin, 1990). There might be what is called 'disciplined subjectivity', as one example in the interpretivist paradigm. This debate, however, rages on and critical researchers such as Roman and Apple (1990, p. 39), for instance, argue that subjectivity and objectivity should not be 'treated as a binary opposition in which the absence of one is seen as the presence of the other' but rather what needs to be acknowledged is 'the reciprocal determinacy that 'subjectivity' and 'objectivity' —the conflicting sets of historically specific power relations and material interests— have upon one another'. Research that is 'openly ideological' (Lather, 1986) or is constructed or seen explicitly as 'an ethical and political act' (Roman & Apple, 1990), and which attempts to address issues of inequalities and injustices, has forced researchers to question and re-examine the subject-object dualism' in new and different ways. It is possible to posit yet another approach, and that is one in which objectivity is interpreted as inter-subjective agreement —giving up any search for objectivity and settling instead for inter-subjectivity. There is no 'truth' to be found through research, but only multiple truths depending on the position taken or occupied in the research setting. But even such a position is not unproblematic since these do not all have equal status given that 'meaning is jointly constructed between researchers and the research subjects in the context of interests that are formed out of contradictory power relations' (Roman & Apple, 1990, p. 40). How researchers think about and resolve the objectivity and subjectivity positions in their individual research is important because it deeply affects how research relationships are created, the research process and procedures set up, what is accepted as data, how and who participates in the analysis and production of theory. The operationalising of choice, negotiation, reciprocity and reflexivity, in a sense creates a requirement for researchers and participants to declare and embrace their individual subjectivities and brings it to the center of the methodology where it is seen as enhancing the quality of a study from a critical perspective.

Context, change and instability

Perhaps what may be considered a silence or gap in discussions about critical research methodology is the question of: how is critical research context related? Does it matter if the methodology is considered in Denmark or South Africa? What does it mean to do critical research concerned with issues of equity and social justice, in societies marked by rapid change, deep and structural inequalities, violence and poverty? The powerful contestations and resultant disruptions, which

produce instability, the heart of a critical research agenda, threaten the very existence of the research situation. What distinguishes countries like South Africa from other countries such as the USA, is the scale of the disruption and instability, and its location at the centre of society, involving mainstream concerns, rather than at the margins. There is an assumption about stability in research, including critical research that assumes a research situation exists in some continuous sense and its participants are present and available to the researcher, once choice, negotiation, reciprocity have been established.

The description of the research methodology for my research could be presented as a relatively normal, steady process. There is little indication of the history of the research process, the material conditions in which it was located, or the transformations that were taking place in the broader context in which the research was happening. Yet the research was marked by severe and consistent disruptions and instability with strikes and protest action. Early in the study, while working with student teachers, the university closed down and the preparation sessions —which were also disrupted— replaced substantive negotiations with teachers in schools, which resulted in a more marginal participation by class teachers in the project work. Later, in schools, during teaching practice, teachers went on strike which impacted directly on how the project was realised. For instance, one project was completely abandoned. Disruptions to carefully conceived research designs are the norm rather than the exception in educational research in South Africa, yet this important feature and its implication for research and emanating recommendations is difficult to discern in research reported in papers and dissertations. Why disruptions are produced in research, how they come to feature in the research process, and what is/can be done with disruptive data has been discussed elsewhere (Vithal, 1998). It may be argued that the stability assumption built into the research situation, in research methods and methodologies, are largely imported from the 'north', from the more 'developed' contexts, and often applied in 'developing' countries in the 'south' which is characterised by rapid and huge transformations in virtually all its institutions (Valero & Vithal, 1998). Such an assertion attempts to bring an analysis of power relations engaged within theory and practice, and within research and education, also to the inequalities in the production, trade, ownership and legitimation of the *means* by which knowledge is produced —the research methodologies themselves in mathematics education. Critical research has emerged in particular contexts, in response to a particular set of concerns and conditions, and is itself implicated within larger global inequalities. The potential for more equitable dialogue is considerable, if for instance, the disruption and instability thrown in such sharp relief in the South African context is seen to bring greater focus on similar situations in other contexts where such concerns may be masked and appear marginal. The challenge for critical researchers is to take seriously instability and discontinuity in research situations, both methodologically and theoretically, and to consider what it means to focus on unsanitised 'disruptive data' not only as a procedural matter but substantively, both practically and theoretically. The politics of knowledge production within critical research itself may be problematised through a focus on context and the concern to theorise disruption and instability in methodology (Vithal & Valero, 2003).

Emancipation, empowerment and hope

In this concluding aspect I return to a central idea of any critical education and research. What distinguishes a critical approach from other approaches to research, Carspecken and Apple (1992, p. 511) write, is that the critical researcher is deeply concerned about 'inequality and the relationship of human activity, culture, and social and political structures.' These concerns guide the questions that are posed and the nature of the inquiry in which the critical researcher acts on the world with others 'in democratic ways so that this world may change'. Critical researchers make explicit that their research agenda is not only about understanding the inequalities and injustices but also includes an openly transformative and emancipatory research agenda which means that they seek to bring some change to the participants' lives' and their contexts. Although this intention can be read in my broad research focus in the choice to select and study a particular approach to mathematics education that is concerned with empowerment and emancipation, this was not a direct or explicit goal in the research.

Nevertheless, I must address the notions of empowerment and emancipation, which have an important position in the theoretical landscape and the imagined hypothetical situation, but raise several difficulties within the arranged situation in the research. The first is that whilst a research project may claim an emancipatory intent or purpose, this cannot be predetermined. The researcher cannot know the direction, nature or content of an empowerment or emancipation nor its impact on the whole life of the participants —what happens in the arranged situation of the mathematics classroom impacts on and is in turn affected by what happens elsewhere both inside and outside the school. Empowerment in some spaces can bring disempowerment in others. Second, is the inescapability of the 'imposition of emancipation'. Someone, usually a researcher, as insider or outsider, selects and decides to involve participants in a research process. The focus on selection based on disadvantage and oppression in critical inquiry leads to the situation of someone in an inevitable position of power defining a group as such and proceeding with a research process. The third is the need for a deeper understanding of the nature of the participation of the researched in all aspects of the research process. The participants do not have the researcher's skills and knowledge and an inherent unequal situation cannot be avoided between the researcher and researched even though the researcher may act to reduce that in specific ways. This is the problem of the researcher knowing best in the research situation. A fourth problem is that of a 'once and for all transformation'. A main focus of critical research is the notion that the transformation that is hoped for occurs in the research process; but what of its sustainability or transient nature? A fifth difficulty lies in seeing the research process itself as constituting the transformation. That is, the processes and practices for studying the change do not necessarily coincide with the processes and practices for making the change. The problem is that of being involved in transforming a situation and simultaneously studying it. Finally whilst no methodology is neutral, no methodology is inherently emancipatory or positivist. Rather it depends on the theoretical assumptions built into it, the way in which it is given meaning in its use

within a research design and the researcher's theoretical leaning[3]. Problematising the aspects of empowerment and emancipation in this way is not intended to discard such notions but rather to see them as located in the situation imagined by all the participants in the research as it inspires and gives reasons and goals for thinking and acting in the arranged situation.

Rather than to speak of research carrying emancipatory intent, it may be useful to speak of research as carrying possibilities and hope, an idea also put forward in the theoretical educational landscape. Although this may be interpreted as weakening a critical approach, it serves equally to address some of the difficulties mentioned above. But the question is what can the 'principle of hope' found in a critical mathematics education (Skovsmose, 1994) mean in a critical approach to research? I was aware that student teachers would be inducted into their first major teaching experience through a radically different pedagogy. The research situation that I arranged with them and through them opened potential for change at a number of levels and in different areas —their role as student teacher, teacher of mathematics with a new approach and so on. Whatever the intent and my direct intervention, changes were difficult to anticipate in contested unstable contexts. But still, it is the hope for change, which inspires and drives initiative and effort to arrange situations. The research situation itself revealed what possibilities —if any— were created for change and what may in fact change, even if temporarily. Distinguishing an actual, arranged and an imagined situation offers a means for talking about a critical approach to research that focuses on possibilities and hope, on potentiality and actuality. Even though critical researchers may enter into the investigation with their epistemological assumptions and political agenda admitted upfront (Kincheloe & McLaren, 1994), sustaining these concerns throughout the research may be far more difficult.

CONCLUSION

These methodological challenges are not new in research discussions. But arguably they have not been sufficiently debated in mathematics education research and only beginning to emerge particularly in research works that attempt to address issues of equity, social justice, democracy and diversity (e.g., Cotton, 1998). In educational and social science research many of these methodological challenges have been raised as issues of ethics particularly in research from a positivist view; and as criteria for evaluating the quality of research in the interpretivist traditions (e.g., Lincoln, 1995) and often with reference mainly to the researcher. However, they have been reified and brought to the center of research in the critical and feminist research frames to become an integral part of research relationships, processes and practices as a whole. Moreover they are challenges that are being extended equally to the researcher as to the research participants while recognizing differentials of identity, power and voice within research. But a key point being made here is that

[3] Survey research, for instance, can be used for purposes of empowerment because even though it assumes a particular relationship between researcher and researched, it can be subverted toward more egalitarian ends (Singh & Vithal, 1999).

they are not taken up in ad hoc ways in research. Rather, researchers attempt to seek coherence and rigour in their research by establishing resonance between their theoretical and methodological frameworks on the one hand and which connects to their research focus and broader educational and socio-political context on the other.

ACKNOWLEDGEMENTS

Although I take responsibility for this chapter, I must express my appreciation to Ole Skovsmose for discussion of many of the key concepts; and to Christine Keitel and Paola Valero for reading and commenting on several drafts of this chapter.

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EPILOGUE

TALKING BACK TO MATHEMATICS EDUCATION RESEARCH

As seen in the many contributions to this book, socio-political approaches to research mathematics education have recontextualised diverse theories from different fields in the social sciences, particularly sociology and cultural studies. Mathematics education researchers have benefited from the dialogue with researchers in other areas, in the search for understanding of the conceptual tools at stake. This epilogue intends to make more visible such conversation through the presentation of Thomas Popkewitz' reading of the chapters in the book. Working daily side by side with one of the most internationally recognised groups of mathematics education research —namely the group at the University of Wisconsin at Madison, USA—, Popkewitz has been interested in keeping a track of his colleagues' scientific endeavour, in particular because his own work analysing school reform in the USA has many common threads with the work on mathematics education reform carried out by that group. In the paper 'Institutional issues in the study of school mathematics: Curriculum research' published in an early collection about mathematics education and culture (*Educational Studies in Mathematics,* volume 19), Popkewitz drew attention to the fact that a cultural understanding of school mathematics required a contextualised analysis of mathematics education in the complexity of institutional practices that constitute it at different micro and macro levels. In more recent publications such as the plenary address in the Third International Mathematics and Society Conference (MES 3) (referenced in Valero, this volume), Popkewitz has presented an analysis of mathematics education in terms of the alchemy of school subjects, and has challenged many of the assumptions at the core of both psychological-mathematical and social approaches to research in mathematics education.

In this closing chapter, Popkewitz weaves together his contribution to the theoretical landscape upon which socio-political perspectives to research in mathematics education built, and highlights the key elements brought forward by the chapters in the previous five dialogic units. His chapter addresses conceptual issues in relation to both mathematics education practices and mathematics education research practices. One central issue is that of power and the politics of knowledge. Two notions of power in relation to forms of knowing are distinguished: power as the establishment of distinctions, domination and exclusion between groups of

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participants in pedagogical practices; and power as the production of systems of reason governing people's inscription in patterns of normality. These two notions of power —similar to those identified by Valero in her discussion about the key elements of a socio-political approach in mathematics education— allow to grasp different aspects of the politics embedded in school mathematics education practices. The chapters in the book adhere to these notions in different ways and illustrate how these tools reveal particular facets of school mathematics practices.

Popkewitz also takes issue with the relationship between mathematics —the field of practice of professional mathematicians— and school mathematics —the field of practice emerging from the alchemy of the former into a field of school practice. In contrast with notions such as that of 'didactic transposition' by Guy Brousseau, the alchemy of school subjects emphasises the inseparability of (mathematical or school mathematical) 'knowledge' from the systems of relationships, meanings and actions that give place to it. School mathematics is more than a transformation of 'professional mathematical knowledge'; it is a completely different space of action, ruled by principles that may have a similarity in content with some areas of mathematics, but that definitely constitutes a distinct social arena. A consequence of these considerations for mathematics education research is a radical redefinition of the objects of study tackled in—and simultaneously constructed by— research.

This last remark leads to a third issue in Popkewitz' paper: Research is implicated in the construction of the very same 'phenomena' or 'facts' that it intends to study. In other words, theories and methodologies do not have an independent existence from the field of 'practice'; rather, they play a direct and important part in the creation of categories, distinctions and legitimate forms of action that operate in practice. Such a contention has at least two implications. On the one hand, the distinction between 'mathematics education practices' and 'mathematics education research', to which different papers in this book have referred to, is an analytical tool rather than a fundamental division between two fields of practice. They are much more connected than what has been suggested by the classical debates about the separation between 'theory' and 'practice'. On the other hand, and as a result of the previous implication, seeing research as a constructor of discursive practices evidences the very political nature of the research endeavour. This is one of the justifications for the need of including a serious and systematic reflection about the researcher's (deliberate or unintended) participation in the shaping of the social space under investigation.

With these discussions, this final chapter rounds the key points that this book intends to raise. Going back to the introduction, the editors were concerned about tackling mathematics education research from socio-political perspectives at the level of the significance of such approach for the study of practices in schools and other institutional settings, but, more important, at the level of its implications for the very same endeavour of doing research. More precisely, the book was organised as a dialogue among authors around the issues arising when doing socio-political research in mathematics education. Popkewitz' contribution eloquently rounds up the conversation and highlights some of the essential conclusions of an examination of mathematics education with theoretical and methodological concerns for issues of power.

THOMAS POPKEWITZ

SCHOOL SUBJECTS, THE POLITICS OF KNOWLEDGE, AND THE PROJECTS OF INTELLECTUALS IN CHANGE

The study of school subjects has shown little interest in the culture, sociology or history of schooling to understand classroom practices. The focus has been didactics or the pedagogical content of teaching. Its organising principles relate to psychological studies of learning and instruction. The dominating question of research has been the effective learning environments related to children's mental processes and a social psychology of children's problem-solving, often talked about as 'situated learning'. This book offers a contrasting and important systematic attempt to bring the scholarship on teaching schools subject within the preview of social, anthropological and political theories. In doing so, it extends and revises a tradition that took the knowledge of teaching and curriculum as a strategy to explore the ways schooling produces and reproduces exclusion related to class, race, and later, issues of gender in the USA and Britain.[1] Further, in a world of research that focuses on microanalysis and empiricism, the contributors to this book continually move between empirical studies of mathematics classrooms and interdisciplinary discussions about assumptions, implications and consequences of teaching and the sciences that explain classrooms.

The choice of mathematics education in this conversation is important. Mathematics is one of the high priests of modernity. Mathematics education carries a salvation narrative of progress into the upbringing practices of schooling. The mathematics of the school is told as a story of progress about the cultured, modern individual whose reason is bounded by the rules and standards of science and mathematics. The narrative is of the enlightened citizen who contributes to the global knowledge society. But mathematics is not only the story of culture but of social progress through equity. The story told is that the proper teaching of

[1] Iconic representations of the history of 'curriculum theory' in this tradition that treat the general formation of the knowledge of schooling are Durkheim (1974), Franklin (1987), Hamilton (1989) Kliebard (1987), and Lundgren (1983); and Young's anthology (1971) that was instrumental in bringing Bernstein and Bourdieu into the literature concerned with the rethinking of the effects of the curriculum.

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mathematics will prevent any future joining of the ranks that deviate from the norm. The chapters of this book engage the ambiguities, anxieties and anomalies of the tales of progress and equity.

My discussion considers the contributions and debates opened through the dialogue of this book. I proceed in the following way. I first consider the general issue of the politics of knowledge in teaching school subjects. Second, the particular politics of mathematics education as a movement from the space of academic mathematics into the social space of schooling is explored, in what I call the alchemy of school subjects. In the third section, two different problematics of change are discussed as organising principles in pedagogical research. The notion of problematisation gives attention to the intellectual traditions in the studies of school subject matter as embodying ordering and governing practices. One is called the equity problematic and the other, the knowledge problematic. The fourth section pursues the politics of research produced through the methodologies of research raised in this book. I conclude with some reflections on the politics of educational research as it relates to change and progress. These foci are drawn from my reading of the contributions in this volume and intend to engage in a conversation about the paths taken.

TEACHING SCHOOL SUBJECTS AND THE POLITICS OF KNOWLEDGE

One notion that travels through today's discussions and the papers of this book is that of mathematical literacy. The concept of literacy, if I can be schematic, places mathematics within issues of progress and inclusion. It is assumed that those who have mathematics knowledge —literacy— are better able to work in modern society, to participate more fully as citizens, and, importantly to engage in critical analyses of social practices. The notion of mathematical literacy assumes a major trajectory of contemporary social thought about modernity. That trajectory of modernity is about the mind as having a critical understanding and competence in mathematics as it functions in the world. From organising one's checking account to understanding the statistical notions of populations in social policy, mathematical skills are believed to be central knowledge components of the modern citizen. But this assumption of mathematical literacy has disenchantments and anxieties that circulate in the book. The anxieties relate to the production of social exclusion as a consequence of inadequate instruction. The study of pedagogical practices, therefore, is to illuminate the practices that divide social class, ethnicity, and gender embodied in the everyday classroom practices and the methods of research.

This concern with the production of the mathematical literacy of the citizen can be contrasted with the notion of equity in most contemporary studies of didactics. Didactics research in school subjects, for example, brings psychologies of the mind and problem-solving into the instructional equation. Research identifies how best to organise content and instructional strategies to produce better learning and, at the same time, to offer remediation for those who have not adequately succeeded. These approaches to didactics research misrecognise, to borrow from Bourdieu, the ways in which the particular distinctions of pedagogical content knowledge embody

norms, distinctions and divisions that qualify and disqualify individuals for action and participation.[2] One can read the studies in this book as directing attention to existing traditions of didactics research as part of the problem of understanding the complexities and nuances of equity as an issue of social exclusion.

The ideas of the French sociologist Bourdieu, the British educational sociologist Bernstein, and to a lesser extent, the French historian/philosopher Foucault, are theoretical sites through which the study of the classroom is revisioned. It is not surprising that Bourdieu and Bernstein are so heavily utilised in these studies. They are two major social scientists whose work traverses the fields of education and general sociology. This scholarship can be related, in part, to what has been called 'the linguistic turns' in the social sciences. This entails a consideration of language and discourse as a material practice. They are material in the sense that language both construes and constructs our sense of *self* in the world through the generation of principles to order action and participation. In particular, each sociologist gives attention to pedagogy as embodying norms, distinctions and classification systems that relate to social differentiations and divisions. I will not summarise the theoretical notions that organise their work nor how they are deployed to have an impact on educational research. Rather, I would like to point to some of the provocative interventions that weave this sociological literature into methods of investigation in mathematics education.

First, there is a recognition that pedagogical strategies are regulative. Regulation relates to the ways in which the patterns of interaction and discourses of pedagogy govern who the child and teacher are and should be. But this regulation as governing has different intellectual trajectories. Bernstein's regulative function of pedagogy is one prominent classification. Pedagogy embodies invisible cultural norms that are inscribed in the selection and organisation of school knowledge. The significance of the cultural norms is that they presuppose a moral order about who the child is. Notions of development, children's 'work habits', and 'abilities' embody a continuum of values about the child who is 'reasonable'.

The distinctions about the characteristics of the child are cultural practices. I call them cultural practices to direct attention to the construction of the performances in the classroom that structure and normalise who the child is and should become. Knowledge or the system of pedagogical reason is a central 'social fact' of inquiry. For example, there is a seemingly naturalness and universality of the concepts of pedagogy and psychology in teaching. The universality is expressed in today's reform as *all* children are to learn! But in fact the distinctions of learning and participation differentiate among students from different cultural and social backgrounds. The politics of school knowledge, as argued in the various chapters, occur in deploying particular norms and distinctions as universal norms. The universalising of cultural norms of particular groups in schooling becomes exclusionary as some children's behaviours and inner characteristics do not match the taken-for-granted norms.

Implicit in the argument is a notion of power. Certain groups dominate through their ability to insert their sensibilities and dispositions as the legitimate knowledge

[2] A counter example, in addition to the contributors to this volume, is Boaler (2000).

of the classroom. Inequities are explained through a sovereign notion of power. Certain social groups dominate and repress other groups through processes of cultural production that are embodied in education. The practices of teaching insert particular cultural and social norms and values as legitimate. The consequence is to limit participation and to produce patterns of marginalisation. Policy and most research about diversity, cultural pluralism and a Eurocentric curriculum evolve around this conception of power (for a review, see Popkewitz & Lindblad, 2000).

A complementary notion of regulation directs attention to the productive side of power.[3] Regulation relates to the *mentalities* or systems of reason through which action and participation are ordered and judged. Foucault's (1979) notion of governmentality, for example, involves a way to think about the principles of reason as an inscription device. Power is not so much a negative power that imposes constrains upon the citizen but one that disciplines the rules of conduct and the boundaries in which existence is to be acted upon. The notion of productive power focuses on the classifications and distinctions of schooling as governing what actions are thought reasonable and who is made into the 'reasonable person'. The distinctions and differentiations create 'maps' that embody the possibilities of alternatives and the paths of progress. To place the child into a 'map' of child development and adolescence, for example, imposes historically constructed notions of ordered stages of child development. This particular way of 'seeing' and thinking performs as a boundary to intern and enclose thinking about what is but also what can be.

But this productive power of knowledge also inscribes a continuum of values about the 'being' of the individual that places some individuals outside of the boundaries. In this sense of power, the system of reason in pedagogy produces the anthropological 'Other' that is talked about in discussions of exclusion. The productive notion of power is brought to bear when Hardy suggests (Cotton & Hardy, this volume) that 'power masquerades as common sense' that leaves participants unaware of the effects of their practices.

These different traditions often overlap in the study of the 'regulation' of the pedagogical practices in generating principles to qualify and disqualify students for participation. Lerman and Zevenbergen argue, for example, that the discursive productions of the classroom embody differing notions of competence among children of different social class even when the same overt behaviours are exhibited. This is produced through different systems of recognition and norms about student competence that are inscribed in the interaction patterns and questioning of the classroom. Similar classroom performances are categorised differentially when examined in relation to social class. Gorgorió, Planas and Bishop's chapter further this discussion of the cultural assumptions that organise the teaching of mathematics by considering Catalonian schools. The seemingly benevolent and progressive reform that emphasises classroom collaboration —i.e., team work— and non-

[3] I use the notion of complementary to recognise the importance of both notions of power in contemporary analysis. The difficulty of contemporary social theory is in rethinking the analytical divides between the two notions of power through the historicity of concepts and interpretative tools. See, e.g., Popkewitz and Brennan (1998); Popkewitz, Franklin and Pereyra (2002).

gendered education involve a clash of cultural values when dealing with Moslem girls in Catalonian schools. The classroom norms that differentiate the 'good' and 'deficit' student embody dominant cultural and political values that place immigrant children as deviant.

These examples of the cultural norms that intersect with mathematics in teaching bring me to the second intervention brought through the book's discussions. Mathematical literacy is not related to some universal notion of knowing mathematics but it embodies particular cultural inscriptions that produce practices of inclusion and exclusion. This is not the same as arguing that inner logical arguments and processes of mathematics are culturally specific. Rather the cultural dimensions are apparent once mathematics is made into a field of a 'literacy', a concept that inscribes normative notions of 'the civilised'. When the conventional academic curriculum is juxtaposed to communal situations that use mathematical notions, it is apparent that the universal knowledge of mathematics is related to a particular type of expertise. Comparative studies of ethnomathematics and the cultural studies of the classroom in Knijnik, Vithal, Meaney, and Gorgorió, Planas and Bishop's chapters, for example, raise questions about the ordering procedures of mathematics education in different cultural and social contexts. The influence of the World Bank in shaping national mathematics curriculum in Brazilian school reform efforts is discussed in Knijnik's chapter. The policy of the World Bank brought into Brazilian schools a particular logic and form that shaped the school content and the pedagogical practices of mathematics education. Knijnik then explores alternative conceptions of mathematics through her work with the Brazilian Landless People Movement to identify different ways to ground the teaching of mathematics in liberation projects.

There is a tension in the portrayal of non-academic ways of thinking about mathematics, however. The portrayal is to show a relation of 'community' uses of numbers to the standards and rules of the school mathematics curriculum. The discussion in ethnomathematics is to keep a balance between community 'uses' and academic knowledge. But the 'logic' of the argument is historically bounded to a particular trajectory of modernisation. By that, I mean that the formulations of mathematics in the curriculum embody a particular way of thinking, ordering and classifying the world that is historically associated with European modernity. That modernity can be thought of as embodying a particular individuality that calculates, plans and assesses one's life according to rational practices. That rationality of a calculated 'self' is a 19th century invention and involves, among others, giving an individual an ordered development of a past, present and future that can be scrutinised and administered (see, e.g., Steedman, 1995). Classifying and ordering children's development of problem-solving abilities is one example. The interior of the mind is made into a calculating relation between past/present/future. Modern pedagogies of mathematics inscribe and configure this calculated mind as the problem of governing pedagogy.

By placing ethnomathematics within a cultural context of modernity, the project of ethnomathematics in recognising non-academic systems of numbers has a duality. The bringing of the different community 'uses' of mathematics into the discourses of pedagogy may function to translate and transport 'native' thinking into the

modernising grammar of schooling. This may occur even while seeking to preserve the community's systems of relevancies. The grammar of schools reconceptualises and calculates the ordering of the child into some path that leads from the present to the future. The essays by Knijnik, Vithal, Meaney, and Gorgorió Planas and Bishop, for example, can be read as offering a caution about modernising projects through focusing on different populations and cultural patterns that stand outside that project. The essays do not necessarily reject the modernisation of the mind, but provide ways to understand its processes and systems of reason comparatively.

The regulatory practice of mathematics education suggests that the universalising language of mathematics is not only about teaching of mathematics. It is also a particular knowledge made plausible and reasonable as part of a particular historical ordering of the world associated with modernity and its current mutations. The classification and ordering system of mathematics are not only 'things' to learn to apply to the world. The 'logics' of mathematics are made plausible within a particular system of reason associated with modernity. As such, it is embodied in an amalgamation of practices through which the world is thought about, seen, talked, and felt. While mathematics in mathematics education is often seen as immune from such considerations of the social and cultural conditions of modernity, the essays illuminate how the universalising language of mathematics gives meaning and differentially orders who the child (and teacher) is through the cultural systems of the classroom.

THE SCHOOL ALCHEMY AND MATHEMATICS

The comparisons of academic mathematics and ethnomathematics produce further considerations in the study of schooling. The classroom studies of the book raise questions about equity through examining how identities and differences are constructed through pedagogy. But the studies leave unproblematic the alchemy of school subjects that translates academic mathematics into the social space of schooling (Popkewitz, 2002). The alchemy of school subjects has a double sense. There are the mathematical representations that serve as devices that students are to use to constitute the social realities for action. Mathematics becomes a tool for students to solve and evaluate problems of their world. And mathematics is a tool in which pedagogy is deployed to construct the child and teacher as historical 'subjects' who act on those representations. The child learning styles, memory, problem-solving skills, understandings are observed and calculated along a continuum of values that signify children's achievement and competence.

The alchemy provides a way to understand this double process of translation and transmogrification of mathematics into mathematics education. It is analogous to reading a text and thus to the cultural implications of literacy itself. To understand how words are used and their meaning, it is not sufficient to look only at a dictionary. The substantive qualities given to words are determined through their deployments in a text and how they relate to other words and thoughts being expressed. But reading a word also requires looking outside of the text to understand what makes the word and text possible in the first place. As a word in a text, the

curriculum uses the symbols and notation systems of mathematics as a reference of affiliation. But the 'things' of mathematics are moved into the social spaces of schooling and reformulated into a new amalgamation of cultural practices and relations from which to understand how the word 'mathematics' is constructed. That new space of the school 'text' involves, for example, the weaving together of the symbols of mathematics with those of class timetables, notions of achievement and pedagogical organisation, theories of childhood, and 'standards' of instruction.
Each social space carries the same name (mathematics) and even overlaps with some of the same 'content'. But the two spaces are different and embody different cultural practices. The power of the transference system into pedagogy, however, is taken as naturally the same as that of mathematics. All involved in the school space find it easy to think, believe and act as if the two social spaces are in fact the same. But they are not! And the question of schooling is, I believe, to understand how pedagogical practices remake and translate the field of mathematics practices into a text that is different and which 'belongs' to the realisations of schooling.

A Museum exhibit called *Art and Primitivism* can help to illuminate the alchemic features of schooling. The *Art and Primitivism* exhibit was to show the influence of African images on modern art by juxtaposing the painting of Picasso, among other artists, with African masks and other cultural objects. The African objects stood on museum pedestals for the spectators to look at and marvel about the lines, textures, and design perspectives of the masks that were thought to inspire the European modernists. But to move the African objects into the museum reclassifies and recontextualises the objects into something different. While appearing as African masks, the masks no longer functioned in relation to the myths and everyday life in which they were fabricated. The masks are narratives through a particular museum space of an 'art' world that re-visions them through modernist discourses of 'culture', aesthetics, and commodification. So it is with mathematics as it stands in the spaces of mathematics education. There is a movement from one social space to another. The alchemy, however, is not necessarily bad and to be prevented. Schooling is a particular place created for children and not the production spaces that produce academic mathematics. My concern is with the particular alchemy that occurs in the transportation and translation to schooling.

Why is the alchemy important? How might it help to rethink mathematics and the study of mathematics education? In certain ways, the studies in this book provide a partial answer. School subjects are not 'merely to teach a particular academic knowledge but functions to differentiate, divide, qualify and disqualify the child for participation. But the alchemy directs attention to the doubleness of the construction of school subjects. First, as the studies of this book illustrate, mathematics is embedded in an amalgamation of cultural practice that produces exclusion through the principles of action and participation. Pedagogy is a normalising practice, I have argued elsewhere, directed to governing the soul, albeit a secular one (Popkewitz, 1991, 1998). Psychology and social psychology are translation tools of pedagogy. The normalising practices of pedagogy further embody displacements and principles for qualifying and disqualifying, and for inclusion and exclusions of individuals for participation and action. The exclusions occur as pedagogy maps of the child through the distinctions that inscribe the characteristics of those who stand outside

of normality and thus who can never be 'of the average' (Popkewitz, 1998b). The interactions patterns and the notions of child development and learning that circulate in classrooms embody racial, gender, and class distinctions (see, e.g., Lesko, 2001).

Second, the alchemy directs attention to the normalisation of the child as related to the representation of mathematical knowledge and the image of expertise brought into the school. Thus, the alchemy not only produces principles for action, but also what constitutes the methods and spaces for action. To accomplish the governing of the soul, mathematics education stabilises and crystallises mathematical knowledge to enable the governing of the child. That is, particular iconic images of expertise are brought into the curriculum that may in fact produce new boundaries to lessen the scope in which participation can occur.

Let me explore this briefly. What constitutes mathematics in mathematics education tends to relate to a particular subfield of scientific mathematics. The curriculum reforms have oblique references to a 'scientific mathematics' that is given legitimacy in the USA through boards of 'scientific mathematics'. While I am not sure at this point what this designation means for the selection of the school subject matter, it may refer to the distinction between applied or applicable mathematics as opposed to mathematics 'for its own sake'.[4] If I compare this designation of 'scientific mathematics' to the formation of the social sciences curriculum, certain anomalies and effects of power are inscribed. In the name of the foundation of the discipline of political science, for example, high school courses were developed in the 1970s that drew on American behavioural political science (see, Popkewitz, 1977). This designated a particular intellectual tradition within political science as the totality of the field of scholarship. It ignored the different systems of reason or paradigms and the debates about who —and what paradigms— are authorised to speak for the discipline. The social studies curriculum crystallised the field with this selection without acknowledging that behavioural political science was only one —although the dominant— tradition of inquiry in that discipline.

The political science curriculum may provide an analogy to mathematics education. Does the selection of a mathematical science obscure different traditions and networks of associations in the field of mathematics? And if so, by making a 'scientific mathematics' as the designated 'hitter' of mathematics education, does this crystallise and fix the notions of mathematics in mathematics education? What are, if any, the consequences of selecting a particular content of a field as its total knowledge —as when behavioural political science was made into the totality of political science? What is left out? How do such selection processes relate to the governing of the child in pedagogy? What becomes not possible for children to know in their engagements with mathematics?

I do not have the answers to these questions, but raise them to give attention to what constitutes disciplinary knowledge in the curriculum, its iconic expressions of expertise, and the uses and limitations of scientific knowledge. Yet there are some partial answers to these questions in Hacking's (2002) discussion of science and mathematics as styles of reason. Hacking argues that mathematics embodies, for example, differing ways of thinking about and creating new objects if one compares

[4] I appreciate Sal Restivo for making this distinction as I thought about this issue.

algorithmic and combinatorial styles of reasoning, on one hand, and spatial reasoning such as geometrical, topological or symmetries on the other hand (p. 2). The different styles of reason function as 'self-authenticating'. That is, each style 'introduces its own criteria of proof and demonstration, and that it determines the truth conditions appropriate to the domains to which it can be applied' (p. 4). Thinking of mathematics in this way directs attention to its practices as more than 'a group of techniques for bringing new kinds of facts to our awareness' (p. 4). The styles of reasoning are ways of finding out the truth that work in an immense world of cultural practices, institutions, authority relations, 'connotations, stories, analogies, memories, fantasies' (p. 9). Approaching science and mathematics as systems of reason that construct its objects and truth statements, Hacking argues, is a way out of the controversies that divide philosophy and education into realist/anti-realist camps about truth and into divisions of subjectivist and objectivist worlds. The different styles of reasoning introduce different registers of debates about the ontological status of the objects 'seen' as true.

For my purpose of thinking about the alchemy of mathematics education, each style of reason in mathematics opens up different objects for scrutiny and provide classificatory schemes by which lives are experienced, truth authenticated, and futures chosen. Thus, when curriculum research examines classroom interactions without examining the domains of objects produced by the styles of reason in mathematics, a particular irony is created. This irony is found in today's reforms that emphasise children's participation in the social construction of knowledge, problem-solving, and collaborative learning. Yet that social construction and collaboration takes-for-granted the ontological status of objects given by the mathematical reason in the curriculum. Participation and agency occur within a restricted and narrow domain of objects as only one style of thought in mathematics is taken for the whole of the world under description. The irony of the alchemy of school subjects is that there is more and more participation for children in a world closed off with less and less alternatives for the opening of new objects for children to scrutinise about their experiences and the styles of reasoning that work on us in determining futures.

GOVERNING, SOCIAL INCLUSION/EXCLUSION AND THE PROBLEMATISATIONS OF EDUCATIONAL RESEARCH

At this point, I want to focus on a discussion posed in the chapter by Cotton and Hardy. In that chapter, they consider the cultural issues of theoretical 'toolkits' as they address issues of equity and inclusion. They compare the different ways that problems, concepts and methods are expressed in a *social justice* and a *Foucauldian knowledge* perspective. I want to pursue the issues that they raise through thinking about the *problematics* of research. My concern is how intellectual traditions of research construct ways of thinking and ordering action, conceive of results, and intern and enclose the possibilities imagined. I draw on two analytical schemes of the *equity problematic* and the *knowledge problematic* that I worked on with S. Lindblad, as we researched the relation of educational governing and social inclusion and exclusion in the European Union (Popkewitz & Lindblad, 2000).

One intellectual tradition of research can be considered as an *equity problematic*. The equity problematic makes the locus of change as the actor who enacts cultural interests. The actor can be structural, such as in the categories of race, class and gender. Or equity can be related to the phenomenological actor whose 'voice' is studied to consider the meanings, motives and intentions of individuals or groups. The question of equity and social exclusion in the equity problematic focuses on who is represented or marginalised in school decision-making and pedagogical processes. The equity problematic is one that maintains a notion of social justice that moves with different set of principles in liberal and Marxist theories.

Governance is a concept to think about 'the means by which an activity or ensemble of activities is controlled or directed, such that it delivers an acceptable range of outcomes according to some established social standard' (Hirst, 1997). Governance evolves around the means to eliminate —at least theoretically— the exclusion of targeted groups of social actors through inclusionary practices, such as those defined by class, gender, race or ethnicity. The equity problematic is embedded in Chronaki's concern with the negative effects of settings related to gender and social class, Ernest's interest in bottom-up participation, and Atweh's discussion of action research as a participatory mechanism.

The knowledge problematic is concerned with the ways in which the subject of research and policy are made into objects of scrutiny and administration. Knowledge, as discussed above, is a practice that orders the world to be known through its systems of classification, differentiation, and division. In this sense, knowledge is a social practice that structures the field of possible action by rendering conduct calculable through inscribing the principles of performance and the modes of subjectification, that is, the rules through which individuals conduct themselves as responsible, self-motivated and competent (Dean, 1995). Whereas the equity problematic focuses on the origins of power, the knowledge problematic concern is with the effects of power through the very 'reason' applied to constitute problem solving. In contrast to the equity problematic focus on the representation of groups or individuals, the knowledge problematic is concerned with the historically embedded rules that produce the subjects as rendered for supervision and action.[5]

Knowledge is then not 'merely' representative of human intention and purpose but produces them through its governing system. Embodied in the developmental norms of the child, notions of learning and achievement, for example, are a continuum of values. These values are about the inner characteristics of the child. They also embody divisions in which the 'Other' resides as those characteristics and capabilities that lie outside of the system of reason and the reasonable person. The system of knowledge excludes through the classifying and differentiating the child who does not embody the characteristics inscribed to act 'reasonably'.

At this point, it is possible to think about the notion of resistance that crosses the knowledge problematic. Resistance, to draw on Foucault (1991, p. 75), problematises the taken-for-grantedness of the constructions of the world, of the

[5] The distinctions are elaborated in Popkewitz (1998a); Popkewitz and Brennan (1998); and Popkewitz, Franklin and Pereyra (2001).

> [...] matter of shaking this false self-evidence, of demonstrating its precariousness, of making visible not its arbitrariness, but its complex interconnection with a multiplicity of historical processes, many of them of recent data.

The two problematics are analytically separated, but I recognise that they sometimes overlap as is evident in many of the essays of this book. But for my purposes here, I want to pursue how the two problematics pose different notions of the politics of schooling, issues of governance, social inclusion and exclusion, and change. The equity problematic studies the access and participation of the categories of groups or populations, typically classified through categories of race, class, and gender. Most educational research moves within this problematic of equity as inclusion. The knowledge problematic considers the construction of the 'qualities' that distinguish and differentiate the individual *being* for action and participation. It is not race, gender, or class that is the central concern of research, but the production of the race-ness, gender-ness, or class-ness of individuality. This problematic focuses on how pedagogical 'reason' constructs subjectivities by normalising certain characteristics and capabilities of the individual. The normalisations are embodied in the qualities of *being* or inner qualities and capabilities of the individual. The normalisation is expressed in the common sense of teaching mathematics when, for example, talking about the dispositions associated with a child's 'self-esteem', the 'stages of child development', or the cognitive, rational characteristics of a 'learner'.

Further, the knowledge problematic places exclusion continually against the background of something simultaneously included. The problem of governance is not, as in the equity problematic, to develop appropriate organisational processes or institutional practices for access and participation. While this politics of representation and access is important, attention is given to the distinctions, differentiations and divisions —the principles of 'reason'— that order participation and its principles that qualify and disqualify individuals. In certain ways, the theories of Bernstein and Foucault, with different epistemological principles, privilege knowledge as a material practice in the construction of distinctions and divisions in schooling.

This brings the argument back to the relation of methods, theories and data. The systems of 'reason' in the methods of science can be viewed as 'maps' that not only 'tell' what is inside 'reason' —e.g., the stages of child development—, but by omission, what characteristics of the child are not included and thus are outside of reason itself and excluded through the ordering and differentiating devices of pedagogy. This is one of the points raised by Skovsmose and Borba when they discuss issues of *what is not there.*

This relation of knowledge to power provides a different way to think about and explore the politics of change in the governance of education and inclusion/exclusion (see, Popkewitz, 1997). The equity problematic focuses on the strategies that deny representation; the knowledge problematic gives attention to the images and narratives that produce the principles of action and participation of the actor. In the equity problematic, resistance tends to be structural and concerned with identifying and struggling against the repressive relations. The knowledge

problematic directs attention to resistance as the diagnosing of the principles that arrange, intern and enclose action and its possibilities. Each problematic, I believe, has importance to the issue of power and one is not necessarily superior to the other. As the chapter by Cotton and Hardy pursue in comparing two different approaches, the equity and knowledge problematics deal with different intellectual trajectories and the politics of schooling that can, at points, overlap but also situate the politics of schooling differently.

CONSTRUCTING THE SUBJECT: METHODOLOGIES AND THE POLITICS OF KNOWLEDGE

On my campus, there are many 'research courses' for graduate students to learn the methods of the education sciences. The courses are classified as methods of statistics —quantitative— and qualitative research. The well-rounded student, it is assumed, knows both methods. But lost in the classifications are technical distinctions about data collection procedures that have little to do with methods. The history and philosophy of science continually bring to our attention that methods are a complex set of arrangements that bring questions of curiosity into a relation with empirical phenomena. Methods are an amalgamation of practices that relate particular ways of asking questions in intellectual schools with theories and conceptualisations as these overlap with the techniques of data collection. This broader notion of methods was illustrated years ago by Ruesch and Bateson (1968). They write that when the fancy words of data collection are taken away, modern social science has only four ways to investigate its human conditions. It can read what people say about their culture; talk to its members —through a survey or an interview—; observe the culture as a neutral observer; or act as a participant observer. These strategies are given value through the theories that make the phenomena of the world into data for inquiry. There is no data until one asks a question to place the events of the empirical world into orderable 'objects' that can then be interpreted.

While methodological discussions often focus on empirical-analytic approaches to social science, such an epistemological reflectivity is also necessary with 'qualitative' research. Kallós (1981) argued that qualitative studies is a technical term that needs to be considered by the ordering procedures through which theoretical concerns are worked through in relation to empirical phenomena. The re-emergence of qualitative methods in the USA social and educational sciences during the 1970s, for example, was methodologically argued as a research strategy to get closer to the *real* world through exploring the meanings people gave to events. But the method responded to and was part of cultural changes that were not simply the efforts to find better research methods. Qualitative methods symbolically construct the classroom as *a community* where values and identity are negotiated in the process of forming collective identities. The research methods embodiment of the notion of *community,* however, was a response to a context of a perceived breakdown in social cohesion and participation of that period (Popkewitz, 1981).

But the methods of research develop their own existence in ways that the very methods and techniques of research construct its subject. The history of social

science and the sociology of knowledge have continually probed how methodological approaches construct its objects of study by ordering what is taken as the 'natural' realities of social and personal life. Danziger (1990), for example, explores the history of psychology to understand how the particular methodological interventions of psychology have determined what and how things are studied and understood. The material consequences of methods as a construction of the subject are evident in the discussion of Wiliam, Bartholomew and Reay. They argue that methods of measurement and evaluation create the subject through the methods applied. The testing apparatus of achievement, for example, builds in differences so as to make comparisons to determine measurement reliability. But this methodological device of testing also constructs what they purport to assess. Validity is an internal device of the test, built on the property of the inferences developed through the test results. The measurement technology involves a hierarchical model that set students of different abilities apart, define the subject taught in schools, and fundamentally change students' experiences of mathematics. The changes in the objects of study had little to do with the overt theories of psychology yet they influence the possibilities of conceptualisation and also how the subject investigated is understood and theorised.

But there is more to the story of methods. The measurement devices are inscription devices that normalise through their methods. Probability theories and populational grouping shape and fashion these techniques in a manner that have implications to whom we are and are to become —for a discussion of statistical reasoning as a normalising practice, see Hacking (1990). Hacking (1995) discusses the populational ordering and classifying procedures of science as constructing human kinds, that is, inventories or profiles of people that can be managed. The achievement levels of schooling provide a determinate classification that has succinct chronological, physiological and psychological characteristics that can be applied to many people. Curriculum practices promote the child who attains mastery of the categories. Programs of remediation and assessment are created that place the child into the social space of 'the human kind' that does not learn. Research programs, for example, describe the achievement levels of the child according to the categories of the national curriculum and what kind of child is 'normal' and outside of normality. The aggregates acquire the abstraction of the sciences or impersonal management to reason about the capabilities and capacities of people (see, e.g., Hacking, 1995, p. 354). But the aggregates also construct biographies of the individual placed in the populational groups. Profiles are 'built' of the 'good' and 'failing' child through the digest of personal facts. These biographies divide and normalise different kinds of people and make them individually governable.

The making of the subject through the methods of research is important both to the sciences of education and policy. State categories are often taken-for-granted in the development of curriculum and research. For example, Cotton and Hardy discuss the particular ways in which the 1995 UK National Curriculum for Mathematics divides mathematics education into four categories and eight levels of attainment. The arbitrary character of the attainment categories remains invisible in evaluation strategies of learning. The levels of attainment are thought as the real achievement of children and the assessment of teaching performance.

This discussion is not to argue that there is no 'real' world outside its discursive constructions. Rather it recognises that science fabricates data through how the empirical world is ordered and made understandable for interpretation. Vithal's discussion of research methods, for example, argues that researchers need a reflexivity that goes beyond discussing the technical procedures for collecting data. Meaney's chapter on Mäori education, as well, brings the political implications embodied in methodological choices into sharp focus. Her concern is to understand and be 'true' to Mäori culture in the study of mathematics education. But the cultural rules of the community are continually transported and translated into the narratives and images of an academic language. The strategies of action research and ethnographic methods, for example, are to 'record' the actual statements of members of the community. The placement of those statements into a research narrative is a retelling of the story. The grammar and syntax is of the 'Other'. In this case, the 'Other' is multiple and includes the discourses of the mathematics, the anthropology community, and a Western redemptive strand in science about the 'improvement' of the situations of those studied. Meaney recognises the continual dilemma of a human science as paths concerned with progress and a (re)construction of experience about 'civilising'.

The relation of methods and techniques places into the forefront the question of 'the case'. It is often assumed that the case is the event studied —the classroom observed or the reform studied. But the Skovsmose and Borba chapter raises the theoretical issue that what constitutes the case is a profound and important question of the research design. They recognise that the case defined by the event may be the weakest notion of case, such as whether a reform program works or the efficacy of one teaching method over another. If I return to the notion of mathematics literacy discussed earlier, is the case to make classroom interactions faithful to the cultural practices of the mathematics community? Or is the case about the position of mathematics instruction in relation to the modernisation of the mind? Or, is mathematics instruction a particular strategy of differentiation and division that relates to social production and reproduction? Depending on how the question of the case is theoretically formed, the constitution of data is determined.

The formulation of the 'case' also brings back the construction of the 'other'. By raising the question of what is 'seen' but also what is not there, Skovsmose and Borba mark out a terrain through which the 'case' of mathematics education related to inclusion and exclusion can be pursued. This notion of what constitutes the case is also in Chronaki's chapter, for example, when she focuses on the insider/outsider debate in research to help illustrate how the methodological strategies of research and classroom relations among participants embody distinctions and hierarchies that inscribe cultural (national) differences as well as those that relate to gender.

The reflections on the methods of research draw attention to the complex set of relations between intellectual traditions, methods and the overt outcomes and interpretations of research. The very practices of ordering and classifying data are inscription devises that construct the subjects as an effect of power. In *The power elite,* C.W. Mills argued that there is a preoccupation in the social sciences with studying the powerless rather than the rules of power. This can also be said in education as it relates to methodological reflections. Science is a strategy that

embodies normalcy and deviance. It targets the saving of populations by preventing their fall from grace (Popkewitz, 1998a). Yet the methodological devices of salvation embody distinctions and divisions that fabricate classificatory maps that place the child to-be-saved as outside of normalcy. While the distinctions of deviance are often told in the history of testing and measurement, the studies of mathematics education make apparent the boundaries of normality/abnormality that travel in the very transactions practices of research as it lacks a reflexivity about its own epistemology.

The above discussion raises a question about a central theme in contemporary research, that is, the division between theory and practice. The frame of reference or system of reason that performs the alchemy is not a realm of talk but fabrications that both construes and constructs principles of action and participation. The reflection on the frames of reference is philosophically embodied in Ernest's discussion of postmodernism and mathematics education. The phrase postmodern covers a broad and contested terrain about knowledge as a cultural practice. It is at a broad level a series of theoretical positions about the construction of subjectivity and historicity in contemporary social science and social life. For some, post-modernism is a way to think about program, children, and classrooms. The philosophical and social/political theories of postmodernity do relate to and challenge the legacy of the critical traditions of the sociology of knowledge and the Frankfurt School of social theory in developing a theory of practice and change (see, e.g., Popkewitz, 2001). A core assumption is that power is exercised less through brute force and more through the systems of knowledge or reason in which the objects of schooling are made comprehensible and capable for action, one that travels through this book and its different traditions of narrating the school. If I take Foucault's notion of resistance discussed above, then there is also a need to rethink programmatic implications of postmodern theories to the practices of schooling without re-inscription of the very modernist projects that postmodernity seeks to replace.

CHANGE AND THE PROJECTS OF INTELLECTUALS

The role of the intellectual in the processes of change is a central issue of the politics of educational research. From the 19th century, science and disciplinary knowledge have provided increasing authoritative knowledge about the organisation of society and individuality. The welfare state is an invention that assumes an expertise of science in state planning and coordination. The assumption is that science can provide the administrative insights into social planning that can produce social and economic progress. In significant ways, the argument of action research posed by Skovsmose and Borba pursues an alternative to this 19th century assumption that links the state with social science. They reject the role of the researcher as a political agent in the sense of having an *a priori* role in deciding what transformations are needed. The researcher, they argue, is one whose studies should instigate and be part of the conversations about future development. Rejecting the intellectual as legislator, they argue that there is no correspondence of concerns of critical mathematics education and changes in the classroom. The purpose of critical

research is to point out that something could be different and confront what is actual with what could be. This places in the foreground the quote by Foucault cited in the chapter by Cotton and Hardy. That quote, I believe, moves the argument further in the direction of critique as a way of considering change. It is re-iterated again in this discussion because of its significance.

> The work of an intellectual is not to mold the political will of others; it is, through the analysis that he does in his own filed [sic], to re-examine evidence and assumptions, to shake up habitual ways of working and thinking, to dissipate conventional familiarities, to re-evaluate rules and institutions and starting from this re-problematization (where he occupies his specific profession as an intellectual) to participate in the formation of a political will (where he has his role as a citizen to play) (Foucault, 1989, pp. 305-306).

Foucault directs attention to the specific intellectual whose interpretations not only diagnose but who refuses to play the role of the intellectual as a governing agent. This role of governing agent is not discussed as such but in relation to narratives of science and pedagogy as producing salvation and redemption. While not foregoing notions of salvation in which inequities are continually challenged, the task of inquiry is to show the contingency of the arrangement that we live. This requires interrogating historically the systems of reason that have played a part in holding those arrangements together and thus to contest the strategies that govern human possibilities. Rose (1999, p. 58) expresses this thought as to disturb 'that which forms that groundwork of the present, to make once more strange and to cause us to wonder how it came to appear so natural'. The role of the intellectual is to develop strategies that are sensitive to the historical and anthropological dimensions in the construction of the technologies of the self, to the appreciation of the diversity of modes of human existence and to the critical possibility to break from the boundaries of thinking in terms of the society that we already know. I recognise, as do some of the chapters in this book, that without rethinking the function of intellectual life and the sciences of education is to conserve the contemporaneity of the present as the very fabrics of change.

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